Qualitative evaluation of the process of functionally stable recovery control of the aircraft in emergencies with an algorithm based on solving inverse dynamic problems

Oleg Mashkov[1], Victor Chumakevych[2[0000-0002-5773-393X]], Vadym Ptashnyk [3[0000-0002-1018-1138]], and Igor Puleko[4[0000-0001-8875-017X]]

[1]State Ecology Academy of Postgraduate Education and Management, Kyiv, Ukraine
mashkov_oleg_52@ukr.net
[2]Lviv Polytechnic National University, Lviv, Ukraine
chumakevich@ukr.net
[3]Lviv National Agrarian University, Lviv, Ukraine
ptashnykproject@gmail.com
[4]Zhytomyr Polytechnic State University, Zhytomyr, Ukraine
pulekoigor@gmail.com

Abstract. The influence of the input data and a Hurwitz matrix on the stability of the system in general is investigated. The factors that influence the behavior and timing of the transient process are analyzed. An expression is obtained for finding the time of the transient process as well as its dependence on the parameters of the Hurwitz matrix.

Keywords: functional stability, recovery control, emergency, inverse dynamic problem, operationally programmed trajectory, transient process

1 Introduction

Functional stability theory originated in the 1980s [1–5] and was used to control complex dynamic objects and computing systems. This theory makes it possible to respond promptly to system and block failures, to redistribute the functions of the failed blocks between those capable of performing the final task [4–12]. The use of this theory was limited by the capacities of computing systems and by the need to reserve the main blocks of the control system. For this reason, this theory was used for the control systems of large aircraft. With progress, the dimensions of control systems elements have decreased and the characteristics of computing systems have improved significantly, so this theory has evolved significantly and has become applicable to complex dynamic control objects of a broader purpose [6–9, 13–20,].

The dynamic object (Fig. 1) is written in the form of linear difference equations:

$$Q(q)y(n) = q^{k+1}P_u(q)U(n) + P_\xi(q)\xi(n),$$

where *y(n)* – initial value;

$U(n)$ – control influence;
$\xi(n)$ – external influence (disturbance);
$Q(q)$, $P_u(q)$, $P_\xi(q)$ – polynomials of q; a_n, b_n, c_n;
q; a_n, b_n, c_n – coefficients of polynomials;
q – delay operator: $q^m\chi(n) = \chi(n–m)$ (sometimes the delay operator q^{-1} is used in the literature, so $q^{-m}\chi(n) = \chi(n–m)$).

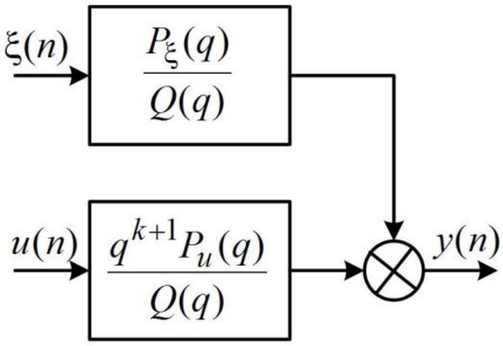

Fig. 1. Block diagram vector of the object parameters

The control device (Fig. 2) is described as:

$$R(q)U(n) = P_r(q)r(n) - P_y(q)y(n),$$

where $r(n)$ – input influence.

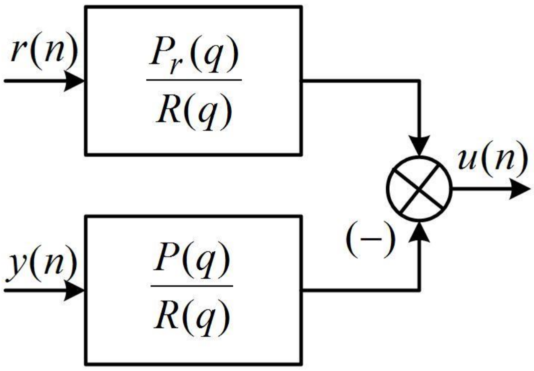

Fig. 2. Block diagram of control device

The system as a whole is described by the equations (Fig. 3):

$$G(q)y(n) = q^{k+1}P_u(q)P_r(q)r(n) + P_\xi(q)R(q)\xi(n),$$

where $G(q)$ – is a characteristic polynomial.

