

Noise Filtration in the Digital Images Using Fuzzy Sets and Fuzzy Logic

Yuliia Pomanysochka^[0000-0003-3868-7621], Yuriy Kondratenko^[0000-0001-7736-883X],
Ievgen Sidenko^[0000-0001-6496-2469]

Intelligent Information Systems Department, Petro Mohyla Black Sea National University,
68th Desantnykiv Str., 10, Mykolaiv, 54003, Ukraine,
jul.pom19@gmail.com, yuriy.kondratenko@chmnu.edu.ua,
ievgen.sidenko@chmnu.edu.ua

Abstract. In this paper, existing methods for filtering noise in digital images are considered. The following noise filtration methods were analyzed: arithmetic averaging filter, geometric averaging filter, median filtering, adaptive median filtration, Gaussian filtration and filtration using fuzzy logic, in particular the fuzzy color preserving Gaussian noise reduction method (FCG filter). Besides, the different types of noise that may occur on a digital image are discussed. All methods were evaluated using metrics like mean squared error, peak signal-to-noise ratio and structure similarity. It has been found that all of the above methods can well filter out only a certain type of noise. Pulse noise on a digital image better removed with median and adaptive median filtering. Gaussian noise better removed with averaging, Gaussian and FGG filters. In this paper, a combination of adaptive median filtering and FGG filter is proposed for removal of combined pulse and Gaussian noises.

Keywords: digital image, filtering noise, fuzzy set, fuzzy logic, mean squared error, peak signal-to-noise ratio, structure similarity.

1 Introduction

Nowadays, almost all of the images are presented in the digital form. They are used in printing, media, medicine, industry, space industry and other areas. Therefore, algorithms and methods for their processing are rapidly developing and demand constant improvements [1-3].

Processing of digital imaging is any change in the data, which is presented in the form of digital images, in order to improve their visual perception by people (for example, correcting color and contrast, correcting small noise) or further processing by information systems (for example, segmentation to the area of certain classes, selection of objects, etc.) [1].

One of the main tasks of digital image processing is to remove noise that may occur while receiving images, transferring them or as a result of data digitization. The process of eliminating various types of noise from images is called filtration [2, 4].

This work is devoted to solving this problem. Both classical filters and those built on the basis of fuzzy logic are considered. The result of the study is a combination of several filters in order to reduce combined noise from digital images.

2 Related Works and Problem Statement

The task of processing images using fuzzy logic techniques was expressed by scientists from the 1990s [1-4]. Initially, research was conducted to create filters for black and white images, then color images, and in recent years there have been developments for filtration of video frames with Fuzzy Logic Methods (FLM) for both black and white and color frames. Most studies focus on two types of noise: impulse (random noise) and Gaussian additive.

The GOA (Gaussian noise reduction) filter [5] reduces the Gaussian noise from the black and white image and uses the fuzzy rules for determination the degree to which the gradient in a certain direction is small (the idea is that a small gradient is caused by noise, while a large gradient is caused by image structure). Fuzzy rules [6] are also used for calculation the value of correction that is used for performing filtration (the contribution of neighborhood pixels depends on their gradient values).

Another filter for black and white images is called FuzzyShrink [7]. It represents the modification of Wavelet filters using FLM. It showed better results than previously created fuzzy filters.

Also for removal impulse noise from black and white digital images FIDRM (Fuzzy Impulse noise Detection and Reduction Method) filter was developed [8]. It uses a similar approach as in the filter GOA, because it also uses a gradient values for denoising the images.

The FRINR (Fuzzy Random Impulse Noise Reduction) filter also eliminated random impulse noise on grayscale images [9]. The detection of noise in FRINR consists of two stages. Firstly, the neighborhood around the pixel is investigated to determine whether the pixel can be regarded as an impulse noise. If so, then fuzzy gradient values are used to determine the degree to which the pixel can be considered as an impulse noise and the degree to which the pixel can be considered free of noise.

