Model of Signals With Double Stochasticity in the Form of a Conditional Cyclic Random Process

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Abstract
The work is devoted to the development of mathematical modeling of digital cyclic signals with double stochasticity, namely, the construction of their mathematical model in the form of a conditional cyclic random process of discrete argument. This model allows to take into account stochasticity of digital cyclic signals both in their morphological statistical analysis and in statistical analysis of their rhythm. Approaches to morphological and rhythm analysis of cyclic signals based on their model are proposed.

Keywords
Cyclic signal, conditional cyclic random process, morphological analysis, rhythm analysis.

1. Introduction

Cyclic processes and phenomena are widespread in various spheres of reality, in particular, they take place in economics, biology, medicine, technology. The study of cyclical processes involving modern information systems requires the preliminary development of adequate mathematical models for them. Many different mathematical models of cyclic processes are known today, including harmonic, periodic and almost periodic deterministic functions, periodically correlated and periodically distributed random processes, linear periodic random process, almost periodically correlated random process, cyclic random process [1-11].

All these models of cyclic signals do not allow for simultaneous consideration of double stochasticity in their structure, namely, stochasticity in the morphological structure of the signal and stochasticity in the rhythmic structure of the signal. Examples of cyclic processes and signals with double stochasticity are cardiac signals of electrical, magnetic and mechanical nature, economic cyclic processes, the processes of the appearance of spots in the Sun, the processes of cracking of the nano coating on the surface of materials.

2. Related works

In [12, 13], the development of a mathematical model of a cyclic signal, which has the means of considering double stochasticity in its structure, namely, stochasticity in the morphological structure of the signal and stochasticity in the rhythmic structure of the signal, was begun. This model is called a conditional cyclic random process. Conditional cyclic random process takes into account the

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cyclicality, stochasticity of the morphological and rhythmic structures of the investigated processes (signals).

3. Overview of the research

Despite the results obtained in [12, 13], a number of problems remain to be solved, in particular, it is necessary to clarify the definition of the class of conditional cyclic random processes and the definition of the random function of the rhythm of the conditional cyclic random process, as well as clarify the essence of the approach to statistical analysis and analysis the rhythm of cyclic signals within their mathematical model in the form of a conditional cyclic random process. It should also be noted that there is no definition of a conditional cyclic random process of discrete argument, which makes it impossible to develop adequate statistical methods for analyzing digital signals.

4. Proposed model

In this paper, we develop (detail) the procedure for constructing a class of conditional cyclic random processes, namely, construct a mathematical model of a cyclic signal in the form of a conditional cyclic random process of a discrete argument, which would consistently reflect its double (morphological and rhythmic) stochasticity and made it possible to perform statistical morphological and rhythm analysis in modern digital data processing information systems.

5. Results & Discussion

We construct a conditional cyclic random process of a discrete argument (conditional cyclic discrete random process). To do this, in accordance with [14], we briefly consider the concept of a cyclic random process $\xi(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D} \subset \mathbf{R}$ of a discrete argument and introduce the notion of a class of isomorphic cyclic random processes of a discrete argument. The time area of definition of a discrete cyclic random process is an ordered discrete set of real numbers $\mathbf{D} = \left\{ t_{ml} \in \mathbf{R}, m \in \mathbf{Z}, l = \overline{1, L}, L \geq 2 \right\}$ whose elements have the following type of linear ordering: $t_{m_1 l_1} < t_{m_2 l_2}$, if $m_2 < m_1$ or if $m_2 = m_1$, and $l_2 < l_1$, in other cases $t_{m_1 l_1} > t_{m_2 l_2}$ ($m_2, m_1 \in \mathbf{Z}$, $l_2, l_1 = \overline{1, L}$, $0 < t_{m,l+1} - t_{m,l} < \infty$). This process is defined on same probabilistic space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$.

Definition 1. A discrete random process $\xi(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D} \subset \mathbf{R}$, is called a cyclic discrete random process, if there is such a discrete function $T(t_{ml}, n)$ that satisfies the conditions of the rhythm function,that finite-dimensional vectors $(\xi(\omega, t_{m_1 l_1}), \xi(\omega, t_{m_2 l_2}), \ldots, \xi(\omega, t_{m_k l_k}))$ and ($\xi(\omega, t_{m_1 l_1} + T(t_{m_1 l_1}, n))$, $\xi(\omega, t_{m_2 l_2} + T(t_{m_2 l_2}, n))$,..., $\xi(\omega, t_{m_k l_k} + T(t_{m_k l_k}, n))$), $n \in \mathbf{Z}$, for all integers $k \in \mathbf{N}$, are stochastically equivalent in the broad sense.

