Simplified Model of Bank Balance Sheet Management

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Abstract. The central issue of bank management is obtaining maximum yield while complying with prudential supervision requirements to reliability and good standing. Particular attention should be given to liquidity risks, whose fully-fledged analysis and management require to approach the bank as a dynamic system. The developed mathematical model includes three asset components (loans; bonds and another low risk securities; liquid assets – accounts, reserves, cash) and two liabilities components (equity and borrowed capital – deposits). Main management parameters of the bank's balance sheet that support choosing adequate combination of returns and liquidity risk include turnover times of the loan portfolio and the securities portfolio, loan and deposit rates, the cash reserve ratio. This approach allows to clearly describe the transformation mechanism of core cash flows and formalize various rules of assets and liabilities management. The findings include analytical expressions allowing to research the impact of main constraints on the bank's yield. Computer-aided implementation of this model may be used for numerical simulation studies of balance sheet items and efficiency of different algorithms of asset allocation decision-making.

Keywords: Bank, Mathematical Modelling, Simulation Algorithm, Balance Sheet, Liquidity Risk, Profitability.

1 Introduction

As elements of the financial system, banks perform a variety of functions, including that of operators maintaining clients' accounts, funds transfers, securities trading, currency exchange etc. However, the main function banks perform as financial intermediaries is transforming the borrowed funds in the loans required by enterprises and customers.

In the context of economic instability, the role of banking risk management becomes of paramount importance. According to GARP (Generally Accepted Risk Principles), the six core categories of banking risks include: *credit risk*, *market risk*, *portfolio concentrated risk*, *liquidity risk*, *operation risk* and *business event risk*.

The banking risks literature pays most of its attention to *credit* risks. A wide range of mathematical models of credit risks that became commercial products (*CreditMetrics™*, *EDFCalc®*, *CreditRisk+*, *CreditPortfolioView™*) are in place, but credit risk modelling, prediction, analysis and management still remain in the highlight.

Market risk modelling has quite a long history dating back to classic studies of *Markowitz*, (1952, 1959) and *Roy* (1952), and is related with optimizing asset portfolios in risk/return terms, when the most common risk management measures include either portfolio variance, or set risk measures generated by *Lower Partial Moments*, $LPM_k(\tau)$ in different combinations of k and τ, such as default probability, mean absolute semi deviation, standard semi deviation etc.

Operation risks are mostly referred to bank management operations, HR management and information technology processes, while *business event* risks are related with external shocks that can be treated as exogenous scenarios in stress testing practices.

Particular attention should be given to *liquidity* risks, whose fully-fledged analysis and management, especially stress testing, require to approach the bank as a dynamical system. At the same time, the existing approaches to analysis and adequacy of bank liquidity rely mostly on static single-period models. Not so much experience has been gained by the banks in what concerns multi-period models, and the simulation models they underlie.

It's also worth noting that depending on the model focus and the scope of solved tasks, the employed mathematical tools vary considerably. Thus, credit risks are simulated using a variety of probability (stochastic) models; linear and non-linear programming models are used for risk-based asset optimization, multi-period models of assets/liabilities management are discrete-time recurrent (difference) equations. In this paper, we will use continuous-time models based on differential equations.

The purpose of this research consists in: (1) developing quite a simple mathematical model of dynamics of the bank cash flows, including the asset management algorithm for liquidity maintenance; this model is supposed to be used for educational purposes to simulate a variety of scenarios of cash inflow and outflow, including stress testing, and to demonstrate response of the balance sheet to changes of controlled parameters values; (2) analytical study of sensitivity of balance sheet items, and profitability of the bank assets, to controlled, regulatory and external factors.

The paper is organized as follows. After the introduction we present the short review of related works. Section 3 provides the rationale for the model with lumped parameters. Section 4 outlines of balance sheet model creation technique and algorithm of computer simulations. Section 5 provides an analytical representation of the balance sheet structure. Results are discussed in final Section 6.