In functionally stable systems, vector of reducing filter parameters (Fig. 4) must change so as to ensure the optimality of the whole system over time.

To solve this problem we will use the method of inverse dynamics problems (Fig. 5).

2 Description of the mathematical model

The mathematical description is based on the matrix equations of the form [4, 6, 8, 18, 19]:

$$\dot{X}(t) = F[X(t), U(t), Z(t)] + P(t); \quad X(t_0) = X_0, \quad t \in [t_0, t_T], \tag{1}$$

where $U(t)$ is a vector of control;
$Z(t)$ is a vector of the object parameters;
$P(t)$ is a vector of disturbances acting on the object;
X_0 is the initial conditions.

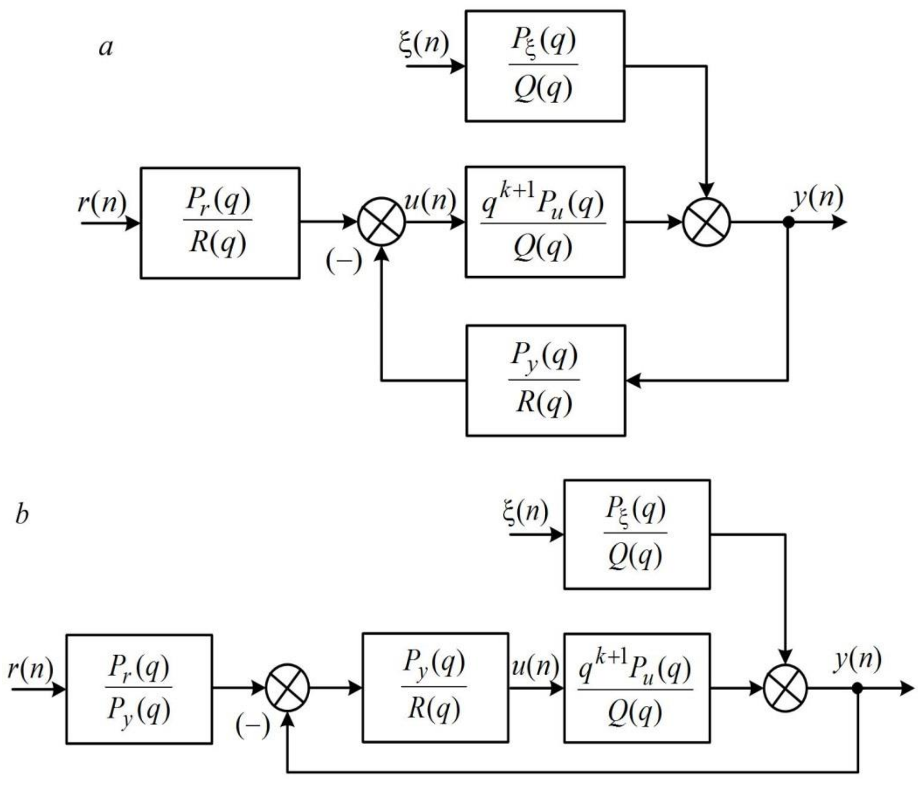

Fig. 3. Structural (a) and equivalent (b) schemes of a closed system

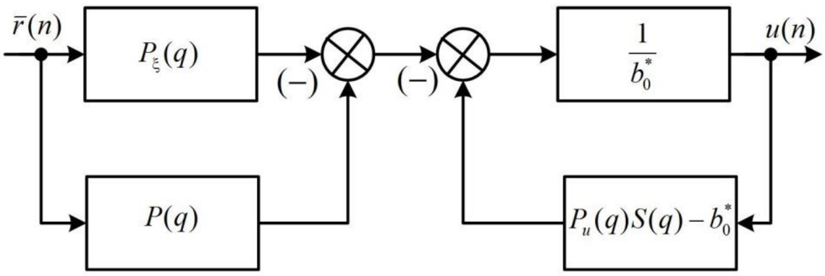

Fig. 4. Optimal reducing filter

The presented theory is intended to provide a given programmatic movement (state) of the object of control and may provide for solving the problems of stabilization, terminal control, and adaptive tracking (Fig. 6). An object that can be described by the system (1) must be on a given trajectory (correspond to a certain state) and provide a minimum of the functionality of quality:

$$J[U_n^0(\cdot), X_n^0(\cdot)] = \min J[U_n(\cdot), X_n(\cdot)]\,, \tag{2}$$

where $X_n^0(t)$, $U_n^0(t)$ are optimal vectors of state and control.