Subsequently, FIDRMC (Fuzzy Impulse noise Detection and Reduction Method for Color images) and HFRMC (Histogram-based Fuzzy Restoration Method for Color Images) filters were developed [10-11]. They focus on removing impulse noise from color images. FIDRMC consists of two phases: the phase of detecting noise and the phase of proper filtration. At the filtering stage, information about the color of a particular neighborhood around the given central pixel is also taken into account.

Next, a Fuzzy Color preserving Gaussian noise reduction method (FCG) [12] was developed to remove Gaussian noise on color digital images. Unlike most other existing methods, the first FCG subfilter distinguishes between deviations in pixel values due to noise from those that are determined by the structures in the image (object boundaries), using the distances between the color components instead of calculating the difference between them.

3 Basic Concepts and Methods of Digital Image Filtering

In the digital image processing, it is assumed that the images represent an $N \times M$ integer table, where the value of each element corresponds to a certain level of brightness. This is the so-called pixel coordinate system [3, 4].

Digital images are generally divided into two classes: vector and raster. Vector image is an image, which is described as a set of graphic primitives. It is drawn by lines on graphic output devices. Raster image is a two-dimensional array and its elements contain color information. It is targeted for bitmap display devices. Noise removal methods work with raster images, so we will not consider the vector ones [2].

Impulse noise is modeled as follows. The appearance of noise emissions in each pixel (i, j) has the probability p and does not depend on the presence of noise in other points or the quality of the image. The pixel brightness value is replaced by the new value d (from 0 to 255). Let $\{x_{i,j}\}$ will be a distorted image. Then

$$x_{i,j} = \begin{cases} d & \text{with probability } p \\ s_{i,j} & \text{with probability } (1-p) \end{cases}, \quad (1)$$

where $s_{i,j}$ is the output brightness of the pixel (i, j) .

If the new value $d = 0$, then the black values of brightness (pepper type noise) are added, if $d = 255$ then the white values of brightness (noise type "salt").

Additive noise is described as

$$g(x, y) = f(x, y) + \eta(x, y), \quad (2)$$

where $f(x, y)$ is an input image; $g(x, y)$ is a noised image; $\eta(x, y)$ is an additive and independent noise with Gaussian or other distribution of probability density function.

Gaussian noise (also called normal noise) occurs on the image as a result of the factors such as noise in electrical circuits, noise of sensors (due to lack of lighting or high temperature). The model of this noise is widely used in the filtration of images and signals [1].

The general principle of image filtering.

Noise reduction is achieved by filtration. The variety of image filtration methods is associated with a variety of mathematical models of signals, noise and filtering optimality criteria. The filtration is carried out in spatial or frequency domains. In the frequency domain, the image must be converted into a frequency representation, for example, by using Fourier transform [13, 14].

All image processing methods discussed in this paper are implemented in a spatial area that is simply a plane containing image pixels. Spatial methods operate directly by pixels of the image, on the opposite of frequency methods, in which operations are performed over the results of the Fourier transform of the image, and not on the image

itself. Typically, spatial methods in a computational sense are more efficient and require less computing resources when implemented [3].

The processed (filtered) image is retrieved during the process of scanning the original image by a filter. If the operator T , executed above the pixels of the noised image, is linear, then the filter is called a linear spatial filter. Otherwise, the filter is nonlinear [14].

Let's consider the basic variants of low-frequency filters. They are implemented by linear operations [15, 16].

A large group of low-frequency filters are averaging (or smoothing) filters. In such filters, a different way of calculation the average brightness value in a window may be applied. Consider the arithmetic and geometric averaging filters.

The arithmetic averaging filter, or "box-box" filter, averages the value of the brightness of the pixel around the neighborhood using a mask with the same coefficients, for example, for a mask size 3×3 , the coefficients are $1/9$, for 5×5 – $1/25$ [17].

With **geometric averaging**, there is a smoothing of an image similar to arithmetic averaging. Such a filter causes a deterioration of the sharpness that is characteristic of all filters in this class, but some objects of the original image are less distorted. This filter, as well as the averaging arithmetic, can be used to suppress the high-frequency additive noise [1, 3].

