

# Intellectualisation of Decision Support Systems for Computer Networks: Production-Logical F-Inference

Yurii Kravchenko<sup>a</sup>, Olesia Afanasyeva<sup>b</sup>, Maksym Tyshchenko<sup>c</sup> and Serhii Mykus<sup>c</sup>

<sup>a</sup> Taras Shevchenko National University of Kyiv, 60 Volodymyrska Street, Kyiv, 01033, Ukraine

<sup>b</sup> Pedagogical University of Krakow, 2 Podchorazych Street, Krakow, 30084, Poland

<sup>c</sup> National Defence University of Ukraine named after Ivan Cherniakhovskyi, 28 Povitroflotskyi Avenue, Kyiv, 03049, Ukraine

## Abstract

The article presents the models of production-logical inference based on algebraic structures. The conducted researches of abstract algebra and production inference models have established the principle of decision-making support in the development and operation of telecommunication systems based on the so-called FS-system. Mathematically, the FS-system is a partial case of algebraic lattices. It is proven that the FS-system, as an abstract structure, with the product-logical properties of its signature, has the advantages of the efficiency of computational implementation over other models. It was established experimentally that such an approach ensures minimization of appeals to external sources and, accordingly, reduction in computing costs. In particular, it was determined that the practical relevance of the findings is that the scientifically substantiated recommendations for an advanced communication management system are designed to consider the possibility of leveraging them in other complex technical systems. Such an approach allows to significantly increase the efficiency of the communication and management process gained by the modernization of hardware and software of the computer system.

Thus, there are grounds to state that it is expedient to utilize the proposed set of mathematical models of the formal description of the production-logical inference of knowledge to automating the decision-making support in telecommunication systems at the design and operation stages.

## Keywords <sup>1</sup>

algebraic system, FS-system, relation, distributivity, production-logical inference.

## 1. Introduction

Studies have shown that at present, the general scientific and applied problem of increasing the efficiency of telecommunication systems becomes of particular importance. An important direction of its solution is the creation and implementation of an appropriate intellectual decision support system (DSS). This system solves various tasks at the design and operation stages. According to experts, this approach will increase the efficiency of telecommunication systems to 15% at no additional cost [1-3]. It should be emphasized that there an advanced way to improve the quality of complex systems is to provide the system with the functional stability that allows to achieve the greatest effect from the complexing. Functional stability of a complex technical system combines the properties of reliability (fail-safe), fault tolerance and survivability. The stability is achieved by application of various existing types of redundancy (hardware, structural, temporal, information, functional, loading, etc.) in a complex system through redistribution of resources. It is fundamental

---

*IT&I-2020 Information Technology and Interactions, December 02–03, 2020, KNU Taras Shevchenko, Kyiv, Ukraine*

EMAIL: kr34@ukr.net (Y. Kravchenko); olesia.afanasyeva@gmail.com (O. Afanasieva); tishenkom1@gmail.com (M. Tyshchenko); serg.mikus@gmail.com (S. Mykus)

ORCID: 0000-0002-0281-4396 (Y. Kravchenko); 0000-0003-1688-9011 (O. Afanasieva); 0000-0003-1266-4106 (M. Tyshchenko); 0000-0002-7103-4166 (S. Mykus)



© 2020 Copyright for this paper by its authors.

Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0).

CEUR Workshop Proceedings (CEUR-WS.org)

that at the design stage no additional redundancy should be introduced and the consequences of emergency situations should be parried by redistribution of already existing resources. So, the priority task of the intellectualized DSS is to support the telecommunication system Functional stability [1, 2, 4, 5].

## 2. Analysis of literary data and problem statement

An analysis of the latest research and publications in which the solution to this problem was initiated allows us to emphasize the positive tendency of the DSS development [1-17] and the need for further research to improve the mathematical and software systems of the systems. The most complex and relevant is the scientific task of developing new mathematical models of the formal description of knowledge, namely, production-logical inference in the automation of the decision support process [18-25]. The given models have well-known advantages: simplicity without loss of completeness due to modularity; efficiency and lack of complexity explaining the inference, which is associated with the presence of analogy with the human cognitive process.

So to ensure functional stability it is necessary to have redundancy. The works [1-4] published data on redundancy in the telecommunication system (table 1).

