New Approach to Computational Cognition and Predicative Competence

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Abstract

Artificial General Intelligence needs fresh methods with cognitive architectures and philosophy of mind. In this context, predicative competence, natural language processing, and cognitive approaches can play a fundamental role in developing a new generation of user-friendly, more autonomous but still safe systems. Understanding this deep layer of thought is vital to develop highly competitive, reliable and effective cognitive robot and brain-inspired system. We present a new approach to computational cognition and predicative competence.

1 Introduction

Newly born babies are born equipped to respond to pleasant sensory experiences. We know that the nasal cavities are developed as early as the second month in the womb. Smelling and tasting begins early during fetal development. By the fifth month of development in the womb the baby is swallowing and sucking. Babies swallow about a half liter of amniotic fluid every 24 hours. The amniotic fluid is then digested by the baby. The nutrients enter the baby's blood. The baby filters out nutrients back to the mother via the umbilical cord. The mother's blood transports it to her kidneys and eliminates the waste. After birth, the umbilical cord is cut. The umbilical cord has been the baby's source of nutrition, connection to momma, her lifeline. It is no wonder that the baby now relies on that early development of smell to find the smell that she has always known. We also know that in the first few days after birth the mother's body produces a sweat similar in scent to amniotic fluid.

Taking into account that the baby has been able to suck and swallow since around five months, it just makes sense that when you put the baby on the mother's chest after birth that the baby would find her way to where her next site of nutrition is, the mother's breast. Your baby can smell the unique scent of mom's breast milk. Babies are born with an instinct to suck and root for food, but a newborn's sense of smell is a strong sense that helps a baby bond with her parents. While baby can smell both of their parents, she can also detect the distinct smell of her mother's milk! By simply holding your baby near your chest, she'll turn her head and root for your breast if she's hungry. Not only, babies can smell their mom from as far away as one to two feet. If you hold the baby and engage the baby with your eyes, while telling the mother to watch what happens, the baby will always turn her head after a few seconds and looks towards the mother. The baby can find her mother simply by smelling her. Babies can focus their eyes only about eight to 10 inches, but they can smell from a much further distance. Familiar and identifiable odors are better remembered than are unfamiliar and less identifiable odors [Rabin and Cain, 1984; Schab and Crowder, 1995]. Baby rotates her head in the direction related to the highest sensed level of her mom's scent.

In the mammalian brain, the development of precise neural circuits is initially directed by intrinsic genetic programming and subsequently refined by neural activity [Katz and Shatz, 1996; Zhang and Poo, 2001; Kirby *et al.*, 2013]. Axons from various olfactory sensory neurons (OSNs) expressing the same olfactory receptor (OR) converge onto a few spatially invariant glomeruli, generating the olfactory glomerular map in the olfactory bulbs (OBs) [Ressler *et al.*, 1994; Vassar *et al.*, 1994; Mombaerts *et al.*, 1996]. OR identity is represented as a unique combinatorial code of axon-sorting molecules at the axon termini, which provides the self-identification tags for OR-specific glomerular segregation. Experimental results indicate that calcium influx associated with neural activity is required for generating the combinatorial code of the axon-sorting molecules. Odor information initially processed by olfactory bulb is sent directly to the piriform cortex and closely interconnected orbital prefrontal cortex [Eichnbaum *et al.*, 1996].

Both of these cortical areas, as well as the olfactory bulb, project heavily to the perirhinal and entorhinal components of the parahippocampal region, which then provides the primary source of olfactory sensory information to the hippocampus itself. In the return pathway, the outputs of hippocampal processing involve direct projections from the parahippocampal region to both the piriform and orbital prefrontal cortices. The hippocampus is part of the limbic system, and plays important roles in the consolidation of information from short-term memory to long-term memory, and in spatial memory that enables navigation. Humans and other mammals have two hippocampi, one in each side of the brain. It contains two main interlocking parts: the hippocampus proper (also called Ammon's horn) and the dentate gyrus. Physiological findings complement the anatomical data indicating that information processing in the olfactory and limbic systems are closely integrated during odor-guided learning and memory. Thus the parahippocampal region appears to play a critical role itself in sustaining memory representations for simple recognition judgements.