For a discrete cyclic random process, the family of its distribution functions satisfies the following equations:

$$F_{k_\xi}(x_1, \ldots, x_k, t_{m_1 l_1}, \ldots, t_{m_k l_k}) = F_{k_\xi}\left(x_1, \ldots, x_k, t_{m_1 l_1} + T(t_{m_1 l_1}, n), \ldots, t_{m_k l_k} + +T(t_{m_k l_k}, n)\right) =$$
$$F_{k_\xi}\left(x_1, \ldots, x_k, y(t_{m_1 l_1}, n), \ldots, y(t_{m_k l_k}, n)\right), x_1, \ldots, x_k \in R, t_{m_1 l_1}, \ldots, t_{m_k l_k} \in D, n \in Z, k \in N. \quad (1)$$

We give the definition of isomorphism with respect to the order and values of cyclic random processes of a discrete argument. Let us have cyclic random processes $\xi_1(\omega, t), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}$ with rhythm function $T_1(t_{ml}, n)$ and $\xi_2(\omega, t'_{ml}), \omega \in \mathbf{\Omega}, t'_{ml} \in \mathbf{D}'$ with rhythm function $T_2(t'_{ml}, n)$. Areas of definition $\mathbf{D}=\left\{t_{ml}\in\mathbf{R},\, m\in\mathbf{Z},\, l=\overline{1,L},\, L\geq 2\right\}$ and $\mathbf{D}'=\left\{t'_{ml}\in\mathbf{R},\, m\in\mathbf{Z},\, l=\overline{1,L},\, L\geq 2\right\}$ these random processes in the general case are different ($\mathbf{D}\neq\mathbf{D}'$).

Definition 2. A cyclic random process $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ with a rhythm function $T_1(t_{ml},n)$ and a cyclic random process $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$ with a rhythm function $T_2(t'_{ml},n)$, will be called isomorphic in order and values, or we say that there is an isomorphism in order and values between these cyclic random processes, if the following properties occur:

1. Isomorphism with respect to the order between domains for determining cyclic random processes (isomorphism between ordered numerical sets $\mathbf{D}$ and $\mathbf{D}'$), namely:

1.a) there is a bijection between $\mathbf{D}$ and $\mathbf{D}'$ (denoted by: $\mathbf{D}\Leftrightarrow\mathbf{D}'$), that is, to any $t_{ml}\in\mathbf{D}$, corresponds to only one $t'_{ml}\in\mathbf{D}'$ ($t_{ml}\to t'_{ml}$), and to any $t'_{ml}\in\mathbf{D}'$ is responsible only one $t_{ml}\in\mathbf{D}$ ($t'_{ml}\to t_{ml}$), and for any different $t_{m_1l_1},t_{m_2l_2}\in\mathbf{D}$ their images $t'_{m_1l_1},t'_{m_2l_2}\in\mathbf{D}'$ are different, and vice versa (the corresponding elements $t_{ml}\in\mathbf{D}$ and $t'_{ml}\in\mathbf{D}'$, and we shall call them bijective related and denote it as follows: $t_{ml}\leftrightarrow t'_{ml}$);

1.b) the type of linear ordering of sets $\mathbf{D}$ and $\mathbf{D}'$ is preserved, that is $\forall t_{m_1l_1},t_{m_2l_2}\in\mathbf{D}$, $\exists t'_{m_1l_1},t'_{m_2l_2}\in\mathbf{D}'$ that $t'_{m_1l_1}\leftrightarrow t_{m_1l_1}$, $t'_{m_2l_2}\leftrightarrow t_{m_2l_2}$ and there is an order relation $t'_{m_2l_2}>t'_{m_1l_1}$, if $t_{m_2l_2}>t_{m_1l_1}$, and vice versa.

2. Isomorphism with respect to the order of cyclic random processes $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ and $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$, namely:

2.a) there is a bijection between random processes $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ and $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$ (denoted by: $\xi_1(\omega,t_{ml})\Leftrightarrow\xi_2(\omega,t'_{ml})$), that is, any pair $(t_{ml},\xi_1(\omega,t_{ml}))$ from a random process $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$, only one pair $(t'_{ml},\xi_2(\omega,t'_{ml}))$ ($(t_{ml},\xi_1(\omega,t_{ml}))\to(t'_{ml},\xi_2(\omega,t'_{ml}))$) corresponds to a random process $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$, and to any pair $(t'_{ml},\xi_2(\omega,t'_{ml}))$ from a random process $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$, only one pair $(t_{ml},\xi_1(\omega,t_{ml}))$ ($(t'_{ml},\xi_2(\omega,t'_{ml}))\to(t_{ml},\xi_1(\omega,t_{ml}))$) corresponds to a random process $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$, and for any of them different $t_{m_1l_1},t_{m_2l_2}\in\mathbf{D}$ their images $t'_{m_1l_1},t'_{m_2l_2}\in\mathbf{D}'$ are different, and vice versa (corresponding pairs $(t_{ml},\xi_1(\omega,t_{ml}))$ and $(t'_{ml},\xi_2(\omega,t'_{ml}))$ we will call them bijective related and denote it as: $(t_{ml},\xi_1(\omega,t_{ml}))\leftrightarrow(t'_{ml},\xi_2(\omega,t'_{ml}))$);