Gauss filter. When defining filters, you can use masks with different weights. It is logical to assume that pixels located closer to the analyzed pixel have a greater effect on the brightness that is calculated during the filtration process. One of the filter that takes into account this fact is the Gaussian filter [6].

Low-frequency filtration methods lead to smoothing the image. They are linear and optimal when removing noise that has a Gaussian distribution. On real images in the boundaries of different objects, the brightness distribution has a different look.

Median filtering. Noises in the form of white or black dots are impulse-type noise. Linear filters do not eliminate them completely, but only locally averaged their values. Noises of this type are removed using non-linear filters, such as the median [2, 4, 5].

A separate class of nonlinear filters for removing noise from a digital image consists of **filters based on fuzzy logic techniques**. Its general idea is averaging the pixel value using the values of neighborhood pixels, taking into account such important structures in the image as the boundaries of the objects and the color component, which the filter should not distort [18-21].

The main problem that this filter solves is that it allows you to distinguish between noise and boundaries of objects in the image, both of which represent a significant change in pixel values. This is possible due to the fact that it calculates the 2-D distance between the various color components. For example, to filter a red component in position (i, j) , the distance between the red and green and red and blue components of some pixel window with the center of (i, j) is used, instead of calculating the average pixel value only by using values from the same red color component [4].

The idea of these simple fuzzy rules [22-24] is to assign large weights to the neighbors of the central pixel of windows that have the same color component as the

central pixel itself. The distance between two pairs is calculated using the Euclidean distance.

Methods for evaluating the quality of the filtration.

The quality of the filtering is usually performed by comparing the original image (without noise) with noised one, and then with the denoised one. In this way, you can see how the image characteristics were improved after applying the filter [25-28].

The metrics of evaluation are the following criteria: MSE (Mean Square Error); PSNR (Peak Signal to Noise Ratio); SSIM (Structural Similarity Image Measurement).

The most universal criterion is MSE, which is determined by the formula:

$$MSE = \sqrt{\frac{1}{M \cdot N} \sum_{i=1}^M \sum_{j=1}^N (v_{i,j} - \bar{v}_{i,j})^2}, \quad (3)$$

where $v_{i,j}$ is a pixel intensity (i, j) of the ideal (original) image without noise; $\bar{v}_{i,j}$ is a pixel intensity (i, j) of the denoised image.

The smaller the value of MSE (that is, the smaller the processed image differs from the ideal one) the better [28].

The next criterion is the PSNR, which is determined using MSE:

$$PSNR = 10 \lg \left(\frac{L_{\max}}{MSE} \right), \quad (4)$$

where L_{\max} is a maximum intensity level in the image.

Also widespread is the measure of structural similarity of images, proposed by Wang [29].

4 Implementation of the Described Filters and the Combined Filter

Authors implemented the filters described earlier and compared the results of their work using image quality filtering criteria such as MSE, PSNR and SSIM. The results are presented in the Tables 1-3 and Fig. 1-2.

Table 1. Comparison of filters for Gaussian noise filtration

| | MSE | PSNR | SSIM |
|-----------------------------|----------|---------|--------|
| Noised image | 605,063 | 20,313 | 0,628 |
| Arithmetic Averaging Filter | 335,33 | 22,879 | 0,820 |
| Geometric averaging filter | 661,538 | 19,925 | 0,773 |
| Gaussian filter | 292,768 | 23,466 | 0,762 |
| FCG filter | 234,341 | 24,432 | 0,846 |
| Median filter | 574,6146 | 20,5370 | 0,6490 |

Thus, the best filter for the removal of Gaussian additive noise is the FGG filter.

