The Use of Digital Visualization Tools to Form Mathematical Competence of Students

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Abstract. The advantages of digital visualization to form mathematical competence of students are exploded in the article. Using a test experiment the problematic field of using the digital visualization tools in process of teaching higher mathematics was identified. Possibilities of involvement of digital visualization tools for conduct mathematical experiment using dynamic geometry environments (DGEs) GeoGebra and advanced graphing calculator Desmos in the study of the fields of "Projective geometry and methods of image", "Mathematics analysis", "Analytical geometry".

Keywords: Visual Thinking, Digital Visualization Tools, Dynamic Geometry Environments, Mathematics Competences, Mathematics Experiment.

1 Introduction

Modern educational technologies aimed to form knowledge and skills, developing cognitive techniques and accelerating student learning are impossible without the use of digital tools. The technology of visualization of educational material, which provides, in addition to the transfer of educational information (images, animations, presentations etc.), psychology-pedagogical techniques of engaging students in the process of learning so called "visual thinking" (Rudolf Arnheim) provides considerable opportunities here. Unlike the use of ordinary spotting, visual thinking is thinking through visual operations, the activity of the mind in special environment, which makes it possible to translate information from one language to another, to understand the connections and relation between objects [1]. This is especially important for conceptual understanding of mathematics, the subject of which is abstract mathematical structures. Modern digital tools allow us to design an interactive environment for the use of visual thinking and thereby influence the thinking "in general"; create favorable conditions for the formation of mathematical competence of students.

However, despite the wide range of digital tools with various functionalities for visualization of mathematical content teaching materials, there is not enough scientific and methodological support for the use of these tools. Therefore, the theoretical substantiation and methodological support of the use of digital visualization tools in teaching mathematics at the University is an urgent task.

2 Literature Review

The use of visualization tools in the educational process of higher school is receiving considerable attention of scientists. Thus, some methodical aspects of the use of visualization in mathematics are considered in works of [2-5]. The possibility of using cloud based learning technologies, in particular, for visualization of mathematics material was substantiated by the authors of this article [6,7]. Despite the large number of studies mentioned and not named here, there is still no scientific justification for the feasibility, description of practices and methodological support for the use of digital visualization tools to shape students' mathematical competence. The purpose of our article is to attempt to partially fill the gap, in particular, to justify the use of digital visualization tools for the formation of mathematical competence of students.

3 Research Methods

In a course of the research, the following methods were used: analysis of scientific literature, determination of categorical-conceptual apparatus; synthesis, generalization, systematization; diagnostics (questionnaire and statistical processing). The research was carried out within the framework of the complex scientific topic of the Computer Science and Mathematics Department of Borys Grinchenko Kyiv University "Theoritical and practical aspects of using mathematical methods and information technologies in education and science", SR № 0116U004625.

4 Research results

Mathematical competence as the ability to formulate, apply and interpret mathematics in a variety of contexts is recognized today as one of the key competences of a person. Eight components of mathematical competence can be divided into two groups [8]: 1) the ability to ask and answer questions in and with mathematics (thinking mathematically; reasoning mathematically; posing and solving mathematical problems; modelling mathematically), 2) the ability to deal with mathematical language and tools (representing mathematical entities; handling mathematical symbols and formalism; communicating in, with and about mathematics; making use of aids and tools).

An analysis of the functionality of basis computer math packages has shown that using them greatly helps to form all those components in students. Nevertheless, are teachers in higher education institutions ready to make full use of these capabilities within fundamental mathematical disciplines? In January 2020 we offered a questionnaire to university teachers on the use of digital visualization tools in a process of teaching mathematics. 41 professors of higher education institutions in Kyiv, Vinnytsa, Ivano-Frankivsk, Nizhyn, Sloviansk, Sumy, Uman, Kharkiv, Kherson and Cherkasy provided answers to the questionnaire. Their analysis showed that almost half of the respondents rarely use or do not use these digital tools at all (31.7% and 14.6%, respectively). This clearly demonstrates the scientific problem of our study, which is to justify the feasibility of using digital visualization tools with the subsequent development of appropriate methodological support.