2.b) cyclic random processes $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ and $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$ are ordered by their domains of definition, and the ordinal types of random processes coincide with the ordinal types of their domains of definition $\mathbf{D}$ and $\mathbf{D}'$. That is, the set of pairs $\{(t_{ml},\xi_1(\omega,t_{ml})),\,t_{ml}\in\mathbf{D}\}$ that form (represent) a cyclic random process $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ is orderly by parameter t_{ml} and has the same order type with a numerical set $\mathbf{D}$, since there is always a bijective mapping of the domain of definition $\mathbf{D}$ to the random process $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ itself, namely the element $t_{ml}\in\mathbf{D}$ is matched by only one pair $(t_{ml},\xi_1(\omega,t_{ml}))$, and vice versa , and for two different $t_{m_1l_1},t_{m_2l_2}\in\mathbf{D}$ matching them pairs $(t_{m_1l_1},\xi_1(\omega,t_{m_1l_1}))$ and $(t_{m_2l_2},\xi_1(\omega,t_{m_2l_2}))$ also different. The same is true for a random process $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$, that is, a linear order from the domain of definition $\mathbf{D}'$ is induced into the random process $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$ itself;

2. c) there is the same type of ordering of cyclic random processes $\xi_1(\omega,t_{ml}),\,\omega\in\mathbf{\Omega},\,t_{ml}\in\mathbf{D}$ and $\xi_2(\omega,t'_{ml}),\,\omega\in\mathbf{\Omega},\,t'_{ml}\in\mathbf{D}'$, namely: for any different pairs $(t_{m_1l_1},\xi_1(\omega,t_{m_1l_1}))$ and $(t_{m_2l_2},\xi_1(\omega,t_{m_2l_2}))$ being in the bijective related with pairs $(t'_{m_1l_1},\xi_2(\omega,t'_{m_1l_1}))$ and $(t'_{m_2l_2},\xi_2(\omega,t'_{m_2l_2}))$ ($(t_{m_1l_1},\xi_1(\omega,t_{m_1l_1}))\leftrightarrow(t'_{m_1l_1},\xi_2(\omega,t'_{m_1l_1}))$, $(t_{m_2l_2},\xi_1(\omega,t_{m_2l_2}))\leftrightarrow(t'_{m_2l_2},\xi_2(\omega,t'_{m_2l_2}))$), the relations of order

$\left(t'_{m_2l_2}, \xi_2\left(\omega, t'_{m_2l_2}\right)\right) > \left(t'_{m_1l_1}, \xi_2\left(\omega, t'_{m_1l_1}\right)\right)$ and $\left(t_{m_2l_2}, \xi_1\left(\omega, t_{m_2l_2}\right)\right) > \left(t_{m_1l_1}, \xi_1\left(\omega, t_{m_1l_1}\right)\right)$ occur, if $t'_{m_2l_2} > t'_{m_1l_1}$ and $t_{m_2l_2} > t_{m_1l_1}$ ($t'_{m_1l_1} \leftrightarrow t_{m_1l_1}$, $t'_{m_2l_2} \leftrightarrow t_{m_2l_2}$);

3. Equality of values of cyclic random processes $\xi_1(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}), \omega \in \mathbf{\Omega}, t'_{ml} \in \mathbf{D}'$ takes place, when their respective arguments $t_{ml} \in \mathbf{D}$ and $t'_{ml} \in \mathbf{D}'$ being in the bijective related ($t_{ml} \leftrightarrow t'_{ml}$), namely, taking into account the property of cyclicity of these random processes, the following equations hold:

$$\mathbf{P}\left\{\xi_1\left(\omega, t_{ml} + T_1\left(t_{ml}, n\right)\right) = \xi_2\left(\omega, t'_{ml} + T_2\left(t'_{ml}, n\right)\right)\right\} = 1,$$
$$t_{ml} + T_1\left(t_{ml}, n\right) \leftrightarrow t'_{ml} + T_1\left(t'_{ml}, n\right), t_{ml} \in \mathbf{D}, t'_{ml} \in \mathbf{D}', n \in \mathbf{Z}. \tag{2}$$