2 Related works

Brief characteristics of some significant studies of the recent years that pay special attention to the dynamic aspect and the role of structural constraints in the course of risk analysis is as follows.

The proposition [1] consists in a dynamic framework which encompasses the main risks in balance sheets of banks in an integrated fashion. The contributions are fourfold: (1) solving a simple one-period model that describes the optimal bank policy under credit risk; (2) estimating the long-term stochastic processes underlying the risk factors in the balance sheet, taking into account the credit and interest rate cycles; (3) simulating several scenarios for interest rates and charge-offs; and (4) describing the equations that govern the evolution of the balance sheet in the long run. The obtained results enable simulation of bank balance sheets over time given a bank's lending strategy and provides a basis for an optimization model to determine bank asset – liability management strategy endogenously

The work [2] presents a dynamic bank run model for liquidity risk where a financial institution finances its risky assets by a mixture of short- and long-term debt. The financial institution is exposed to insolvency risk at any time until maturity and to illiquidity risk at a finite number of rollover dates. Both insolvency and illiquidity default probabilities in this multiperiod setting are computed using a structural credit risk model approach. Numerical results illustrate the impact of various input parameters on the default probabilities.

The paper [3] analyzes capital requirements in combination with a particular kind of cash reserves, that are invested in the risk-free asset, from now on, compensated reserves. It considers a dynamic framework of banking where competition may induce banks to gamble. In this set up, one can capture the two effects that capital regulation has on risk, the capital-at-risk effect and the franchise value effect. In [4] a discrete-time infinite horizon banking model is considered to examine the interaction between risk weighted capital adequacy and unweighted leverage requirements, their differential impact on bank lending, and equity buffer accumulation in excess of regulatory minima.

The concept [5] consists in developing a dynamic structural model of bank behaviour that provides a microeconomic foundation for bank capital and liquidity structures and analyses the effects of changes in regulatory capital and liquidity requirements as well as their interaction. The stylized bank balance sheet comprises two classes of assets, loans and liquid assets, and four classes of liabilities, deposits, long- and short-term debt, and equity. Decisions on how to adjust these asset and liability classes are taken by risk-neutral managers in a discrete time, infinite horizon setting.

3 Substantiation of the Model Aggregation Method

In general, loan dynamics $x(t,\tau)$ is described with the transport equation [6]

$$\partial x / \partial t + \partial x / \partial \tau = -\varepsilon(\tau) x + u(t,\tau) \tag{1}$$

where t is the current time, τ is the time counted down from the date of loan issue (loan "age") , $u(t,\tau)$ is the loan issue function, $\varepsilon(\tau)$ is the rate of loan repayment.

Since loans are usually issued on some standard term T_k (one day, one week, one month, three months, six months, one year etc.), the equation (1) may be presented as a set of same-type equations k=1,2,3,…

$$\partial x_k / \partial t + \partial x_k / \partial \tau = -\varepsilon_k x_k \tag{2}$$

with boundary conditions of $x_k(t,0) = u_k(t)$, each of which has an analytical solution

$$x_k(t.\tau) = u_k(t-\tau)\exp(-\varepsilon_k \tau) \quad (3)$$

Therefore, on the date t, the total volume of loans issued for the term T_k are equal to

$$x_k(t) = \int_0^{T_k} x_k(t,\tau)d\tau = \int_0^{T_k} u_k(t-\tau)\exp(-\varepsilon_k \tau)d\tau \quad (4)$$

and their dynamics is described as

$$dx_k / dt = u_k(t) - \varepsilon_k x_k - u_k(t-T_k)\exp(-\varepsilon_k T_k) \quad (5)$$