But what is the role of the hippocampus itself in odor memory? Hippocampal function is not required for the acquisition of biases towards single odors, although it is necessary for some simultaneous discriminations involving closely juxtaposed odors [Eichenbaum *et al.*, 1989]. The hippocampus is also not necessary for the maintenance of single odor memories during performance of an olfactory recognition memory task [Otto and Eichenbaum, 1992]. In humans it is generally agreed the hippocampus plays a role in "declarative memory", our record of facts and events that are subject to conscious recollection and explicit, typically verbal expression. By contrast, the hippocampus is not involved in unconscious form of memory including the acquisition of skills and the adoption of dispositions towards responses to stimuli expressed implicitly through changes in response speed or bias in response selection. These defining features of the kind of memory supported by the hippocampal region in humans have been enormously valuable in clarifying the brain system for declarative memory in humans. At the same time, features of conscious recollection and explicit memory expression present a formidable challenge to the creation of bio-models that could be useful in revealing the neural circuits that mediate declarative memory.

Our insight is that odor memory at human primordial stage plays the role of elementary orientation clue for baby and this orientation clue is strictly connected anatomically to the hippocampal system. As Piaget already noticed, the scent of her/his mother plays a role in a baby's ability to collect and organize her/himself spatially. An emerging, unconscious "body sense" helps them orient themselves in relation to her/his mother first, then to other people and objects, and eventually to develop their own spatial references [Piaget, 1936]. Furthermore, we argue that from that elementary orientation clue, our full spatial, logical and predicative competence, sustained by our declarative memory, can emerge later, growing up. One-year-old babies may not be able to speak or to have mathematical knowledge, but they are able to think logically, according to new research that shows the earliest known foundation of our ability to reason. The type of reasoning in question, process of elimination, is formally called "disjunctive syllogism" [Halberda, 2018].

The shared, living sign is begetting its language by the unified toil of human perception and action, by the active contemplation between the shams of the inner, and the pretences of the outer still dwelling in the nondual dichotomy of the resonant cycle of tuned action and perception (oriented action and perception) [Fadiga *et al.*, 1995]. As a matter of fact, strong coupling between processes related to perception and action emerges in the human brain as a consequence of learning a sensorimotor task [Rizzolatti and Craighero, 2004]. A reflexive relationship is bidirectional with both the cause and the effect affecting one another in a loop relationship in which neither can be assigned as causes or effects.

Spatial concepts such as a sense of distance are learned through movement and exploration, based on self-orientation [Rizzolatti and Sinigaglia, 2010]. Spatial learning must be experienced over and over again until it is internalised and automatic. Spatial orientation is one of the key capacities which must be mature if a child is to learn to read and write easily [Piaget, 1945]. Spatial awareness can be defined as "an awareness of the body in space, and the child's relationship to the objects in space." This is based on spatial orientation, which is the skill that allows children to understand and execute requests for them to "line up at the door" or "put their backs to the wall," and to learn to read and write easily [Gallese and Lakoff, 2005].

Even the original concept of "shared space" in society has emerged in Paleolithic times from the concept of orientation. Its appearance cannot be interpreted as a random event, but it must be seen as the result of considerable brainwork. This emergent logic structure can be based upon the discovery of personal orientation in space. From there, all higher thought structures can be developed. Our ancestors were not concerned with concepts of measure and metric, but allowed for geometric considerations such as reflections, rotations, combinations and commutations, aeons later synthetized into Clifford Algebras (CA) [Ablamowicz, 2000].

Historically, the development of geometric thinking became manifest primarily in the painted caves of the Homo Sapiens of the Upper Paleolithic by implicitly oriented drawings showing the combination of single lines at right angle (plane line cross) and X plus circle [Bednarik, 1990]. As soon as you investigate into the origins of culture, you come upon the survival formulas of our Paleolithic ancestors. Among those symbols there is a concept of orientation that can be followed forward until the times of Descartes. Those geometrical shapes are just the vestiges of early artists resonating with their visions, thoughts and culture.

Understanding this deep layer of thought is vital to developing highly competitive, reliable and effective cognitive architectures for intelligent, brain-inspired system and for Artificial General Intelligence [Fiorini, 2019a]. Nevertheless, the lack of understanding of culturally and historically situated conventions makes it difficult, if not impossible, to access the original meaning behind iconography in the past [Fiorini, 2019b].