For isomorphic with respect to the order and values of cyclic random processes $\xi_1(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}), \omega \in \mathbf{\Omega}, t'_{ml} \in \mathbf{D}'$ the equality of their (k-dimensional) distribution functions $F_{k_{\xi_1}}(x_1, ..., x_k, t_{m_1l_1}, ..., t_{m_kl_k})$ and $F_{k_{\xi_2}}(x_1, ..., x_k, t'_{m_1l_1}, ..., t'_{m_kl_k})$ holds, and when the corresponding sets of their arguments $t_i + T_1(t_i, n)$ and $t'_i + T_2(t'_i, n), i = \overline{1,k}, n \in \mathbf{Z}$ being in the bijective related, namely, the following relations exist:

$$F_{k_{\xi_1}}\left(x_1, \ldots, x_k, t_{m_1l_1} + T_1\left(t_{m_1l_1}, n\right), \ldots, t_{m_kl_k} + T_1\left(t_{m_kl_k}, n\right)\right) =$$
$$= F_{k_{\xi_2}}\left(x_1, \ldots, x_k, t'_{m_1l_1} + T_2\left(t'_{m_1l_1}, n\right), \ldots, t'_{m_kl_k} + T_2\left(t'_{m_kl_k}, n\right)\right),$$

$$x_i \in \mathbf{R}, t_{m_il_i} \in \mathbf{D}, t'_{m_il_i} \in \mathbf{D}', t_{m_il_i} + T_1\left(t_{m_il_i}, n\right) \leftrightarrow t'_{m_il_i} + T_2\left(t'_{m_il_i}, n\right), i = \overline{1,k}, k \in \mathbf{N}, n \in \mathbf{Z}. \tag{3}$$

Suppose we have a class $\mathbf{\Theta}$ of all possible cyclic random processes of a discrete argument given on a probabilistic space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ and on one of the sets of a class **SetOf_D** of all possible sets of type $\mathbf{D} = \left\{t_{ml} \in \mathbf{R}, m \in \mathbf{Z}, l = \overline{1,L}, L \geq 2\right\}$. On the set (class) $\mathbf{\Theta}$ we define an equivalence relation $\varphi \subset \mathbf{\Theta}^2$, namely, a binary relation $\varphi(\xi_1(\omega, t_{ml}), \xi_2(\omega, t'_{ml}))$, where $\xi_1(\omega, t_{ml}), \xi_2(\omega, t'_{ml})$ are arbitrary discrete cyclic random processes with $\mathbf{\Theta}$. The equivalence relation $\varphi \subset \mathbf{\Theta}^2$ can be defined in different ways, in particular, by postulating isomorphism with respect to the order and values between cyclic random processes with $\mathbf{\Theta}$.

Defining equivalence relation $\varphi \subset \mathbf{\Theta}^2$ in a class $\mathbf{\Theta}$ generates a breakdown $\mathbf{D_\Theta} = \{\mathbf{\Theta}_\beta, \beta \in \mathbf{B}\}$ a class $\mathbf{\Theta}$ into isomorphic subclasses in order and values of cyclic random processes of the discrete argument, that is, for the elements of partition $\mathbf{D_\Theta} = \{\mathbf{\Theta}_\beta, \beta \in \mathbf{B}\}$, the following ratios occur:

$$\bigcup_{\beta \in \mathbf{B}} \mathbf{\Theta}_\beta = \mathbf{\Theta},\ \mathbf{\Theta}_\beta \neq \varnothing,\ \mathbf{\Theta}_{\beta_1} \bigcap \mathbf{\Theta}_{\beta_2} = \varnothing,\ \beta_1 \neq \beta_2, \beta_1, \beta_2 \in \mathbf{B}. \tag{4}$$

We select one arbitrary element from the partition $\mathbf{D_\Theta} = \{\mathbf{\Theta}_\beta, \beta \in \mathbf{B}\}$ namely, some class of equivalence (isomorphism) $\mathbf{\Theta}_\xi \in \mathbf{D_\Theta}$ of cyclic random processes of a discrete argument. This class includes isomorphic discrete cyclic random processes, which are different in order and values, differing only in their rhythm functions, and the transition from one process to another can be ensured by the action of the corresponding scale transformation operator. Namely, two arbitrary cyclic random processes $\xi_1(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}), \omega \in \mathbf{\Omega}, t'_{ml} \in \mathbf{D}'$ with $\mathbf{\Theta}_\xi$ connected through direct $\mathbf{G}_{y_{12}}[\cdot]$ and inverted $\mathbf{G}_{y_{21}}[\cdot]$ scale conversion operators, namely:

$$\xi_2(\omega, t'_{ml}) = \mathbf{G}_{y_{12}}[\xi_1(\omega, t_{ml})], \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}, t'_{ml} \in \mathbf{D}', \tag{5}$$
$$\xi_1(\omega, t_{ml}) = \mathbf{G}_{y_{21}}[\xi_2(\omega, t'_{ml})], \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}, t'_{ml} \in \mathbf{D}'. \tag{6}$$