Then, the loan portfolio dynamics x(t)

$$dx / dt = u(t) - \varepsilon^* x - \sum_k u_k(t-T_k)\exp(-\varepsilon_k T_k) \quad (6)$$

where $x(t)$ is the loan portfolio volume, $u(t)= \Sigma_k u_k(t)$ is the total flow of issued loans, $\varepsilon^*=(\Sigma_k \varepsilon_k x_k)/x$ is the weighted average rate of loan repayment

$$x(t) = \sum_k x_k(t) = \sum_k \int_0^{T_k} u_k(t-\tau)\exp(-\varepsilon_k \tau)d\tau \quad (7)$$

Let's present the output flow as

$$u_k(t-T_k)\exp(-\varepsilon_k T_k) = u^*{}_k(t) + \Delta u_k(t) \quad (8)$$

where $u^*{}_k(t)$ is the current average amount of loans issued for T_k period taking into account their repayment (amortization), $\Delta u_k(t)$ is the loan deviation from the mean value

$$u^*{}_k(t) = x_k(t) / T_k \quad (9)$$

It follows from (7) and (9) that the loan portfolio may be presented as

$$x(t) = \sum_k u^*{}_k(t) T_k \quad (10)$$

Let's define the loan portfolio turnover time T_x as

$$T_x = x(t) / \sum_k u_k(t-T_k)\exp(-\varepsilon_k T_k) = \sum_k u^*{}_k(t) T_k / \sum_k [u^*{}_k(t) + \Delta u_k(t)] \quad (11)$$

While $\Delta u_k(t)$ values can vary within a wide range, the total deviation $\Sigma_k \Delta u_k(t)$ from the average flow is insignificant in the stable bank, i.e.

$$\sum_k \Delta u_k(t) / \sum_k u^*{}_k(t) << 1 \quad (12)$$

Then, to a first approximation, the turnover time T_x is a weighted average of loan term T_k

$$T_x = \sum_k \Delta u^*{}_k(t) T_k / \sum_k u^*{}_k(t) \quad (13)$$

The turnover time T_k in its meaning is similar to duration $D_k(t)$, which is the weighted average maturity of asset or liability, but it is calculated much easier.

In case of constant flow of payments $u^*_k(t)$ and ε_k=0, the duration is obvious to be equal to one half of the turnover time $D_k(t)$= T_k/2. In case of payment flow, which is decreasing as it nears the time of repayment, duration is growing. Thus, in case of $\varepsilon_k T_k$=1, i.e., when the debt is reducing by the loan maturity by e=2.72 times, D_k= 0,582T_k.

Finally,

$$dx/dt = u(t) - x/T_x(t) - \varepsilon x \quad (14)$$

where $u(t)$ is the net flow of loans, $T_x(t)$ is the loans turnover time, ε is the loan amortization (repayment) rate.

Expressions similar to (1)-(14) can be found when describing dynamics of the deposits provided for the term $y(t,\tau)$ with the only difference being that the negative member $\varepsilon_k x_k$ meaning loan repayment is replaced with the positive member meaning accrual of deposit interest $\rho_k y_k$.

4 The Simplified Aggregated Model of the Bank

To present the logics of operations of the banking institution, let us consider the simplest high-level model of dynamics of the core financial flows, which, nevertheless, describes key aspects of its operations. In the most compact form, which is convenient for a mathematical study, this model is further stated as a system of ordinary differential equations.

When choosing the state vector let's limit ourselves with five aggregated balance sheet items, only four of which are independent in accordance with the principle of equality of assets and liabilities (Table 1). Shareholder's own capital (equity) usually acts as the balancing variable.

The exogenous variable – borrowed and attracted funds (term deposits and demand deposits of individuals and legal entities, clients' account balances, interbank borrowing) serves as the principal source of bank's funds and the starting point of the model.

Dynamics of term deposits y_1 and demand deposits y_2 within the aggregated model is described with same-type equations (1), so for the sake of simplicity, these components of liabilities are combined $y = y_1+y_2$, while parameters $T_y(t)$ and ρ_y are weighted average

$$dy/dt = v(t) - y/T_y(t) + \rho_y y \quad (15)$$

where $v(t)$ is a net cash inflow to the deposit accounts, $T_y(t)$ is the time of liabilities (deposits) turnover, ρ_y is the interest accrued on deposits. Here it is suggested that the interest is paid simultaneously with the withdrawal of the deposit, though the model may also use another approach, when the interest is withdrawn as far as it is accrued.