Among the mentioned reasons for non-systematic use of digital visualization tools are most often indicated: lack of methodological support (73.7%); high time spent on preparation for such classes (47.4%). 26.3% of those, who answered this question, admit that they do not have enough digital technology, and 15.8% believe that it is unnecessary to use digital tools when teaching mathematics. It was also found that the overwhelming majority of respondents (63.4%) use the relevant software products solely to identify study material or increase student interest. Thus, the survey confirms the lack of consensus on the usefulness or benefits of using digital visualization tools to successfully teach mathematics.

Do teachers need methodical support for the use of digital visualization tools in the teaching of mathematical disciplines? Questionnaire replies indicate that only 13.5% of respondents do not think that they need some methodical help (and this is taking into account those who basically do not use digital technology). Instead, 73% see this problem as relevant for them.

After analyzing the results of the survey, we focused on one of the most relevant but little-used practices – the use of digital visualization tools for mathematical experimentation.

The undeniable advantage of software rendering environments is that they allow to create dynamic images, as well as automate bulky calculations and constructions within mathematical experiment. Digital visualization at the stage of observing the behavior of some mathematical object or their totality allows us to intuitively come to a hypothetical conclusion. The next step us to find a way to prove or disprove a hypothesis (visual representation often helps to "see" the idea of proof), rigorous mathematical proof, exploration of the realm (limits) of truth, special cases, etc.

As an example, let's consider the mathematical experiment in DGEs GeoGebra when studying the topic "Projective Coordinates of a point" (discipline "Projective Geometry and Image Methods").

The notions of projective coordinate system and projective coordinates of a point on an extended Euclidean line are not intuitively felt for students, as they differ significantly from the usual Cartesian coordinates. To prevent these concepts and their properties from being a misunderstood mathematical abstractions for students, it is advisable to use a computer-aided geometry experiment. Not only will it help to create a visual image of projective line (p) as a projection of affine plane to the line, but it will also allow students to express hypothesis about the characteristic features and properties of the projective coordinate system and coordinates of a point on the projective line.

Thus, by changing the position of the points of the projective rapper on the line, the points O, the length of the basis vectors and the vector **m**, students see (Fig.1):

1) that a projective coordinate system can be an arbitrary three points, since it is always possible to construct its generating basis and the vector **e** in it;
2) if the projective coordinate system on the projective line is given, then the coordinates of the point M in it do not depend on the choice of the point O – the center of the basis that generates this projective coordinate system;

3) that the coordinates of a point on a projective line are determined to a constant factor, since all vectors collinear to the vector **m**, produce the same point M on the projective line p.

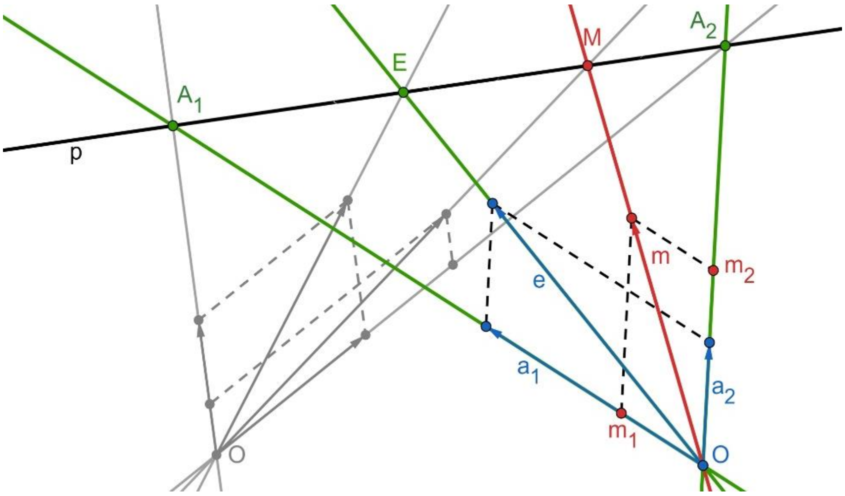

Fig. 1. Projective coordinates of a point

Here is another example of using a dynamic geometric model when studying a topic: "Continuous Function" (discipline "Mathematical analysis 1"). Let's solve the problem: is there a versatile triangle in which the smallest median is equal to the highest height?