2 From Orientation to Logic

From the combination of single lines at right angle (plane line cross) and X plus circle, we obtain our reference diagram to study orientation in plane and in space, with four quadrants (divided into two octants each) numbered in counterclockwise fashion (1, 2, 3, 4) and five transformation flips F12, F22, F32, RD, LD (Fig. 1). According to CA, given a unit vector n, we can consider the reflection of a vector a in the hyperplane orthogonal to n. Even more importantly, from the Cartan-

Dieudonné theorem, rotation is the product of two successive reflections. For instance, compounding the reflections in the hyperplanes defined by the unit vectors n and m results in a rotation in the plane defined by $n \wedge m$ [Altmann, 1986]:

$$a'' = mnanm \equiv Ra\widetilde{R} \qquad (01)$$

where we have defined $R = mn$ and the tilde denotes the reversal of the order of the constituent vectors $\widetilde{R} = nm$. The object $R = mn$ generating the rotation in (01) is called a "rotor". It satisfies the relation:

$$\widetilde{R}R = R\widetilde{R} = 1 \quad . \qquad (02)$$

In 2012, researchers proposed an approach to exploit the properties of CA and GA (Geometric Algebra) rotation operators, called rotors, to code sentences through the rotation of an orthogonal basis of a semantic space [Augello *et al.*, 2012]. The experimental results have shown that this method is efficient to sub-symbolically encode both the semantics of the words and the structure of the sentence, intended as the order in which words appear in the phrase. Nevertheless, it did not take full advantage of all the intrinsic symmetries that CA can exploit. In fact, the full group of symmetries of a regular polygon, which includes rotations and reflections, is called "dihedral group".

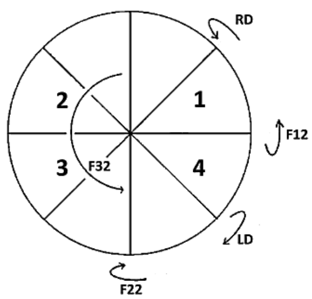

Figure 1. From the combination of single lines at right angle (plane line cross) and X plus circle we obtain our reference diagram to study orientation in plane and in space, with four quadrants numbered in counterclockwise fashion (1, 2, 3, 4) and five transformation flips F12, F22, F32, RD, LD (see text).

The notation for the dihedral group differs in geometry and abstract algebra. In geometry, D_n or Dih_n refers to the symmetries of the n-gon, a group of order $2n$. In abstract algebra, D_{2n} refers to this same dihedral group. In mathematics, the binary cyclic group of the n-gon is the cyclic group of order $2n$, $\boldsymbol{C_{2n}}$, thought of as an extension of the cyclic group $\boldsymbol{C_n}$ by a cyclic group of order 2. It is the binary polyhedral group corresponding to the cyclic group [Coxeter, 1948]. The binary cyclic group, as a subgroup of the Spin group, can be described concretely as a discrete subgroup of the unit quaternions of GA. Nevertheless, in this article the geometric convention is used mainly, due to its minimal educational resources requirement.

We call the elementary plane rotation "group generator" a. Therefore a_4^n, with any natural number n represent rotations in the x-y plane, $n = 1$ means rotation by 90°, $n = 2$ by 180°, $n = 3$ by 270°, and $n = 4$ by 360°. Obviously $a_4^5 = a_4^1$ which means that the plane rotation group is a cyclic group of order 4, namely Z_4. In the 3D spatial interpretation of our reference diagram, a_4 can be interpreted as a rotation by 90° in counter-clockwise direction around the vertical z-axis (front-view) combined with a reflection at the x-y-plane.

Now it is possible to carry out several transformations from our reference diagram in Fig. 1, without changing its location and orientation. This study can be achieved by many different approaches, by GA [Hestenes and Sobczyk, 1984], abstract algebra, crystallography by Schönfließ symbols [Schmeikal, 1993], Miller indices, iconic display of binary connectives [Peirce, 1902], logic alphabet [Zellweger, 1982, 1992], matrix representation, etc.

Here we follow the combinatorial approach for its extreme simplicity and minimal educational tools requirement. Therefore, any of our transformations of the quartered circle can be represented by a permutation of four objects, being the counter-clockwise numbered four quadrants of the disk and the spatial flips as reported from Fig. 1 (for the plane case we assume that quadrant number label front-view and rear-view is the same). Those permutations are labeled P_i and R_j for "flips" and "rotations" respectively, while E is the "identity element."