The main issue of liabilities simulation is that $v(t)$ is a random process. In case of crisis developments, the inflow $v(t)$ is decreased, and $T_y(t)$ is reduced as a result of outflow of funds from customer accounts and withdrawal of term deposits (if the latter is provided for under the agreement conditions).

One can state three approaches to prediction and simulation $v(t)$.

- *Scenario approach*. A set of possible (suggested) exogenous time-varying functions $v(t)$ (scenarios) are specified.
- *Statistic approach*. To build $v(t)$, one of the methods of forecasting of time series is used.
- *Bayesian approach*. It is based on combining the scenario approach with one or multiple random variables. Depending on the value taken by this random variable, different scenarios of cash inflow and outflow takes place in the certain time interval.

Table 1. Stylized Aggregated Balance Sheet of a Commercial Bank, %.

Assets		Liabilities	
Loans, x	60	Equity, c	10
Bonds and other investment securities, b	15	Debt (term and demand deposits, customer accounts and borrowing), y	90
Correspondent accounts, reserves, cash, s	25		
Total assets, A	100	Total liabilities, L	100

The bank's loan portfolio is generated with the attracted (borrowed) funds and loans dynamics describes according to (14).

Usually, when a loan is approved, a deposit account (loan facility) is opened at the same time on the liabilities side, with the borrower withdrawing funds in installments as required from this account, but for the sake of simplicity only the resultant flows are taken into account in the model.

Borrowing demand g(t) can either exceed the funds at the bank's disposal $h(t)$, or be insufficient. That is why

$$u(t) = \min\{g(t), h(t)\} \qquad (16)$$

where $g(t)$ is the lending demand, $h(t)$ is the bank's funds planned to be allocated as loans.

Approaches to simulation of the lending demand $g(t)$ are similar to that described above for deposit inflow simulation $v(t)$.

Pursuant to the banking risk management policy, only part $\gamma_x<1$ of the available funds is allocated for lending

$$h(t) = \gamma_x(t)q(t) \tag{17}$$

where $q(t)$ is estimated available funds of the bank (inflow less outflow of funds).

Other bank funds are spent to purchase other earning assets, or can be allocated to increase funds in the correspondent accounts and as cash $s(t)$ thus used as the reserves aimed to mitigate liquidity risks.

Most part of available funds of the bank, including non-demanded funds intended for lending $max\{0, h(t)\text{-}g(t)\}$ is placed by the bank in the portfolio assets – investment securities, mostly in the bonds, and traded risk assets (stock).At the same time, available securities are paid off or sold. This mechanism can be described as follows

$$db/dt = w(t) + \max\{0, h(t) - g(t)\} - b/T_b(t) \tag{18}$$

where $b(t)$ is investment in securities, $w(t)$ is bank's funds planned to be used for purchasing portfolio assets

$$w(t) = \gamma_b(t)q(t) \tag{21}$$

where γ_b is a part of the funds spent on purchasing the securities, $T_b(t)$ is the turnover time of the securities portfolio.

The key issue of asset management is the algorithm of allocating the bank's funds that, in case of reasonable management, is supposed to depend on the estimated net inflow $q(t)$.

This algorithm may be presented as follows. The available investment resources of the bank $q(t)$ are calculated as the resultant between the inflow (released funds, interest income, deposit growth, redemption of securities) and output flow (growth of reserves, interest expenses, bad loans, operating and other expenses)

$$q(t) = dy/dt - dr/dt + (1-\xi(t))x/T_x(t) + b/T_b(t) + \varepsilon x + \rho_x x + \rho_b b - \rho_v y - z(t) \tag{20}$$

where ρ_x, ρ_y, ρ_b are interest rates, of loans, deposits and securities, respectively, r(t) is the reserve, z(t) is the planned operating expenses and other bank payments, $0<\xi(t)\leq 1$ is a random process that characterizes bank loss from the bad loans.