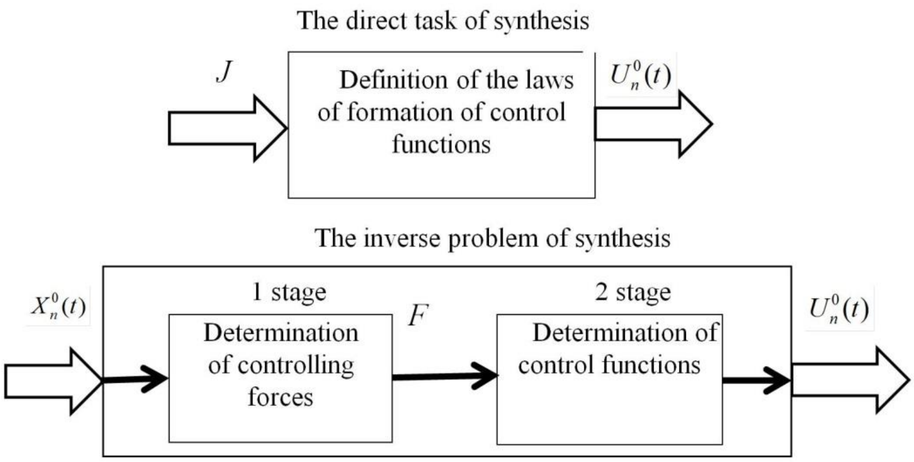

Fig. 5. Block diagram of the synthesis problem statement

3 Statement of the problem

In Refs. [18–20], examples of synthesis of functionally stable automatic systems for stabilization of motion of dynamic objects based on the solving inverse problems of dynamics are presented. To evaluate the quality of such systems, it is necessary to evaluate their resilience and the characteristics of the transients.

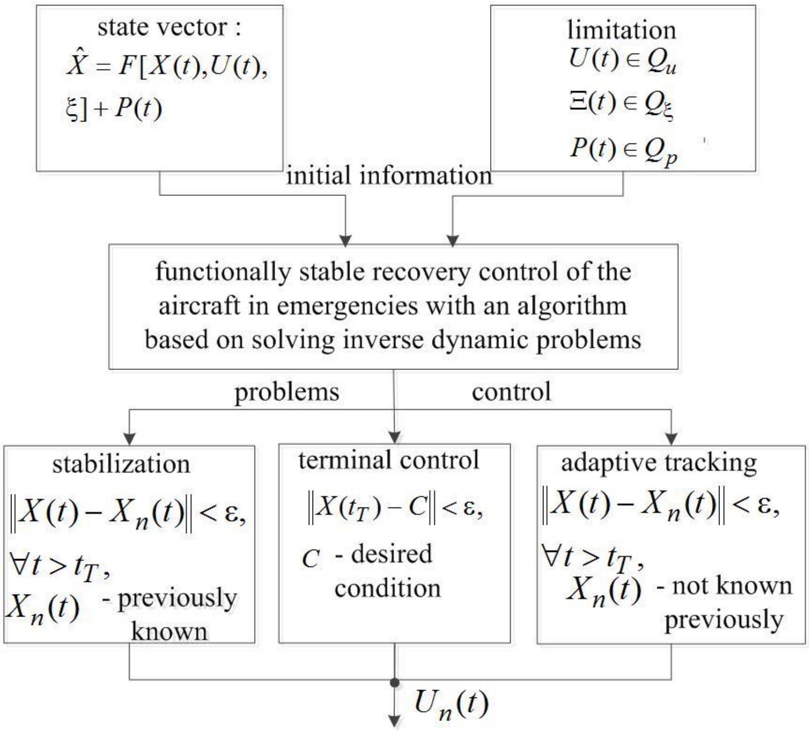

Fig. 6. Block diagram "Statement of the problem"

The system of matrix differential Eq. 1 can be represented as a structural diagram (Fig. 7).