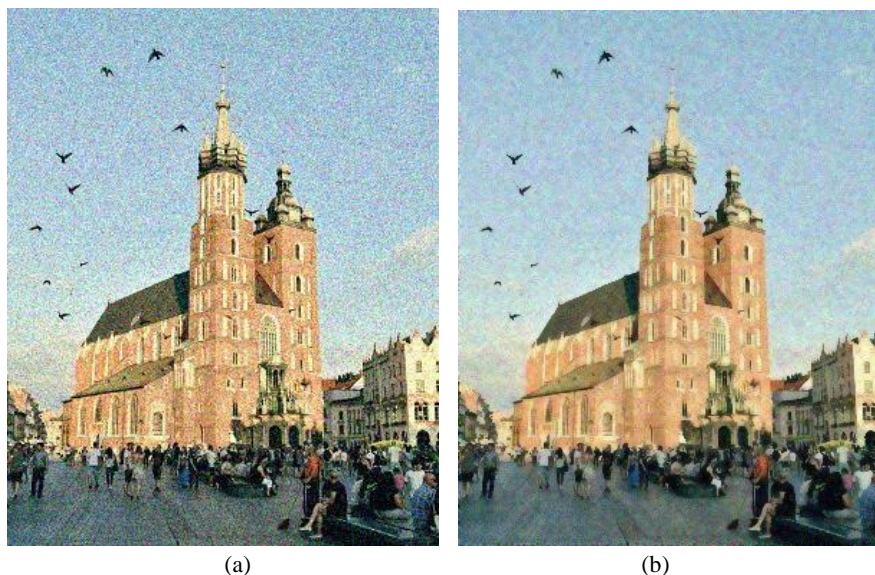


Fig. 1. Input noisy image (a) and result of image processing by the FCG filter (b)

Table 2. Comparison of filters for impulse noise filtering

| | MSE | PSNR | SSIM |
|-----------------------------|-----------|---------|--------|
| Noised image | 1055,659 | 17,896 | 0,562 |
| Arithmetic Averaging Filter | 401,3021 | 22,0961 | 0,7773 |
| Geometric averaging filter | 6246,1072 | 10,1747 | 0,2394 |
| Gaussian filter | 488,2479 | 21,2444 | 0,6948 |
| FCG filter | 571,9151 | 20,5575 | 0,6696 |
| Median filter | 207,3446 | 24,9639 | 0,9248 |

So, the best filter to remove impulse noise is the median filter.

It should be noted that more often on images there is a combination of several noises, specifically Gaussian additive and impulse noises. We checked the efficiency of the methods for filtering such combined noise.

Table 3. Comparison of filters for filtering of combined noise

| | MSE | PSNR | SSIM |
|-----------------------------|-----------|---------|--------|
| Noised image | 1613,751 | 16,052 | 0,446 |
| Arithmetic Averaging Filter | 483,2866 | 21,2888 | 0,7225 |
| Geometric averaging filter | 6460,1289 | 10,0284 | 0,2208 |
| Gaussian filter | 735,2863 | 19,4662 | 0,5941 |
| FCG filter | 535,3383 | 20,8445 | 0,6739 |
| Median filter | 586,7834 | 20,4460 | 0,6512 |

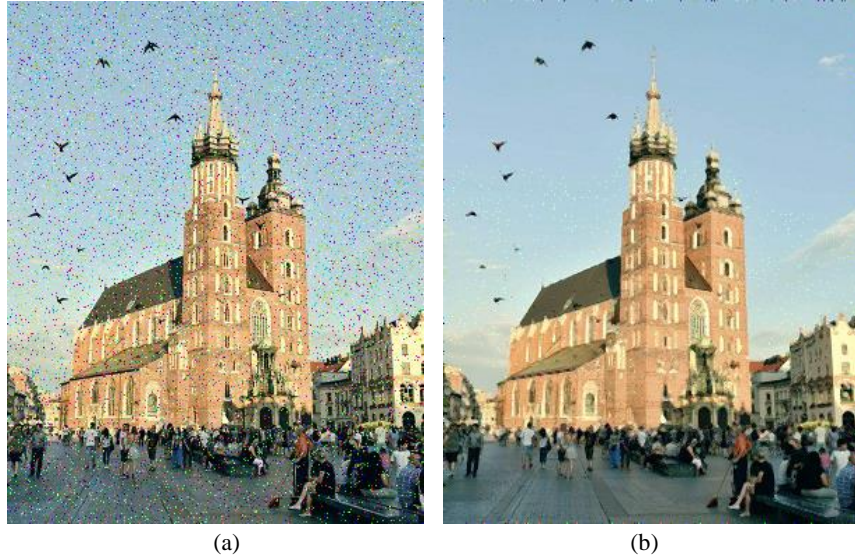


Fig. 2. Input noisy image (a) and result of image processing by median filter (b)

Consequently, we can see that the above filters poorly remove the combined noise from the images. The best result is shown by averaging arithmetic filter, but it also is unsatisfactory.