Let's provide explanations on certain equation (20) elements.

The principal part of the funds attracted by the bank must be secured with required reserves. In Russia, the required reserves are withdrawn from the banks, placed in non-interest bearing account in the Bank of Russia and can be used to cover the liquidity shortage, only if the set of conditions is met (averaging mechanism). Besides, the bank must establish excess reserves for possible bad loans and as security of current payments. Excess reserves represent any vault cash that banks hold that is in excess of the required reserves amount. Banks typically have a low incentive to maintain excess reserves because cash earns the rate of return of zero.

Primary reserves, as combined with the government bonds (secondary reserves), create the required liquidity cushion that ensures bank stability against adverse changes of the external conditions.

Further, as a separate component, we'll single out the reserves available to support liquidity as percent of the attracted funds, with this percent (above the required reserves) may be regulated by the bank itself

$$r = ay \tag{21}$$

where a is the cash reserve ratio.

Taking into account the bank may modify the reserve percentage in a flexible manner

$$dr/dt = ady/dt + yda/dt \tag{22}$$

Formula (20) allows to design the criterion of and assess solvency of the bank. Reduced resources q(t) alert solvency reduction. This can occur, when the deposit outflow starts exceeding their inflow, i.e. dy/dt becomes negative, the amount of credit default rate ξ grows and operating expenses z(t) increase. If q(t) becomes negative, it means that the bank starts shifting to reduced liquidity, which can finally lead to insolvency and bankruptcy, when the equity becomes negative. In this way equation (20), taken together with (14) and (22), returns the necessary condition (lower limit) of the financial stability and we'll define the financial stability headroom of the bank by the ratio χ that may be used as the stability indicator (criterion)

$$\chi(t) = [(1-\xi(t))x/T_x(t) + b/T_b(t) + \varepsilon x + \rho_x x + \rho_b b]/[z(t) + \rho_v y - (1-a)dy/dt] \tag{23}$$

As an expert evaluation, we can offer the following scale: $1<\chi<1,5$ – low stability, $1,5<\chi<3$ – medium stability, $\chi>3$ – high stability. Apparently, this parameter is fluctuating throughout the bank operations going down during economic recession featuring reduced lending demand and outflow of funds from depositors' account.

To make current payments, the bank needs available cash in the correspondent accounts and in its cash office. These most liquid components of the assets (primary reserves), including reserves for credit losses, are united in variable s(t).

As shown above, under pressure, in case of economic shocks or the bank's high-risk lending policy, the value q(t) may turn out to be negative thus resulting in termination of loan business, suspension of acquiring other assets and reduction of s(t). When the bank's financial situation improves, including as a result of state support measures, resolution and capitalization increase, the flow of resources reverses sign and the liquidity adequacy s(t) must be restored.

With this taken into account, both previously introduced variables – lending cash flows h(t) and portfolio investment cash flows w(t) should be adjusted as follows

$$h(t) = \gamma_x(t)\max\{0; \mathrm{sgn}(0, s-r)\}\max\{0, q(t)\} \tag{24}$$

$$w(t) = \gamma_b(t)\max\{0; \mathrm{sgn}(0, s-r)\}\max\{0, q(t)\} \tag{25}$$

and in the cash dynamics equation s(t) it is necessary to provide possible switching between the modes of expenditure and replacement

$$ds/dt = \text{sgn}(0, r-s)\max[0, q(t)] + \min[0, q(t)] + dr/dt \qquad (26)$$

As mentioned above, the bank's equity is a balancing variable, i.e. c=x+s+b-y and

$$dc/dt = \rho_x x + \rho_b b - \rho_v y - z(t) - \xi(t)x/T_x(t) + \min[0, q(t)] \qquad (27)$$

Bank's equity grows due to profit (less the income tax and dividends paid to the shareholders). For the sake of simplicity, taxes are not accounted in this model. The dividends are also considered not to be distributed, and all profit is allocated to increase the equity value.