**Table 1**  
Redundancy analysis in the communication system

Subsystems	Redundancy				
	Hardware	Temporal	Software	Informational	Structural
Topology	0,15-0,45	0,2-0,4	0,1-0,25	0,1-0,5	0,2-0,4
Communication lines	0,2-0,25	0,2-0,4	0,2-0,5	0,1-0,5	0,2-0,25
Communication nodes	0,2	0,1-0,2	0,1-0,2	0,2-0,5	0,1-0,2
Control	0,05- 0,1	0,1-0,2	0,1-0,2	0,1- 0,3	0,1-0,15

## 3. Research goal and tasks

The research was aimed at substantiating the new approach in the production models of knowledge extraction, which, unlike existing ones, uses the advantages of algebraic systems in computer programs and provides the necessary quality and efficiency with a significant reduction of computational resources.

To achieve this goal, the following tasks were solved:  
 Developing of the corresponding mathematical models.  
 Computer simulations.

## 4. Productive-logical inference models

Intellectualized DSS implements the so-called productive-logical model of knowledge representation. The rationale for this is as follows. In fact, nowadays a sufficient number of models of knowledge representation, used for implementation of knowledge-based systems, have been developed. It is most expedient to represent knowledge by means of rules of the "if-then" type (phenomenon - reaction). The production system can be considered the most widespread model of

knowledge representation. It is possible to provide many examples of real knowledge-based systems, as well as examples of systems where the language of frame-type knowledge representation was adopted to describe the local knowledge [7–9, 11, 12]. If the product system is considered as a model of knowledge representation, then the rules can be added a clear meaning if from the human point of view they are seen as the means of direct description of the method of logical conclusion for addressing issues in the subject area. The distinctive feature of knowledge representation with high modularity is simplicity of extension, modification and cancellation. In addition, on the computer side, it is possible to define a simple and precise mechanism for using knowledge with high homogeneity. These two distinctive features are the reasons why the knowledge representation method is so widespread. The concept is supposed to consider the basic structures of the product systems and various aspects related to the implementation of knowledge-based systems.

As you know, a production system consists of three main components: a set of rules used as a knowledge base, known as a rule base; working memory (or short-term memory), which stores the prerequisites related to the specific tasks of the subject area, and output results; inference engine, that applies the rules according to the content of working memory. In reality, various additional tools are needed to build working (operating) systems [8, 9]. The major issue in the product systems is to investigate (check) the availability of special data concerning the conditional part of the rule in the working memory. Evaluation of the conditional part by searching and comparing has a wide scope of application, in some respects, but in certain instances such direct evaluation is insufficient.

In a real-case scenario, when using product systems these rules are extended as needed (e.g., they use OR connection in the conditional part, enter the conditional part with calculations based on the working memory content and other operations or enter the final part, that indicates the output without the working memory content), so the rules can be considered classical.

Such rules represent the output result established between the content of the working memory, which is referenced from the conditional part, and the content specified in the final part [18].

Process description of the logical output results allows to identify a false rule when a proof of conclusion failed, despite the fact that the opposite should have happened, or an incorrect output was obtained. It is an effective tool for systems development as it allows to explore their possibilities and identify how to expand them.

It is known that an algebraic system  $\langle F_F, \Omega_F, \Omega_R \rangle$  (in some literature, an algebraic structure) is a non-empty set  $F_F \neq \emptyset$  with a given set of operations  $\Omega_F$  and relations  $\Omega_R$  satisfying some axioms. A set  $F_F$  is called the carrier of the algebraic system; sets  $\langle \Omega_F, \Omega_R \rangle$  are called the signature of an algebraic system.

The main task of abstract algebra (in modern scientific literature is simply algebra) as a section of mathematics is the study of the properties of axiomatically given algebraic systems. The term "abstract" emphasizes that objects of study are abstract structures, such as groups, rings, fields, and modules, in contrast to algebraic expressions that are studied in elementary algebra [18].

The general idea in forming the inference models is to provide the signature of the algebraic system of production-logical properties of the general scheme "if... then". The article proposes to consider the original algebraic system, the so-called FS-system, developed and proposed as a theoretical justification for the mathematical support of the telecommunication system's DSS. The abbreviation FS comes from "Functional Stability". It is the FS system that is part of the mathematical support for the formation of a property such as functional stability. In any abnormal situation associated with negative external and internal factors, the property of functional stability allows a complicated system to continue to be in a workable state due to the use of redundancy.