To solve the problem, we assume that the programmatic movement is asymptotically stable.

The following conditions are imposed on this class of controlled dynamic systems.

1. There is a subspace $R \in R^m \times R^n$ and a unique function $U : R \times \Xi \to R^m$ such that for any pair of points $\{X, Z\} \in R$ and the point $\{\xi\} \in \Xi$, the identity is satisfied

$$Z \equiv F[X, U(x, z, \xi), \xi], \tag{3}$$

i.e., Eq. 1 can be solved with respect to control on the subspace R.

The condition (3) guarantees the existence and uniqueness of an ideal programmatic control law of the form $U_n = U_n(X_n, \dot{X}_n, \Xi)$, which ensures the accurate implementation of programmatic movement provided $X_n(t_0) = X_0$; $P(t) = 0$; $(X_n(t), \dot{X}_n(t)) \in R$ $\forall t \in [t_0, t_T]$.

2. The following restrictions are imposed on the initial and constant external disturbances [1–4, 18, 19]:

$$\|X(t_0) - X_n(t_0)\| < \delta_0; \quad \|P(t)\| < C_p, \tag{4}$$

where δ_0, C_p are positive parameters.

3. There is a Hurwitz matrix Γ with prime eigenvalues γ_i

$$\dot{X} = \Gamma X \ , \tag{5}$$

where $X(t_0) = X_0$ is the determined initial vector of state;

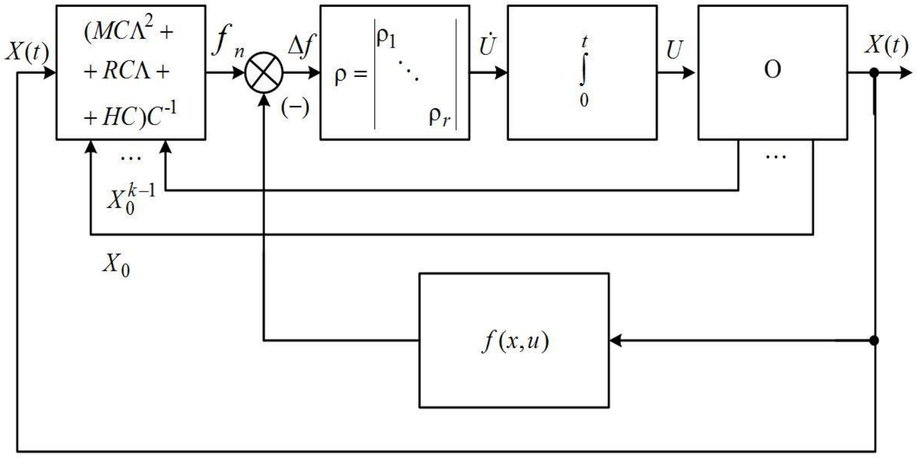

Fig. 7. Structural diagram of functionally stable control system by the method of inverse dynamic problems

γ_i are the roots of the characteristic equation of the system, $i = \overline{1,n}$, for any $X \in R^n$, the condition holds:

$$(X, \dot{X}_n(t) + \Gamma(X - X_n(t))) \in R, \ \forall t > t_0 \, . \tag{6}$$

4. For $t \in [t, \infty]$, the control law

$$U(t, X) = U[X, \dot{X}_n + \Gamma(X - X_n), \Xi] \, . \tag{7}$$

For any disturbances that satisfy the condition (4), the movement $X(t)$ ($X(t_0) \in R$) asymptotically approaches a determined programmatic movement $X_n(t)$, i.e., it provides the asymptotic stability of the programmatic movement in general.

From the expressions (1), (3), (6), we find

$$F[X(t), U(t), \Xi] = \dot{X} = \dot{X}_n(t) + \Gamma(X - X_n(t)) \, . \tag{8}$$

Thus, we can write it down

$$\dot{X} - \dot{X}_n(t) = \Gamma(X - X_0) \, . \tag{9}$$

Due to the selection of the Hurwitz matrix Γ, the required transient process is provided [1–4, 14]. For the system (9), the roots of the characteristic equation determine the stability of the system in general. If Re $\gamma_i < 0$, $i = \overline{1, n}$, the roots have negative real parts, the trivial solution of the system is unstable. When the roots do not have a positive real part, but at least one with a zero real part, the system is on the limit of stability.