Therefore, it is necessary to develop a tool for the removal of the combined type of noise. To remove impulse noise, a median filter will be used, for the Gaussian noise – filter FCG, which has been experimentally shown to be better than other filtration methods.

This should be done using two approaches: sequential applying of the above filters; combination of both methods in one adaptive filter [30-33].

The combined adaptive filter will work according to the following algorithm.

1. Create three windows individually for components R, G and B.
2. Checking the central pixels in each window:
 - calculating the average intensity of the window;
 - if the central pixel is impulse noise (that is, its value differs from the average by more than 50), go to step 3;
 - if the central pixel is not impulse noise, go to step 4.
3. Modify the value of the central pixel in the window according to the median filter algorithm.
4. Modify the value of the central pixel in the window according to the algorithm of the FGG filter.

Table 4. Evaluation of image processing results by proposed approaches

| | MSE | PSNR | SSIM |
|---|----------|--------|-------|
| Noised image | 1613,751 | 16,052 | 0,446 |
| Image processed by sequential use of filters | 445,310 | 21,644 | 0,730 |
| The image processed by the combined adaptive filter | 425,972 | 21,837 | 0,741 |

Thus, it can be seen that the combination of adaptive median and FCG filters into a single combined adaptive filter is appropriate and effective, because this filter is better than the sequential applying of these filters according to all criteria [34, 35].

In order to be sure of the effectiveness of the combined method of filtering noise in digital images, it was decided to conduct a comparative analysis of all considered filters for three types of noise: impulse, Gaussian, and combined. The analysis was carried out on 10 color images with different detail level, colors, contrast and other characteristics.

Table 5. The resultant comparison table of all methods

| | Number of points | | |
|---|-------------------------|--------------------------|--------------------------|
| | Impulse noise filtering | Gaussian noise filtering | Combined noise filtering |
| FCG filter | 20 | 66 | 32 |
| Combined use of median filter and FCG | 56 | 36 | 69 |
| Sequential use of median filter and FCG | 55 | 31 | 58 |
| Gaussian filter | 30 | 44 | 21 |
| Geometric averaging filter | 10 | 26 | 10 |
| Arithmetic averaging filter | 40 | 64 | 51 |
| Median filter | 69 | 13 | 39 |

So it was proved the effectiveness of using the combination of median and FCG filters to remove the combined noise. However, it should be emphasized that this filtration method is worse for images that are distorted individually by additive Gaussian noise or impulse noise.

5 Conclusions

In this paper, it was demonstrated that classical filtration methods, as well as those that apply fuzzy logic approaches, cannot cope with the removal of the combined noise type in images (a combination of impulse noise and an additive Gaussian). These conclusions were made by calculating MSE, PSNR and SSIM criteria for processed images.

Therefore, it was needed to develop an approach that would show an effective result for the removal of the combined noise type. A combination of a median filter and a FCG filter was proposed for solving this problem. The results were verified by processing 10 color images. It was experimentally proved the effectiveness of using the proposed approach.