According to [3], the action of scale conversion operators $\mathbf{G}_{y_{12}}[\cdot]$ and $\mathbf{G}_{y_{21}}[\cdot]$ on cyclic random processes $\xi_1(\omega, t_{ml}), \omega \in \mathbf{\Omega}, t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}), \omega \in \mathbf{\Omega}, t'_{ml} \in \mathbf{D}'$ is fully defined (given) by its scale conversion functions $y_{12}(t_{ml}), t_{ml} \in \mathbf{D}$ and $y_{21}(t'_{ml}), t'_{ml} \in \mathbf{D}'$ which are increasing functions, as follows:

$$\xi_2(\omega, t'_{ml}) = \xi_2(\omega, y_{12}(t_{ml})) = \xi_1(\omega, t_{ml}),\ \omega \in \mathbf{\Omega},\ t_{ml} \in \mathbf{D},\ t'_{ml} \in \mathbf{D}'. \quad (7)$$

$$\xi_1(\omega, t_{ml}) = \xi_1(\omega, y_{21}(t'_{ml})) = \xi_2(\omega, t'_{ml}),\ \omega \in \mathbf{\Omega},\ t_{ml} \in \mathbf{D},\ t'_{ml} \in \mathbf{D}'. \quad (8)$$

According to work [3], between the functions of rhythm $T_1(t_{ml}, n)$ and $T_2(t'_{ml}, n)$ cyclic random processes $\xi_1(\omega, t_{ml}),\ \omega \in \mathbf{\Omega},\ t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}),\ \omega \in \mathbf{\Omega},\ t'_{ml} \in \mathbf{D}'$ of the equivalence classes $\mathbf{\Theta}_\xi$ that are linked through scale conversion operators $\mathbf{G}_{y_{12}}[\cdot]$ and $\mathbf{G}_{y_{21}}[\cdot]$ have the following dependencies:

$$T_2(y_{12}(t_{ml}), n) = y_{12}(t_{ml} + T_1(t_{ml}, n)) - y_{12}(t_{ml}),\ t_{ml} \in \mathbf{D},\ n \in \mathbf{Z} \quad (9)$$

$$T_1(y_{21}(t'_{ml}), n) = y_{21}(t'_{ml} + T_2(t'_{ml}, n)) - y_{21}(t'_{ml}),\ t'_{ml} \in \mathbf{D}',\ n \in \mathbf{Z} \quad (10)$$

Statistical analysis of any two cyclic random processes $\xi_1(\omega, t_{ml}),\ \omega \in \mathbf{\Omega},\ t_{ml} \in \mathbf{D}$ and $\xi_2(\omega, t'_{ml}),\ \omega \in \mathbf{\Omega},\ t'_{ml} \in \mathbf{D}'$ of the equivalence class $\mathbf{\Theta}_\xi \in \mathbf{D}_{\mathbf{\Theta}}$ by their two multi-cycle realizations $\xi_{1_\omega}(t_{ml})$ and $\xi_{2_\omega}(t'_{ml})$ should give approximate results, namely, similar statistical estimates of the probabilistic characteristics of the cyclic signals.

For the formal identification of a concrete discrete cyclic random process, the class $\mathbf{\Theta}_\xi$ submit as a set labeled with a parameter λ isomorphic in order and values of cyclic random processes $\mathbf{\Theta}_\xi = \{\xi_\lambda(\omega, t^\lambda_{ml}),\ \omega \in \mathbf{\Omega},\ t^\lambda_{ml} \in \mathbf{D}_\lambda,\ \lambda \in \mathbf{\Lambda}\}$ given on the same probabilistic space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$.

The parameter λ gains values from some set $\mathbf{\Lambda}$ ($\lambda \in \mathbf{\Lambda}$), whose power is equal to the power of the class $\mathbf{\Theta}_\xi$. Any, elements with $\mathbf{\Theta}_\xi$ differ only in the discrete area of their definition, and as a consequence, their discrete functions of rhythm. Actually the definition area $\mathbf{D}_\lambda = \left\{ t^\lambda_{ml} \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\}$ of any discrete cyclic random process $\xi_\lambda(\omega, t^\lambda_{ml}),\ \omega \in \mathbf{\Omega},\ t^\lambda_{ml} \in \mathbf{D}_\lambda$ with $\mathbf{\Theta}_\xi$ uniquely identifies (distinguishes, marks) it among other random processes from $\mathbf{\Theta}_\xi$. Also, the area of definition $\mathbf{D}_\lambda = \left\{ t^\lambda_{ml} \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\}$ of a particular λ-process $\xi_\lambda(\omega, t^\lambda_{ml}),\ \omega \in \mathbf{\Omega},\ t^\lambda_{ml} \in \mathbf{D}_\lambda$ from $\mathbf{\Theta}_\xi$ completely determines its rhythm function $T_\lambda(t^\lambda_{ml}, n)$, namely:

$$T_\lambda(t^\lambda_{ml}, n) = t^\lambda_{m+n,l} - t^\lambda_{ml},\ m, n \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2,\ t^\lambda_{ml} \in \mathbf{D}_\lambda \quad (11)$$