The algorithm of scenario simulations (after replacing derivatives with finite differences) has the form:

1. Exogenous functions (scenarios) setting v(t), T_y (t), g(t), ξ(t)
2. Setting constant coefficients ε, ρ_x, ρ_b, ρ_y,
3. Setting the calculation step Δt
4. Setting initial values of the balance sheet state variables c(t), y(t), x(t), s(t), b(t)
5. Setting initial values of the control variables γ_x(t), γ_b(t), a(t), T_x(t), ρ_a(t)
6. A new value of the deposits y(t+Δt), Eq. (15)
7. A new value of the reserves r(t+Δt), Eq. (22)
8. Operating expenses z=ρ_a(t)A
9. Available investment resources q(t), Eq. (20)
10. A new value of the cash and other liquid assets s(t+Δt), Eq. (26)
11. The lending planned h(t), Eq. (24)
12. The planned volume of securities purchase w(t), Eq. (25)
13. Issued loans u(t), Eq. (16)
14. A new value of the loan portfolio x(t+Δt), Eq. (14)
15. A new value of the securities portfolio b(t+Δt), Eq. (18)
16. A new value of the own capital c(t+Δt), Eq. (27)
17. Stability indicator χ(t) Eq. (23)
18. Return to step 4 with t: = t+Δt

The capital adequacy ratio is used as the main structural constraint. In this model, the constraint takes on the form as follows

$$c(t)/[(1-f)A] = c(t)/\{(1-f)\{c(t)+y(t)]\} \geq \theta \qquad (28)$$

where f is the share of the risk-free assets, θ is the capital adequacy ratio (θ=0.08 according to the recommendations of the Basel Committee on Banking Supervision, θ=0.1 for Russian banks).

Therefore

$$c(t) \geq \{(1-f)\theta/[1-(1-f)\theta]\}y(t) \qquad (29)$$

Further, k ratio is more convenient to use as the adequacy ratio

$$k \geq (1-f)\theta/[1-(1-f)\theta] \qquad (28)$$

For example, f=0.3, θ=0.1, then k= 0.075.

The built model describes dynamics of the main variables of the bank's condition, allows to simulate mechanisms of management and transformation of cash flows and study sensitivity of the balance sheet items and bank profit to the management efforts and external factors, including stress. Thus, it can be considered as a backbone for the theoretical and analytical research. At the same time, aggregating balance sheet items, use of the integral parameters of turnover of assets and liabilities, and the assumption of the continuous smooth character of the used functions prevent from showing some important aspects of the bank's operations. The next step in enhancing adequate description of the bank's operations is using the distributed parameter models [6].

5 Structural Constraint Impact on the Bank Performance

Just like any other financial organization attracting funds of people and companies, every bank acts in the context of tight restrictions imposed by the external regulator (in Russia it is the Bank of Russia, in the USA it is the Federal Reserve System), and internal rules. These restrictions are aimed to maximize mitigation of various banking risks, but at the same time they considerably affect the structure and performance of assets.

Suppose that the bank is stable for some period of time, i.e., its amount and structure of assets and liabilities remain unchanged, while profit is fully distributed and its equity does not grow. In this case, one can analytically study impact of different parameters on bank's financial performance, provided the restrictions imposed on the balance sheet structure by the supervisory body are met.

Then the derivatives and several terms in equations (14)-(15), (18), (26)-(27) are set to zero, and one can completely define the balance sheet components via the model ratios.

The borrowed capital is determined by the product of the cash inflow rate by the modified turnover time T^*_y

$$y^* = v/(1/T_y - \rho_y) = vT^*_y \tag{31}$$

and the equity, pursuant to the constraint (18), must be at least

$$c^* = ky^* \tag{32}$$

where k=(1-f)θ/[1-(1-f)θ].