References

1. Gonzalez, R.C., Woods, R.E.: Digital image processing. 2nd ed. Englewood Cliffs, NJ (2002).

2. Tomasi, C. Manduchi, R.: Bilateral filtering for gray and color images. In: Proc. 6th IEEE Int. Conf. Computer Vision, pp. 839-846. Bombay, India (1998).
3. Rakshit, P., Konar, A.: Principles in Noisy Optimization. Applied to Multi-agent Coordination. Springer, Singapore (2018). DOI: 10.1007/978-981-10-8642-7.
4. Sundararajan, D.: Digital Image Processing. A Signal Processing and Algorithmic Approach. Springer, Singapore (2017). DOI: 10.1007/978-981-10-6113-4.
5. Van De Ville, D., Nachtgael, M., Van der Weken, D., Philips, W., Lemahieu, I., Kerre, E.: A new Fuzzy Filter for Gaussian Noise Reduction. In: Proceedings of Visual Communications and Image Processing (SPIE), vol. 4310, pp. 1-9. San Jose, CA, United States (2000). DOI: 10.1117/12.411782.
6. Kondratenko, Y.P., Encheva, S.B., Sidenko, E.V.: Synthesis of Intelligent Decision Support Systems for Transport Logistic. In: 6th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications, vol. 2, pp. 642-646. Prague, Czech Republic (2011). DOI: 10.1109/IDAACS.2011.6072847.
7. Schulte, S., Huysmans, B., Pizurica, A., Kerre, E.E., Philips, W.: A New Fuzzy-Based Wavelet Shrinkage Image Denoising Technique. In: Blanc-Talon, J., Philips, W., Popescu, D., Scheunders, P. (eds.) ACIVS 2006, LNCS, vol. 4179, pp. 12-23 (2006).
8. Kovalevsky, V.: Modern Algorithms for Image Processing. Apress, Berkeley (2019). DOI: 10.1007/978-1-4842-4237-7.
9. Schulte, S., De Witte, V., Nachtgael, M., Van der Weken, D., Kerre, E.E.: Fuzzy Random Impulse Noise Reduction Method. Fuzzy Sets and Systems 158, 270-283 (2007).
10. Bredies, K., Lorenz, D.: Mathematical Image Processing. Birkhäuser, Basel (2018). DOI: 10.1007/978-3-030-01458-2.
11. Mu, Z.-L., Li, C.-M., Yang W.-G.: Improvement of Spatially Selective Noise Filtration. J Inner Mongolian Petrochemical Industry 11, 147-149 (2006).
12. Boyle, R., Sonka, M., Hlavac, V.: Image processing analysis and machine vision. University Press, Cambridge (2008).
13. Buades, A., Morel, J.: A Non-local algorithm for image denoising. In: IEEE Computer Society Conference on Computer Vision and Pattern Recognition, pp. 60-65. San Diego, CA, USA (2005). DOI: 10.1109/CVPR.2005.38.
14. Chan, R., Ho, C., Nikolova, M.: Salt-and-pepper noise removal by median-type noise detectors and detail-preserving Regularization. IEEE Transactions on Image Processing 14(10), 1479-1485 (2005).
15. Tkachenko, A.N., Brovinskaya, N.M., Kondratenko, Y.P.: Evolutionary adaptation of control processes in robots operating in non-stationary environments. Mechanism and Machine Theory 18(4), 275-278 (1983). DOI: 10.1016/0094-114X(83)90118-0.
16. Mitra, S., Sicuranza, G.: Nonlinear image processing, Academic Press (2001).
17. Nachtgael, M., Schulte, S., Van der Weken, D., De Witte, V., Kerre, E.E.: Gaussian Noise Reduction in Greyscale Images. International Journal of Intelligent Systems Technologies and Applications 1(3/4), 211-233 (2006).
18. Kondratenko, G.V., Kondratenko, Y.P., Romanov, D.O.: Fuzzy Models for Capacitive Vehicle Routing Problem in Uncertainty. In: 17th International DAAAM Symposium on Intelligent Manufacturing and Automation: Focus on Mechatronics & Robotics, pp. 205-206 (2006).
19. Kondratenko, Y., Kondratenko, G., Sidenko, I.: Intelligent Decision Support System for Selecting the University-Industry Cooperation Model Using Modified Antecedent-Consequent Method. In: Medina, J. et al. (eds.) Information Processing and Management of Uncertainty in Knowledge-Based Systems: Theory and Foundations. 17th International