Thus, the set **SetOf_D** of all possible domains for determining isomorphic in order and values of cyclic random processes of a discrete argument from a class $\mathbf{\Theta}_\xi$ can also be represented as a parametric set, namely:

$$\mathbf{SetOf\_D} = \{\mathbf{D}_\lambda,\ \lambda \in \mathbf{\Lambda}\} = \left\{ \left\{ t^\lambda_{ml} \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\},\ \lambda \in \mathbf{\Lambda} \right\} \quad (12)$$

In view of the above considerations, there is a bijection between the equivalence class $\mathbf{\Theta}_\xi = \{\xi_\lambda(\omega, t^\lambda_{ml}),\ \omega \in \mathbf{\Omega},\ t^\lambda_{ml} \in \mathbf{D}_\lambda,\ \lambda \in \mathbf{\Lambda}\}$, the set of domains of definition **SetOf_D**, and the set of rhythm functions $\mathbf{SetOf\_T} = \{T_\lambda(t^\lambda_{ml}, n),\ t \in \mathbf{R},\ n \in \mathbf{Z}, \lambda \in \mathbf{\Lambda}\}$ isomorphic in order and values of discrete cyclic random processes with $\mathbf{\Theta}_\xi$, namely, $\mathbf{\Theta}_\xi \Leftrightarrow \mathbf{SetOf\_D} \Leftrightarrow \mathbf{SetOf\_T}$. With such ambiguous mappings between classes of functions, the bijective related elements, there will be those elements of sets $\mathbf{\Theta}_\xi$, **SetOf_D** and **SetOf_T** which have the same parameter λ, namely: $\xi_\lambda(\omega, t^\lambda_{ml}) \leftrightarrow \mathbf{D}_\lambda \leftrightarrow T_\lambda(t^\lambda_{ml}, n)$.

The above mathematical objects formally take into account the fact that the model of a cyclic signal, for example, an electrocardiogram signal is a cyclic random process, and also reflect the experimental fact that the estimates of the functions of a rhythm of an electrocardio signal on its different realizations are significantly different, however, the statistical characteristics of the signal on its various realizations are approximate. However, such a mathematical description is necessary to supplement the new probabilistic space, which will make it possible to describe the rhythm of the electrocardio signal as a random function of rhythm within the framework of the theory of random processes. Such a description will eliminate the existing contradiction between the descriptions of rhythm and morphological structure in existing models of cyclic signals and processes.

To do this, consider a stochastic experiment that is described by some probabilistic space $(\Omega', \mathrm{F}', \mathrm{P}')$ that is stochastically independent of $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$. We introduce a random object $\lambda(\omega') = \lambda, \omega' \in \Omega', \lambda \in \Lambda$ as a measuring function with definition area $\mathbf{\Omega}'$ and value range $\mathbf{\Lambda}$. In this case, ω'-implementation of a random object $\lambda(\omega')$ is a parameter λ that determines the corresponding cyclic random process $\xi_\lambda(\omega, t^{\lambda}_{ml}),\ \omega \in \mathbf{\Omega},\ t^{\lambda}_{ml} \in \mathbf{D}_\lambda$ with the definition area $\mathbf{D}_\lambda$ and the rhythm function $T_\lambda(t^{\lambda}_{ml}, n)$.

That is, you can enter the following three random objects that are given in the probabilistic space $(\Omega', \mathrm{F}', \mathrm{P}')$, namely: conditional cyclic random process of a discrete argument $\xi(\omega, t_{ml}(\omega')),\ \omega' \in \mathbf{\Omega}',\ \omega \in \mathbf{\Omega},\ t_{ml}(\omega') \in \mathbf{D}(\omega')$; the random discrete domain of its definition $\mathbf{D}(\omega') = \left\{ t_{ml}(\omega') \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\}$, given on a probabilistic space $(\Omega', \mathrm{F}', \mathrm{P}')$ and taking values from the set **SetOf_D**, and the random function of the rhythm $T(t_{ml}(\omega'), n)$, is a conditional cyclic random process of a discrete argument $\xi(\omega, t_{ml}(\omega'))$ that takes its values (deterministic functions of the rhythm) from the set **SetOf_T**.

We give a definition to these three probabilistic objects. First, we define the conditional cyclic random process of a discrete argument.

Definition 3. A random object $\{\xi(\omega, t_{ml}(\omega')) \in \mathbf{\Theta}_\xi,\ \omega' \in \mathbf{\Omega}',\ \omega \in \mathbf{\Omega},\ t_{ml}(\omega') \in \mathbf{D}(\omega')\}$ that is given on stochastically independent probabilistic spaces $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ and $(\Omega', \mathrm{F}', \mathrm{P}')$ is called a conditional cyclic random process of a discrete argument if, for each ω', its corresponding ω'-realization $\left\{\xi_{\omega'}(\omega, t^{\omega'}_{ml}),\ \omega \in \mathbf{\Omega},\ t^{\omega'}_{ml} \in \mathbf{D}_{\omega'}\right\}$ belongs to a class $\mathbf{\Theta}_\xi$ isomorphic in order and values of cyclic random processes of the discrete argument.