For a logical and understandable further introduction, we will introduce the basic notation and definition. Let  $R$  be a binary relation on a certain set  $F$ , which is transitive and reflexive. It is known that in discrete mathematics, for partially ordered sets there is the notion of a "minimal element" (for it, there is no smaller element) and "the smallest element" (it is least of all). The classic terms of "closure" and "reduction" are interpreted in a procedure for minimizing computational resources in the automation of logical programming, namely minimization of memory and the number of operations.

It is also known that there is a loop  $R^*$  of arbitrary relation to the transitivity property, or the transitive closure. The transitive reduction  $R'$  of the ratio  $R$  means the minimum ratio is such that its

transitive closure coincides with the transient closure of  $R$ . The literature [17] presents algorithms for constructing a transitive reduction of oriented graphs. This is an arbitrary graph with the minimum possible number of nodes, the transitive closure of which coincides with the transitive closure of this graph. It is proved that this problem is a computational equivalent to the construction of the transitive closure, and also the unity of the transitive reduction of the acyclic graph is established. Also, in [18] it is proved that the construction of the least transitive reduction of an arbitrary graph is an NP-complete problem.

**Definition 1.** FS-system  $\langle F_F, \Omega_F, \Omega_R \rangle$  is a system of algebra (algebraic structure, algebraic system), in which, along with operations over any pair of its elements ( $\wedge$  ("intersection") and  $\vee$  ("union")) an additional binary relation is given with the production-logical properties.

The relation is called production-logical if it has reflectivity, transitivity, and other properties that are determined by a particular model. One of these properties is distributivity. Informally, distributivity refers to the possibility of logical inference in parts and to combine its results on the basis of operations  $\wedge$  and  $\vee$ . Assigned to the FS-system, the relation  $R$  is called  $\wedge$ -distributive, if  $(A, B), (A, B_1), (A, B_2) \in R$  leads to  $(A, B_1 \wedge B_2) \in R$ . And called  $\vee$ -distributive, if  $(A, B), (A_1, B) \in R$  leads to  $(A_1 \vee A_2, B) \in R$ . The relation is called distributive if both of these properties are specified. In order to simplify the symbols  $\leq, \geq, \text{and}, \neg$ , the theorems of plural operations  $\subseteq, \supseteq, \cap, \text{and } \cup$  are used instead, and the elements of the FS system are indicated by uppercase letters. Also, for simplicity, the assumption is made that it is advisable to refer only to one of the two properties mentioned above under the distributive relationship.

Let the FS-system have some binary relation  $R \in \Omega_R$ . The set of all atoms of the FS system, the relationship pairs  $R$  contained in the elements, will be called the set of atoms that the relation  $R$  relies on. Constructed on the associations of these atoms subset of the initial FS-systems we denote  $F_F, F_R$ . Obviously,  $F_R$  is a subsystem (subset) in  $F$ .

For a given  $R$  we introduce a binary relation  $\supseteq_R$ , a subset of partial order  $\supseteq$  that the elements of all pairs  $\supseteq_R$  belong to  $F_R$ . We also denote  $\supset_R$ . A subset  $\supseteq_R$  that does not contain pairs of reflections (type  $(A, A)$ )

**Definition 2.** The binary relation  $R$  in the FS system is called production-logical, or logical, if it contains  $\supseteq_R$  and also is distributive and transitive.

Thus, in the FS-system, along with operations on any pair of its elements, a rule is given for their logical, so-called "comparison" or inference. The mathematical formalization of this rule allows us to develop management algorithms for automating the decision support process in a telecommunication system. The purpose of this management is to respond to non-emergency situations by redistributing redundancy.

We propose a logical rule of the so-called "comparison". First, we draw attention to the fact that the closure of a set is a collection of all points of a given set. The closure of the set generally contains three types of points: isolated points; boundary points belonging to this set; boundary points that do not belong to this plurality themselves. Thus, the closure is the adherence to this set of all its boundary points. A closed set coincides with its closure.

The point of touch of a set is such a point (not necessarily belonging to the given set itself), any neighborhood of which contains at least one point from this set. It turns out that if the point belongs to this set, then it is already a point of contact (since it itself is always included in its own neighborhood, no matter how small this suburb was). We recall that in the theory of sets, the transitive closure of a binary relation on a plural is the least transitive relation on the set that includes it. In mathematics, the set is closed with respect to some operation, if the result of this operation on elements of the set will always be the element of this set.

In the theory of FS-systems, as in the theory of ordinary sets, it is expedient to consider the transitive closure and the reduction of the relation. Therefore, the task of finding a logical circuit and logically reducing relations on hierarchical sets, or different types of algebraic systems, is relevant.