- Conference, IPMU 2018, vol. 854. pp. 596-607. Springer, Cham (2018). DOI: 10.1007/978-3-319-91476-3_49.
20. Piegat, A.: *Fuzzy Modeling and Control*. Springer-Verlag, 2001.
 21. Kondratenko, Y.P., Altameem, T.A., Al Zubi, E.Y.M.: The optimisation of digital controllers for fuzzy systems design. In: *Advances in Modelling and Analysis, AMSE Periodicals, Series A 47*, pp. 19-29 (2010).
 22. Trunov, A.N.: Transformation of operations with fuzzy sets for solving the problems on optimal motion of crewless unmanned vehicles. *Eastern-European Journal of Enterprise Technologies* 4(4), 43-50 (2018). DOI: 10.15587/1729-4061.2018.140641.
 23. Schulte, S., De Witte, D., Kerre, E.E.: A Fuzzy Noise Reduction Method for Color Images. *IEEE Transactions On Image Processing* 16(5), 1425-1436 (2007). DOI: 10.1109/TIP.2007.891807.
 24. Kondratenko, Y., Kondratenko, N.: Computational Library of the Direct Analytic Models for Real-Time Fuzzy Information Processing. In: *IEEE Second International Conference on Data Stream Mining & Processing (DSMP)*, pp. 38–43. Lviv, Ukraine (2018). DOI: 10.1109/DSMP.2018.8478518.
 25. Kam, H.S., Tan, W.H.: Noise Detection Fuzzy (NDF) filter for removing salt and pepper noise. In: Badioze Zaman, H., Robinson, P., Petrou, M., Olivier, P., Schröder, H., Shih, T.K. (eds.) *Visual Informatics: Bridging Research and Practice. IVIC 2009. Lecture Notes in Computer Science*, vol. 5857, pp. 479-486. Springer, Berlin, Heidelberg (2009).
 26. Najeer, A.J., Rajamani, V.: Design of hybrid filter for denoising images using fuzzy network and edge detecting. *American Journal of Science Research* 3, 5-14 (2009).
 27. Kondratenko, Y., Kondratenko, V.: Soft Computing Algorithm for Arithmetic Multiplication of Fuzzy Sets Based on Universal Analytic Models. In: Ermolayev, V. et al. (eds.) *Information and Communication Technologies in Education, Research, and Industrial Application. Communications in Computer and Information Science (ICTERI)*, vol. 469, pp. 49-77. Springer, Switzerland (2014). DOI: 10.1007/978-3-319-13206-8_3.
 28. Khandewal, P., Kaur, G.: Comparative study of different image enhancement technique. *International Journal of Electronics and Communication Technology* 7(2), 116-121 (2016).
 29. Wang, Z., Bovik, A.C., Sheikh, H.R., Simoncelli, E.P.: Image quality assessment: From error visibility to structural similarity. *IEEE Transactions on Image Processing* 13(4), 600-612 (2004).
 30. Kundra, H., Aashima, E., Verma, E.M.: Image enhancement based on fuzzy logic. *International Journal of Computer Science and Network Security* 9(10), 141-145 (2009).
 31. Sidenko, I., Filina, K., Kondratenko, G., Chabanovskyi, D., Kondratenko, Y.: Eye-tracking technology for the analysis of dynamic data. In: *IEEE 9th International Conference on Dependable Systems, Services and Technologies (DESSERT)*, pp. 509-514. Kyiv, Ukraine (2018). DOI: 10.1109/DESSERT.2018.8409181.
 32. Palagin, A.V., Opanasenko, V.N.: Design and application of the PLD-based reconfigurable devices. In: Adamski, M., Barkalov, A., Wegrzyn, M. (eds.) *Design of Digital Systems and Devices, Lecture Notes in Electrical Engineering*, vol. 79, pp. 59-91. Springer, Berlin, Heidelberg (2011).
 33. Belim, S.V., Seliverstov, S.A.: Hierarchy analysis method as a way to detect impulse noise on images. *Information Technologies* 4, 251-258 (2015).
 34. Newman, M.E.: Analysis of Weighted Networks. *Physical Review* 70(5), 056-131 (2004).
 35. Belim, S.V., Kutlunin, P.E.: Impulse noise detection in images using a clustering algorithm. *Herald of Computer and Information Technologies* 3, 3-10 (2016).