	P_0	P_1	P_2	P_3	R_0	R_1	R_2	E
P_0	E	R_1	R_2	R_0	P_3	P_1	P_2	P_0
P_1	R_1	E	R_0	R_2	P_2	P_0	P_3	P_1
P_2	R_2	R_0	E	R_1	P_0	P_3	P_1	P_2
P_3	R_0	R_2	R_1	E	P_1	P_2	P_0	P_3
R_0	P_3	P_2	P_0	P_1	R_1	R_2	E	R_0
R_1	P_1	P_0	P_3	P_2	R_2	E	R_0	R_1
R_2	P_2	P_3	P_1	P_0	E	R_0	R_1	R_2
E	P_0	P_1	P_2	P_3	R_0	R_1	R_2	E

Figure 2. Complete group multiplication table for finite noncommutative permutation group G (see text).

The important property of these elements is that they can be associated or multiplied according to their algebraic property. They form a finite noncommutative algebraic group, which we can call "G" and which a multiplication table can be computed from as in Fig. 2. Group G is isomorphic to geometric group $\boldsymbol{D_4}$, the spatial rotation-group of a square in space or "dihedron group." The complete subset of elements of a group G which commute with all elements of G form the "centre of the group" G, denoted $\boldsymbol{Z}$(G). The center of the dihedral group, $\boldsymbol{Dn}$, is trivial when n is odd. When n is even, the center consists of the identity element together with the 180° rotation of the polygon.

As a matter of fact, considering the transformation flips from Fig. 1, the basic orientation-group contains two more proper subgroups of order 4, K1 and K2. They are isomorphic with the Kleinian Fourgroup (Kleinsche Vierergruppe) $\boldsymbol{Z}_2$ x $\boldsymbol{Z}_2$. The Klein Fourgroup is the smallest non-cyclic group, and every non-cyclic group of order 4 is isomorphic to the Klein four-group. The cyclic group of order 4 and the Klein Fourgroup are therefore, up to isomorphism, the only groups of order four. All other cyclic subgroups of the basic orientation-group are of order 2.

If we center the regular polygon at the origin, then elements of the dihedral group act as linear transformations of the plane. This lets us represent elements of $\boldsymbol{D_4}$ as matrices, with composition being matrix multiplication, by submultiplication tables, as reported in Fig. 2 and 3.

	E	R_1	P_1	P_0
E	E	R_1	P_1	P_0
R_1	R_1	E	P_0	P_1
P_1	P_1	P_0	E	R_1
P_0	P_0	P_1	R_1	E

	E	R_1	P_2	P_3
E	E	R_1	P_2	P_3
R_1	R_1	E	P_3	P_2
P_2	P_2	P_3	E	R_1
P_3	P_3	P_2	R_1	E

Figure 3. On the left the Kleinian Fourgroup K1 and on the right the Kleinian Fourgroup K2 from the basic orientation-group $\boldsymbol{D_4}$, considering the transformation flips (K1, P_0 and P_1; K2, P_2 and P_3) from Figure 1.

So far, we have considered $\boldsymbol{D_4}$ to be a subgroup of ***O(2)***, i.e. the group of rotations (about the origin) and reflections (across axes through the origin) of the plane. However, notation ***Dn*** is also used for a subgroup of ***SO(3)*** which is also of abstract group type ***Dn***: the proper symmetry group of a regular polygon embedded in three-dimensional space (if $n \geq 3$). Our reference diagram may be considered as a degenerate regular solid with its face counted twice (front-view plus rear-view). An example is given in Figure 4.

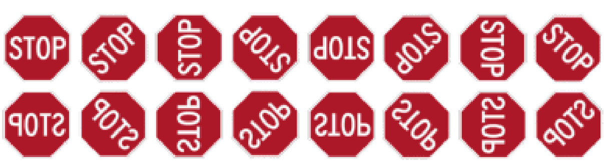

Figure 4. Front-view and rear-view for the stop sign as an example of degenerated regular polyhedron. The sixteen elements of dihedral group $\mathbf{D_8}$ on a stop sign: The first row shows the effect of the eight rotations, and the second row shows the effect of the eight reflections.

In 3D space, all the possible rotational symmetries of an object, as well as its possible orientations about the origin are described by the rotation group ***SO(3)***. The rotation group ***SO(3)*** has as a universal cover the group ***SU(2)*** which is isomorphic to the group of unit quaternions ***Sp(1)***. This is a double cover since the kernel has order 2. The interested reader to dig deeper into the CA and quaternion approaches to logic is referred to [Schmeikal, 1998].