Suppose in the triangle ABC (Fig. 2): AB < BC < AC; BM – is the smallest median; CH – is the highest height. Fix the lengths of sides AB and BC. Then, moving the point C in a circle with center B, "come across" such a position at which BM = CH (≈0,55). This suggests a hypothetical affirmative answer to the question of the problem.

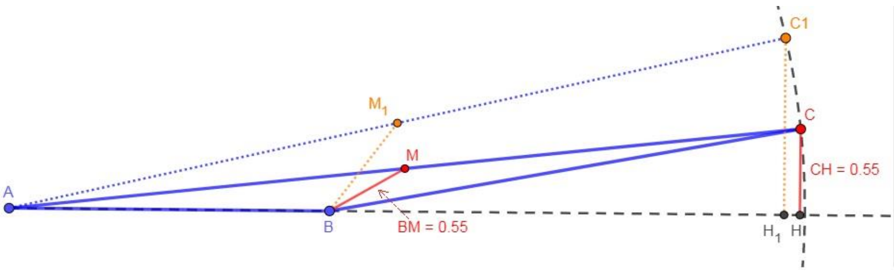

Fig. 2. A dynamic model of the dependence of the median on height

To prove this hypothesis let's continue the experiment. Note, that with small oscillators of point C (a slight change in the magnitude of the ABC angle) we have slight changes in both the height of CH and the median BM and therefore in their differences. That is there is a reason to believe that y = CH – BM is a continuous function of the angle ABC (find an analytic expression of this function and make sure that the assumption is correct).

By changing position of point C, we can see that if an angle ABC is expanded, then CH = 0 and $y < 0$. If however the angle ABC is straight, then CH coincides with BC, BM < CH and $y > 0$. Due to the continuity of the function, there exists a position of point C (and hence the triangle is ABC), that y = CH – BM = 0, or the same as BM = CH.

A computer mathematical experiment can be effectively used to verify the solution obtained. Here is an example of studying the topic “Trigonometric Fourier series” (discipline “Mathematical analysis 2”).

As is known when finding Fourier coefficients, we have to calculate cumbersome integrals, so there is a high probability of technical errors. Graphic visualization of the found feature development in the Fourier series allows to control the correctness of the analytical result. In Fig.3 we have visualization in the graphing calculator Desmos of the development of the function $f(x) = \frac{p}{4} - \frac{x}{2}$ in Fourier series in sinus (Fig. 3a), cosines (Fig. 3b), sines and cosines (Fig. 3c) on segment $[0;\ p]$. The images allow us to conclude that the row of cosines coincides with a given function much faster than the row of sinuses itself, and an error is made in the schedule for sines and cosines.