**Definition 3.** Let  $R$  relation be assigned to the FS-system  $F$ , then the ordered pair  $A, B \in F$  is logically bound by the  $R$  relationship (written as  $A \xrightarrow{R} B$ ) if one of the following conditions is true:

$$A, B \in R; \quad (1)$$

$$A \supseteq_R B; \quad (2)$$

$$\text{there are such } B_1, B_2 \in F, B_1 \cup B_2 = B, \text{ that } A \xrightarrow{R} B_1, A \xrightarrow{R} B_2; \quad (3)$$

$$\text{there is an element } C \in F \text{ such that } A \xrightarrow{R} C \text{ and } C \xrightarrow{R} B. \quad (4)$$

This definition 3 on the given  $R$  forms another binary relation  $\xrightarrow{R}$  on the FS system  $F$ . As can be seen from this definition, any logical bond  $A \xrightarrow{R} B$  is formed on the basis of a finite subset of the pairs of relations  $F$ . It is also worth noting that the relations  $R$  and  $\xrightarrow{R}$  make sense for all elements of the FS system.

Definition 3 is the inference rules of the FS system. When inferring a logical connection  $A \xrightarrow{R} B$ , the inference step will be called the usage of exactly one rule, possibly simultaneously to some finite set of elements of the FS system. Formally this is as follows:

$$\text{if } A_i \xrightarrow{R} B_i^1, A_i \xrightarrow{R} B_i^2, i = 1, \dots, n, \text{ then } A_i \xrightarrow{R} B_i^1 \cup B_i^2, i = 1, \dots, n;$$

$$\text{or } A_i \xrightarrow{R} C_i, C_i \xrightarrow{R} B_i, i = 1 \dots n, \text{ then } A_i \xrightarrow{R} B_i, i = 1, \dots, n.$$

The level of recursion in the ratio  $A \xrightarrow{R} B$  is called the number of inference steps needed to get this connection. In addition, only usage of the rules given in definition 3 is taken into account.

Since in the general case the connection  $A \xrightarrow{R} B$  can be obtained not by a single set of rules, then it is expedient only to evaluate its maximum recursion level without specifying its exact value. With regard to the steps of withdrawal of the ratio  $A \xrightarrow{R} B$  we will use the terms "initial", "last", "next" and so on. This implies the progress in the direction of direct logical inference, that is, from the pairs of initial relation  $R \cup \supseteq_R$  to the pair of relations  $\xrightarrow{R}$ .

**Definition 4.** Let  $R$  be a logical relation to  $F$  and  $A, B \in F$ . Then, if  $A \xrightarrow{R} B$  is true, then  $(A, B) \in R$ .

The justification for this is done by induction by  $m$ , the upper estimate of the recursion rate in the relation  $A \xrightarrow{R} B$ . For  $m=0$ , one of the conditions (1) – (2) of definition 3 takes place. Case (1) directly indicates the necessary statement. If (2) is true, then  $A, B \in F$  as well, since the logical relation  $R$ , by definition, contains such pairs.

Let us assume further that the definition is true for some  $m \geq 0$ , and prove its assertion at the level  $m+1$ . In this case, new options for consideration may be given by rules (3) – (4).

If (3) occurs, then, by induction, the level of recursion in the corresponding ratios  $A \xrightarrow{R} B_1$ ,  $A \xrightarrow{R} B_2$  does not exceed  $m$ , therefore  $A, B_1 \in R$ ,  $A, B_2 \in R$ . Then, through the property of the distributivity of the logical relation  $R$ , we obtain  $(A, B) \in R$ . The option of the origin of  $A \xrightarrow{R} B$  connection from condition 4) is considered similarly.

It is not difficult to conclude that there is a logical closure for an arbitrary binary relation  $R$  in the FS system and it coincides with the set  $\xrightarrow{R}$  of all pairs logically related by the relation  $R$ .

The production-logical relations are the mathematical basis for solving applied problems associated with the automation of logical inference and other aspects of logical programming. In connection with this circumstance, there are questions of the automatic transformation of relations, in which the logic closure remains unchanged. Such transformations can be used, for example, to bring the knowledge base to the form which is convenient for research and implementation. The next reasonable step is to introduce the concept of equivalent relations.

Two relations  $R_1, R_2$  are called equivalent if their logic matches are the same. For such relations, we use the notation  $R_1 \approx R_2$ . An equivalent transformation of this relation is called a certain replacement of a subset of its pairs, which leads to an equivalent relation.