According to the condition (6), the matrix is chosen with prime eigenvalues, then there are positive numbers C and γ such that

$$\mathrm{Re}\,\gamma_i < -\gamma,\ \|X(t) - X_n(t)\| < C\|X(t_0) - X_n(t_0)\| e^{-\gamma(t-t_0)},\ \forall t > t_0. \tag{10}$$

Thus, the programmatic movement in general will be asymptotically stable.

4 Results and Discussions

Let us give an estimate of the maximum time of the transient process

Case 1. external influences are absent $\pi(t) = 0$.

Let the law of control that guarantees for any $\xi \in \Xi$, $\pi(t) \in Q_\pi$ (ε is the closeness of real and programmatic movements [4, 6, 18, 19], starting from the end time moment $t_n > t_0$) be synthesized, i.e.,

$$\|X(t) - X_n(t)\| < \varepsilon,\ \ \forall t \geq t_n. \tag{11}$$

The expression (11), given the expression (10), can be represented in the form:

$$C\|X(t_0) - X_n(t)\| e^{-\pi(t_n - t_0)} \leq \varepsilon, \tag{12}$$

hence

$$\gamma(t_n - t_0) \leq \ln\frac{C}{\varepsilon}\|X(t_0) - X_n(t_0)\|. \tag{13}$$

Denote as $T_n = t_n - t_0$ the time of the transient process in the system.

The time T_n of the transient process can be estimated using the expression (13)

$$T_n \leq \gamma^{-1} \ln\frac{C\|X(t_0) - X_n(t_0)\|}{\varepsilon}. \tag{14}$$

Case 2. External disturbances occur $\pi(t) \neq 0$.

Let the controlled object be described by the equation $\dot{X}(t) = F[X(t), U(t), \xi] + \pi(t)$, $X(t_0) = X_0$, $t \in [t_0, t_n]$. Moreover, $X \in Q_X$, $\dot{X}_n(t) + \Gamma(X - X_n) \in Q_X$.

The control U is of the chosen form: $U(t,X)=U[X,\dot{X}_n+\Gamma(X-X_n),\xi]$, $t\in[t_0,\infty)$.

The equations of the closed-loop system have the form: $\dot{X}(t)-\dot{X}_n(t)=\Gamma[X(t)-X_n(t)]+\pi(t)$.

Suppose that $X_n(t)$ and $\dot{X}_n(t)$ both lie on sets Q_X and $Q_{\dot{X}}$ with stocks δ_1 and δ_2, respectively (Fig. 8), when

$$\delta_1>C\|X_0-X_n(t_0)\|,\ \delta_2>C\|X_0-X_n(t_0)\|\cdot\|\Gamma\|, \tag{15}$$

or given $\|\Gamma\|\neq 0$.