Piaget applied the Klein Fourgroup to binary connectives, so that a given connective is associated first with itself (in an identical (I) transformation) and then with its algebraic complement (its inverse (N) transformation), also with its order opposite (its reciprocal (R) transformation) and, finally, with the combination of its N and R transformations (that Piaget calls its "correlative" or C transformation) [Inhelder and Piaget, 1955]. This correlative corresponds to what logicians usually call the "dual" (D) transformation [Robert and Brisson, 2016]. The Piaget-Klein Group Cayley Table is reported in Figure 5.

x	**I**	**N**	**R**	**D**
I	I	N	R	D
N	N	I	D	R
R	R	D	I	N
D	D	R	N	I

Figure 5. Piaget-Klein Group Cayley Table.

As you can see, it is isomorphic to the Kleinian Fourgroups of Figure 3. Piaget used an algebraic form of logic which was very different from de Morgan's or Boole's "law of thought." Inhelder and Piaget considered a metastructure of algebra that could itself be represented at the level of algebra, in particular as a symmetry group. For Boolean logic this group is isomorphic with $\boldsymbol{Z}_2$ x $\boldsymbol{Z}_2$.

As a matter of fact, today logicians refer to the square of opposition (SOO) or the Square of Apuleius or Buridan Square or the semantic square. The SOO has its origin in the four marked sentences to be employed in syllogistic reasoning, representing the relations between the four basic Aristotelian categorical propositions: Universal Affirmative (A), Universal Negative (E), Particular Affirmative (I), Particular Negative (O), arranged in a square structure (Figure 6).

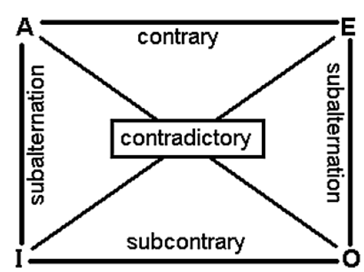

Figure 6. The classic semantic Square of Opposition (SOO) structure.

Gottlob Frege's "Begriffsschrift" also presents a square of oppositions, organised in an almost identical manner to the classical square, showing the contradictories, subalternates and contraries between four formulae constructed from universal quantification, negation and implication [Frege, 1879]. SOO are considered as important basic components of logical competence of human rationality [Beziau and Payette, 2012].

In the 19th century, George Boole argued for requiring existential import on both terms in particular claims (I and O), but allowing all terms of universal claims (A and E) to lack existential import. This decision made Venn diagrams particularly easy to use for term logic. The SOO, under this Boolean set of assumptions, is often called the "modern SOO". In the modern SOO, A and O claims are contradictories, as are E and I, but all other forms of opposition cease to hold; there are no contraries, subcontraries, or subalterns.

X	**e**	**a**	**b**	**c**
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Figure 7. Multiplicative Klein Group Cayley Table.

Thus, from a reductionist, modern point of view, it may make sense to talk about "the" opposition of a claim, rather than insisting as older logicians did that a claim has several different opposites, which are in different kinds of opposition with the original claim. Boolean laws of thought is contained in the symmetry of the original concept of space. In other words, the symmetry of Boolean logic is a proper subgroup with index 2 of the basic two-dimensional orientation group corresponding to INRD by $\boldsymbol{Z}_2$ x $\boldsymbol{Z}_2$. The group Fourgroup is the resultant of the direct product of two copies of the cyclic group of order 2. The Klein four-group (Figure 7) and the cyclic group of order 4 (Figure 8) are therefore, up to isomorphism, the only groups of order 4. Both are abelian groups in mathematics.

X	**e**	**a**	**b**	**c**
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

Figure 8. Cyclic Group Cayley Table.

CA reflections are so fundamental that they are already hardwired even into our current number and polynomial systems to generate automatically optimized expression representation for language of languages by numeric words. For instance, considering the simplest CA reflections hardwired into Rational Numbers, according to *CICT* (Computational Information Conservation Theory) [Fiorini, 2016, 2017], Solid Number (SN) D = 101 generates a SN family of order 25 (25 irreducible family members with cycle length 4) or SN_{25} for short, formed by 25 cyclic numeric words of length 4, that can be arranged into 5 simple cyclic groups plus 20 full cyclic groups, which coupled two-by-two originate 10 pure Klein Fourgroups [Fiorini, 2018a]. As a matter of fact, comparing Fig. 7 to Fig. 8, it is clear that any Klein Fourgroup can be formed by the coupling of two irreducible cyclic groups of length 4.

We are just at the beginning of a new journey to achieve a deeper rational awareness of the root meaning for human cognitive resources.

3 Cognition and Predicative Competence

According to Piaget, the Klein Fourgroup is the formal structure being at work in adult reasoning on propositions: so, treating conveniently algebraic complements (N) and order reciprocals (R) in an integrated structure, by a valid treatment of duals (D), would guarantee people to achieve predicative proficiency.