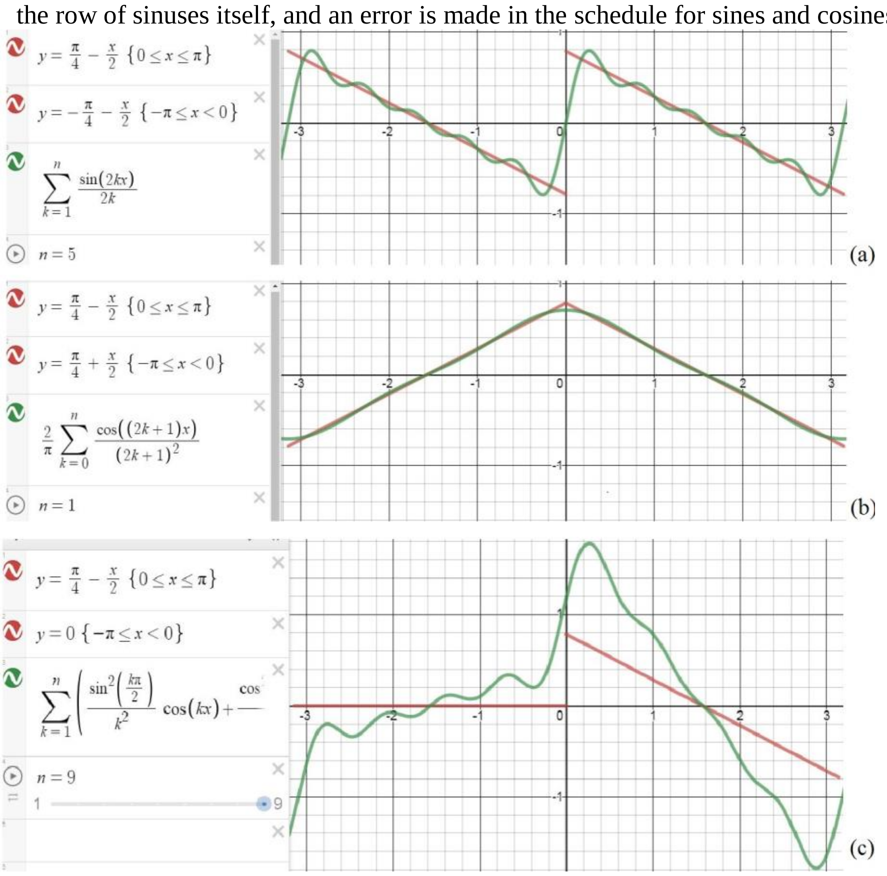

Fig. 3. Fourier series functions $f(x) = \frac{p}{4} - \frac{x}{2}$

Finally, give the last example (discipline “Analytical geometry”, topic: “Curves of the second order”). The dynamic model of conic sections (section of a cone by plane) allows static, apparently completely different objects with significant qualitative differences (curves of the second order: circle, ellipse, hyperbola, parabola, pair of straight lines, point) to be interpreted only as separate manifestations (states) of one dynamic phenomenon.

Note also that there is no clear scientific evidence that mathematics students’ education outcomes depend on the use (or non-use) of digital visualization tools or the benefits of visual-technical ways of presenting information over others. However, the practice has shown that visualization of mathematical abstractions using digital tools help “weak” students to better understand and learn these abstractions. Students with a high level of mathematical competence are easier to do without “physical” visualization, but are able to use it when they needed.

5 Conclusions

Based on the elucidation of the essence and structure of mathematical competence of students and the analysis of the functionality of the basic systems of computer mathematics, it is established that their use contributes to the formation of all components of mathematical competence. At the same time, the survey of university mathematics teachers outlined the current problem of substantiating the feasibility of using digital visualization tools and the development of appropriate methodological support.

The implementation of the first part of the task is carried out based on examples that reveal the possibilities of using digital visualization tools GeoGebra for a mathematical experiment.

The experiment was carried out using computer mathematics systems (Desmos, GeoGebra) within the disciplines “Projective Geometry and Image Methods”, “Analytical Geometry” and “Mathematical Analysis”. Specific examples show the feasibility of using digital visualization tools in teaching the following topics: “Projective coordinates of a point”, “Continuity of function”, “Trigonometric Fourier series”, “Second-order curves”.

Thus, judicious use of digital tools in the study of mathematical disciplines improves conceptual understanding of mathematics, supports intuition about predicting a possible result, hypothesis, facilitates the search for a method (idea) of formal proof (of refutation), replaces (if necessary) technically sophisticated calculations, transformations, computer-based constructions, allows to check (confirm) results obtained analytically. All this contributes to the formation of mathematical competence of students.

The development of methodological support of use of digital visualization tools to form mathematical competence of students will be the subject of our further scientific researches.

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