$$\delta_1-C\|X_0-X_n(t_0)\|>0, \tag{16}$$

$$\delta_2\|\Gamma\|^{-1}-C\|X_0-X_n(t_0)\|>0. \tag{17}$$

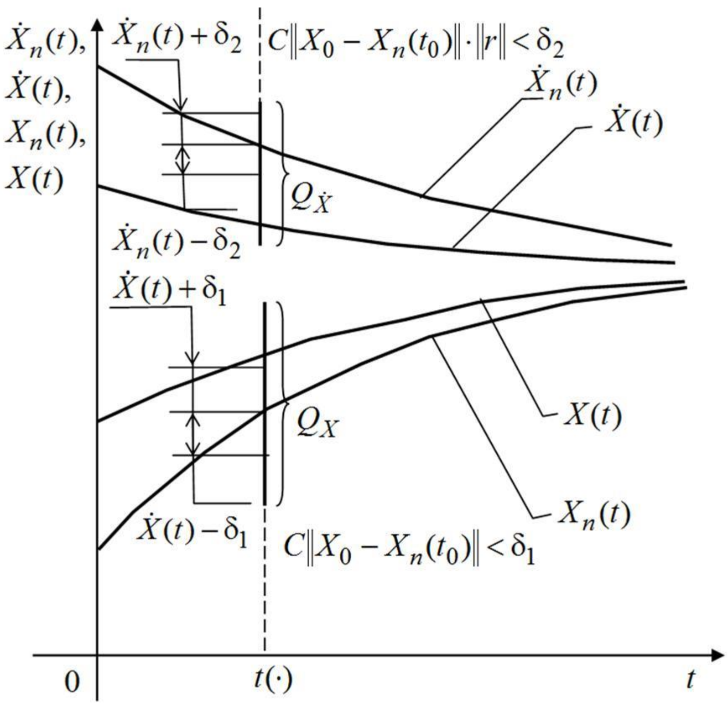

Fig. 8. Graph of transient processes in a functionally stable system of recovery control by the method of inverse dynamic problems

Let us introduce the denotations

$$X=\min[\delta_1-C\|X_0-X_n(t_0)\|,\delta_2\|\Gamma\|^{-1}-C\|X_0-X_n(t_0)\|], \tag{18}$$

$$C\gamma^{-1}C_{\pi} < \min(x, \varepsilon)\,, \tag{19}$$

$$\varepsilon > \|X(t) - X_n(t)\|\,. \tag{20}$$

Given the accepted denotations, the following condition can be obtained

$$\|X(t) - X_n(t)\| \le \left\|e^{r(t-t_0)}\right\| \cdot \|X_0 - X_n(t_0)\| + C\gamma^{-1}C_{\pi}\,. \tag{21}$$

Hence, given (20), it follows: $C\|X_0 - X_n(t_0)\|e^{-\gamma(t-t_0)} + C\gamma^{-1}C_{\pi} < \varepsilon$.

Solving the obtained inequality with respect to $t = t_n$, we find $\varepsilon - C\gamma^{-1}C_{\pi} > C\|X_0 - X_n(t_0)\|e^{-\gamma(t_n - t_0)}$.

$$t_n \le t_0 + \gamma^{-1}\ln\frac{C\|X_0 - X_n(t_0)\|}{\varepsilon - C\gamma^{-1}C_{\pi}}\,. \tag{22}$$

The time of the transient process:

$$T_n \le \gamma^{-1}\ln\frac{C\|X_0 - X_n(t_0)\|}{\varepsilon - C\gamma^{-1}C_{\pi}}\,. \tag{23}$$

Comparing the expression (23) with the expression (14), we can conclude that in the presence of external disturbances, the time of transient process increases

$$T_n(C_{\pi} \ne 0) = T_n(C_{\pi} = 0) + \gamma^{-1}\ln\frac{\varepsilon}{\varepsilon - C\gamma^{-1}C_{\pi}}\,. \tag{24}$$

With great disturbances (increase in C_{π}), the time of transient process increases significantly.

To reduce the time of the transient process in the system, it is necessary to increase γ by means of an appropriate choice of control.

5 Conclusions

For a multidimensional controlled object, satisfying $\det C \ne 0$, the structure of the control algorithm does not explicitly contain the equation of motion of the object. The proposed approach to the construction of control algorithms allows obtaining algorithms without using detailed equations of the controlled process. Moreover, it is sufficient to use as a mathematical model the generalized equations that reflect the fundamental laws of motion.

To synthesize the aircraft control algorithms, complete nonlinear equations of motion can be used without linearizing them. The resulting algorithms are also nonlinear. Their structure is adequate to the structure of mathematical models of controlled processes, and the parameters of these algorithms are determined by the parameters of mathematical models of assigned motion trajectories.

In essence, the construction of aircraft motion control algorithms along the assigned trajectory has two aspects. The first one is related to the direct formation of the vector of the required control force $f^*[x]$, and the other is related to the calculation of the values of the elements of the vector of the control function $U_n(t)$ that creates the necessary force f^*. The calculation relations according to which f^* and $U_n(t)$ are calculated form the contents of the motion control algorithm.

The coefficients of the control system are determined by the basic parameters of the motion of the object, as well as by the parameters of the assigned program trajectory. This allows changing the parameters of the programmatic trajectory in the object movement.

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