In any molecular proposition made of two atoms (P and Q), the resulting truth-table contains four truth-values, so that there are 16 different logical functions coded by 16 different binary operators [De Giacomo and Fiorini, 2019]. Eight of these operators appear in two genuine Klein Fourgroups [Cummins, 1995, 1997]. The 8 remaining binary operators appear in 4 "crushed", one-dimensional, Klein groups of only two molecules.

If we assume Piaget′s theory on adult human classical propositional logic was right, then the crushes of these groups capture the explanatory power of human reasoning fallacies [Robert and Brisson, 2016]. For a more detailed analysis and examples on predicative fallacies, the interested reader is referred to Fiorini [2018b]. On the other hand, the "genuine" groups contain four distinct molecules, as being the four different basic Klein transformations (I,N,R and D) with their multiplicative table as reported in Fig. 5.

Piaget was indeed very right when he took INRD to be the basic structure of logic rather than Boolean algebra. In fact, treating conveniently neutral elements (I), algebraic complements (N) and order reciprocals (R) in an integrated structure, by a valid treatment of duals (D), would guarantee people to make logically valid classical inferences on propositions to achieve predicative competence. When dealing with genetic structures of cognition Boolean algebra is not the right way to represent them. In fact the symmetry of Boolean laws of thought is already contained in the symmetry of the original concept of space.

But the formal rationality provided by the SOOs is not spontaneous and, therefore, should not be easy to learn for adults. This is the main reason why we need reliable and effective training tools to achieve full logic proficiency and predicative competence, such as the EPM (Elementary Pragmatic Model) and E^2PM (Evolutive Elementary Pragmatic Model) [Fiorini, 2017]. EPM extension as E^2PM represents the latest contribution to current EPM modeling and simulation, offering an example of new forms of evolutive behavior by inter- and trans-disciplinarity modeling (e.g. strategic foresight, uncertainty management, embracing the unknown, creativity, etc.) for the children of the Anthropocene [McNeill and Engelke, 2014].

4 Conclusion

We have shown that classical and modern logic can be derived from the primeval concept of orientation developed by newborns since their birth, growing through evolutive perturbation and evolutive information sources [Fiorini, 2019a]. Evolutive information is an elusive idea whose specific and contingent understanding involves interdisciplinary, trans-disciplinary, cultural and ontological multi-perspectives, to arrive to a nondual pragmatist, semiotic-processes, philosophical framework to be named "Cybersemiotics" [Brier, 2008], by simply applying a bottom-up approach, using technoscience from below, by a deep learning approach. To really understand evolutive information, we need to analyze the strong, resonant coupling between processes related to action and perception which emerges in the human brain as a consequence of learning sensorimotor task [Fadiga *et al.*, 1995].

Artificial General Intelligence needs fresh methods with cognitive architectures and philosophy of mind. We have shown that elementary geometric reflections (not rotations) can be assumed as the basic components of human orientation and cognition to create meaningful representations, and CA is the fundamental tool to reliably handling even logical transformations. In fact, the Klein Fourgroup structure generates squares of opposition, and an important component of human rationality resides in the diagram of the SOOs. It can be used even to explain human reasoning fallacies.

Furthermore, CA reflections are so fundamental that they are already hardwired even into current number and polynomial system representations to generate automatically optimized components for language of languages by numeric words [Fiorini, 2016]. The major trouble is that we are not aware at rational level of their concealed, powerful properties.

But the formal rationality provided by the SOOs is not spontaneous and, therefore, should not be easy to learn for adults. This is the main reason why we need reliable and effective training tools to achieve full logic proficiency, such as the EPM and E^2PM [De Giacomo and Fiorini, 2019]. Treating conveniently the four fundamental transformation (I,N,R and D) would guarantee people to make logically valid classical inferences on propositions and to achieve predicative competence.

Taking into consideration the development of E^2PM, our approach is quite flexible and can even evolve into the basic architectural blocks to build human-centered symbiotic autonomous system (HCSS) by purposive actors within continuous change [Fiorini, 2019b], and for intelligent tutoring, starting from an automated learning and teaching of logic and predicative competence. In this case, we need a model of the learner, as a lay person, so that combined with a model of the expert, an automated tutor can be built [Nkambou *et al.*, 2010], as a system of strategies to help transform the learner into an expert. In future paper we will address this topic in more detail.

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