In the steady mode, pursuant to (29), non-working assets are minimum and equal to reserves

$$s^* = r^* = ay^* \tag{33}$$

Investment in low-income but reliable (low-risk) assets such as government bonds are aimed to ensure financial stability of the bank and mitigate risks. The amount of these investments must correlate with the bank's equity.

Then this component of the assets can be determined as

$$b^* = nc^* \tag{34}$$

Further, we find the loans value from the balance condition,

$$x^* = c^* + y^* - s^* - b^* \tag{35}$$

As a result, the bank's balance-sheet may be presented analytically:

Table 2. Analytical Representation of Balance Sheet

Assets $A=(1+k)vT^*_y$	Liabilities $L=(1+k)vT^*_y$
Loans $x^* = [1+k(1-n)-a]vT^*_y$	Equity $c^* = kvT^*_y$
Bonds and other investment securities $b^* = nkvT^*_y$	Debt (term and demand deposits, customer accounts and borrowing) $y^* = vT^*_y$
Liquid assets (reserves, correspondent account, cash) $s^* = avT^*_y$	

Interest income (margin) of the bank m taking into account the estimated loan loss ratio $E\xi$ is calculated as

$$m = \rho^\wedge{}_x x^* + \rho_b b^* - \rho_y y^* = \{[1 + k(1-n) - a]\rho^\wedge{}_x + \rho_b nk - \rho_y\} y^* \tag{36}$$

where $\rho^\wedge{}_x = \rho_x - (E\xi)/T_x$.

Operation expenses z may be interpreted as some imputed rate ρ_a of the bank asset servicing $z=\rho_a A$, then the pre-tax profit p amounts to

$$p = m - z = \{[1 + k(1-n) - a]\rho^\wedge{}_x + \rho_b nk - \rho_y - \rho_a(1+k)\} y^* \tag{37}$$

Return on assets

$$ROA = p / A = \{[1 + k(1-n) - a]\rho^\wedge{}_x + \rho_b nk - \rho_y - \rho_a(1+k)\} / (1+k) \tag{38}$$

Return on equity

$$ROE = p / c^* = ROA(1+k) / k \tag{39}$$

6 Conclusion

The aggregated model of the bank as a dynamic system with lumped parameters allows to clearly show the transformation mechanism of core cash flows and formalize various rules of assets and liabilities management. Computer-aided implementation of this model may be used for computational studies of efficiency of different asset management algorithms.

Main controlled parameters of the bank's balance sheet that support choosing adequate combination of yield and liquidity risk include: T_x – the loan portfolio turnover time, T_b – the securities portfolio turnover time, ρ_x – the loan rate, ρ_y – the deposit rate, a – the cash reserve ratio.

In the near-stable situation it's not difficult to derive simple analytical expressions allowing to research the impact of these parameters on the bank's yield and liquidity risks. Thus, one can use formulae (25)-(26) to study impact of the control parameters, including a variety of ratios, on the yield, and correlate it with the loan portfolio risks denoted by $E\xi$ value in this model. The liquidity risk depends on the assets/liabilities turnover time ratio (T_x and T_y). Since T_y value is not used in expressions (25)-(26), then the ratio T_x/T_y is an independent parameter that can be used when analyzing the bank's standing in risk/return terms. Let us note that the loan and deposit interest rates affect the respective cash flows $v(t)$ and $g(t)$ and must be taken into account when simulating these random processes.

It is seen from (25)-(26) that k equity ratio to the amount of attracted and borrowed funds significantly impacts the return on equity, but barely affects the return on assets that depends mostly on their structure and interest margin.

The suggested model can be easily extended through drilling-down to the financing sources (demand/term/savings deposit etc.) and asset allocation methods. To dramatically enhance the model adequacy, it is necessary to take into account the time structure of loans and term deposits [6].

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