Definition 4. A random function $T(t_{ml}(\omega'), n),\ \omega' \in \mathbf{\Omega}',\ t_{ml}(\omega') \in \mathbf{R},\ n \in \mathbf{Z}$ given on a probabilistic space $(\Omega', \mathrm{F}', \mathrm{P}')$ is called a random rhythm function of a conditional cyclic random process of a discrete argument, if for each ω', corresponding to its ω'-realization $T_{\omega'}(t^{\omega'}_{ml}, n),\ t^{\omega'}_{ml} \in \mathbf{D}_{\omega'}$ belongs to the class **SetOf_T**, each element of which satisfies the conditions of the rhythm function, namely: 1) a group of conditions: 1.a) $T_{\omega'}(t^{\omega'}_{ml}, n) > 0$ if $n > 0$ ($T_{\omega'}(t^{\omega'}_{ml}, 1) < \infty$); 1.b) $T_{\omega'}(t^{\omega'}_{ml}, n) = 0$, if $n = 0$; 1.c) $T_{\omega'}(t^{\omega'}_{ml}, n) < 0$, if $n < 0$, $t^{\omega'}_{ml} \in \mathbf{D}_{\omega'} \subset \mathbf{R}$; 2) for any $t^{\omega'}_{m_1 l_1} \in \mathbf{D}_{\omega'}$ and $t^{\omega'}_{m_2 l_2} \in \mathbf{D}_{\omega'}$ for which $t^{\omega'}_{m_1 l_1} < t^{\omega'}_{m_2 l_2}$, for function $T_{\omega'}(t^{\omega'}_{ml}, n)$, strict inequality is fulfilled $T_{\omega'}(t^{\omega'}_{m_1 l_1}, n) + t^{\omega'}_{m_1 l_1} < T_{\omega'}(t^{\omega'}_{m_2 l_2}, n) + t^{\omega'}_{m_2 l_2}$, $\forall n \in \mathbf{Z}$; 3) the function $T_{\omega'}(t^{\omega'}_{ml}, n)$ is the smallest modulo ($|T_{\omega'}(t^{\omega'}_{ml}, n)| \leq |T^{\gamma}_{\omega'}(t^{\omega'}_{ml}, n)|$) of all such functions $\left\{T^{\gamma}_{\omega'}(t^{\omega'}_{ml}, n),\ \gamma \in \mathbf{\Gamma}\right\}$, which satisfy the conditions 1 and 2 above.

Definition 5. A random object $\mathbf{D}(\omega') = \left\{ t_{ml}(\omega') \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\}$ given on a probabilistic space $(\Omega', \mathrm{F}', \mathrm{P}')$ is called a random domain for determining a conditional cyclic random process of a discrete argument if, for each ω', its corresponding ω'-realization $\mathbf{D}_{\omega'} = \left\{ t^{\omega'}_{ml} \in \mathbf{R},\ m \in \mathbf{Z},\ l = \overline{1, L},\ L \geq 2 \right\}$ is a discrete subset of real numbers whose elements satisfy the following conditions: $t^{\omega'}_{m_1 l_1} < t^{\omega'}_{m_2 l_2}$ if $m_2 < m_1$, or if $m_2 = m_1$, and $l_2 < l_1$, in other cases $t^{\omega'}_{m_1 l_1} > t^{\omega'}_{m_2 l_2}$ ($m_2, m_1 \in \mathbf{Z}$, $l_2, l_1 = \overline{1, L}$, $0 < t^{\omega'}_{m,l+1} - t^{\omega'}_{ml} < \infty$).

Conditional cyclical random process $\{\xi(\omega, t_{ml}(\omega')), \omega' \in \mathbf{\Omega}', \omega \in \mathbf{\Omega}, t_{ml}(\omega') \in \mathbf{D}(\omega')\}$ of a discrete argument allows simultaneous consideration of both stochasticity of the signals taken into account in their statistical morphological analysis and stochasticity of the rhythmic structure of the investigated signal taken into account in the rhythm analysis.

Morphological statistical analysis is reduced to the statistical analysis of any of its ω'-realization $\left\{\xi_{\omega'}\left(\omega, t_{ml}^{\omega'}\right), \omega \in \mathbf{\Omega}, t_{ml}^{\omega'} \in \mathbf{D}_{\omega'}\right\}$ as a cyclic random process with a determined rhythm function $T_{\omega'}\left(t_{ml}^{\omega'}, n\right), t_{ml}^{\omega'} \in \mathbf{D}_{\omega'}$ in accordance with known methods of statistic processing of cyclic random processes [1-4]. In particular, such an analysis is reduced to a statistical evaluation of the stationary and stationary-related random processes $\left\{\varphi_l\left(\omega, t_{ml}^{\omega'}\right), \omega \in \mathbf{\Omega}, t_{ml}^{\omega'} \in \mathbf{D}_{\omega'}, m \in \mathbf{Z}, l = \overline{1, L}\right\}$ embedded in a cyclic random process L. Each such random stationary process is given on a discrete set $\mathbf{D}_{\omega'}^{l} = \left\{t_{ml}^{\omega'} \in \mathbf{R}, m \in \mathbf{Z}, l = const\right\}$ that is embedded in $\mathbf{D}_{\omega'}$ and describes (models) the l-th phase of the investigated electrocardio signal. The values of a stationary discrete stationary random process $\varphi_l\left(\omega, t_{ml}^{\omega'}\right), \omega \in \mathbf{\Omega}, t_{ml}^{\omega'} \in \mathbf{D}_{\omega'}, m \in \mathbf{Z}, l = const$ are defined as follows:

$$\varphi_l\left(\omega, t_{ml}^{\omega'}\right) = \xi_{\omega'}\left(\omega, t_{ml}^{\omega'}\right), \omega \in \mathbf{\Omega}, t_{ml}^{\omega'} \in \mathbf{D}_{\omega'}^{l}, m \in \mathbf{Z}, l = const \quad (13)$$

Heart rhythm analysis is reduced to a statistical analysis of elements of the random domain of determining $\mathbf{D}(\omega') = \left\{t_{ml}(\omega') \in \mathbf{R}, m \in \mathbf{Z}, l = \overline{1, L}, L \geq 2\right\}$ a conditional cyclic random process of a discrete argument $\{\xi(\omega, t_{ml}(\omega')), \omega' \in \mathbf{\Omega}', \omega \in \mathbf{\Omega}, t_{ml}(\omega') \in \mathbf{D}(\omega')\}$, or to a statistical analysis of its random rhythm function $T(t_{ml}(\omega'), n), \omega' \in \mathbf{\Omega}', t_{ml}(\omega') \in \mathbf{R}, n \in \mathbf{Z}$. The random rhythm function $T(t_{ml}(\omega'), n)$ is completely determined by the elements of the random area $\mathbf{D}(\omega')$ according to the formula:

$$T(t_{ml}(\omega'), n) = t_{m+n,l}(\omega') - t_{m,l}(\omega'), m, n \in \mathbf{Z}, l = \overline{1, L}, t_{m,l}(\omega') \in \mathbf{D}(\omega') \quad (14)$$

In particular, when $n = 1$, the rhythm function $T(t_{ml}(\omega'), 1)$ is calculated as follows:

$$T(t_{ml}(\omega'), 1) = t_{m+1,l}(\omega') - t_{m,l}(\omega'), m \in \mathbf{Z}, l = \overline{1, L}, t_{m,l}(\omega') \in \mathbf{D}(\omega'). \quad (15)$$

If in the heart rhythm analysis we take the random function of the rhythm $T(t_{ml}(\omega'), 1)$ of a conditional cyclic random process $\xi(\omega, t_{ml}(\omega'))$, keeping a clear relation to the heart cycle phase and cardiac cycle number, then we present the mathematical model of the rhythmocardio signal as a vector of random sequences:

$$\mathbf{V}_L(\omega', m) = \left\{\Delta T_l(\omega', m), \omega' \in \mathbf{\Omega}', l = \overline{1, L}, m \in \mathbf{Z}\right\}, \quad (16)$$

where each l-th component of the vector is a random sequence $\Delta T_l(\omega', m)$, whose value is equal to the value of a random rhythm function $T(t_{ml}(\omega'), 1)$ at times $t_{ml}(\omega')$ from a discrete set $\mathbf{D}_l(\omega') = \{t_{ml}(\omega') \in \mathbf{R}, m \in \mathbf{Z}, l = const\}$ that is embedded in $\mathbf{D}(\omega')$ and describes the time distances between the same type l-th phases of the investigated electrocardio signal in its two adjacent cycles, namely:

$$\Delta T_l(\omega', m) = T(t_{ml}(\omega'), 1) = t_{m+1,l}(\omega') - t_{m,l}(\omega'),$$
$$m \in \mathbf{Z}, l = \overline{1, L}, t_{m,l}(\omega') \in \mathbf{D}(\omega'). \quad (17)$$

6. Conclusions

The procedure of mathematical model of cyclic digital signals in the form of conditional cyclic random process of discrete argument is developed in the work. The class of isomorphic in order and values of cyclic random processes of a discrete argument is given. This model allows to take into account stochasticity of cyclic signals both in their morphological statistical analysis and in statistical analysis of their rhythm. The definition of a random rhythm function of a conditional cyclic random process of a discrete argument is given.

The developed model eliminates the logical contradiction between the known probabilistic mathematical models of digital cyclic signals and their rhythm models and underpins the development of new highly informative methods of statistical anaysis of cyclic signals. Based on the new mathematical model in future research, it is necessary to clarify the probabilistic structure of cyclic signals with double stochaticity, which will allow the use of new diagnostic features in computer diagnostic systems.

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