Therefore, the following is true: let  $R$  be the relation in FS-system and  $A_j \xrightarrow{R} B_j, j = 1, \dots, m$ . Then the relation  $R' = R \cup \{(A_j, B_j) | j = 1, \dots, m\}$  is equivalent to  $R$ .

Proof. Let us first notice that relations  $R$  and  $R'$  operate with the general large number of atoms of the FS system, hence  $\supseteq_R = \supseteq_{R'}$ . Further, by definition, any logical relation is its own logical closure. This conclusion also applies to the relation  $\xrightarrow{R}$ . Further, it follows that for any  $R_1 \subseteq R_2$  the following is true:  $\xrightarrow{R_1} \subseteq \xrightarrow{R_2}$ . In our case of the construction of the relation  $R'$  inclusions  $R \subseteq R' \subseteq \xrightarrow{R}$  are made.

Turning to the logical closures and taking into account what is said above, we find that relations  $R$  and  $R'$  have a common logical closure  $\xrightarrow{R}$ .

Let us consider the logical reduction model. It is known that in the case of software implementation of information technology DSS it is appropriate to avoid unnecessary duplication of code or data. In applications of logic systems, there is also the question of their equivalent minimization. In mathematical logic, the minimal system of axioms is called the basis. Problems of the existence of rules of bases for different logic were considered in [18]. The theory of artificial intelligence emphasizes the fact of additional possibilities of minimization in systems of production type.

Since the FS system is a mathematical model of the algebra of the production system, the obvious solution to the minimization problem is to provide a binary representation, when only a unique part of the information about this relation is stored in the computer memory, and other information can be obtained from the general properties of logical relations. By the unique part a relation  $R_0 \subset R$  is meant which is selected according to certain criteria of relations, from which  $R$  turns out as a logical closure.

Next, let us turn to the method of reduction. We should recall that the method of reduction is aimed at reducing from complex to simpler. The logical reduction of the relation  $R$  in the FS system is called its equivalent minimum relation  $R_0$ . Minimality in this definition is understood in the sense of partially ordered large quantities. Firstly, for this  $R$  the logical reduction  $R_0$  may not be the only one. Secondly, after being excluded from  $R_0$  at least one pair, the resulting relation is not equivalent to  $R$ .

On the basis of studies of other algebraic systems, it is expedient to conclude that these results can be transferred to the FS-system as well: namely, that the logical closure of this relation  $R$  is a transitive closure of some other relation  $\overset{\square}{R} \supseteq R$ . This result is useful for developing efficient algorithms for constructing logic closures and reductions. It allows us to reduce the study of questions of locating a logical closure and reduction of the order relations to considering the corresponding properties of transitive relations.

For an arbitrary binary relation  $R$  in the FS system, let us consider the relation  $\overset{\square}{R}$  constructed from  $R$  by the sequential implementation of the following steps:

- 1) add to  $R$  all pairs of the type  $(A, A)$ , where  $(A \in F_R)$ , and indicate the new relation  $R_1$ ;
- 2) add to  $R_1$  all pairs of type  $(A, B)$  with elements of the type  $A = \bigcup_i A_i$ ,  $B = \bigcup_i B_i$ , where all  $(A_i, B_i)$  ( $i = 1, \dots, n$ ) belong to  $R_1$ ;
- 3) combine the obtained relation with the inclusion relation  $\supseteq_R$ .

From the previous material of the article it turns out that the relation  $\overset{\square}{R}$  is equivalent to  $R$ . This is proved as follows. Let  $R$  be a binary relation in the FS system. Then, if  $A \xrightarrow{R} B$  then  $(A, B) \in \overset{\square}{R}$ . Also let  $A \xrightarrow{R} B$  take place. If the indicated relationship occurred directly from the condition of production-logical properties, then we have  $(A, B) \in \overset{\square}{R}$  at once.

We still have to consider the case of applying step 3). All the reflexive pairs necessary for this can be prepared at the very beginning of the inference process. In this case, pairs of type  $A \supseteq_R B$  will not be required. So step 3) can only finish this process.

If we compare the specified 2 stages of the inference with the sequence of construction of the relation  $\overset{\square}{R}$ , then it turns out that the inference  $A \xrightarrow{R} B$  corresponds to the construction of a certain subset  $\overset{\square}{R}$ , which proves the inclusion  $(A, B) \in \overset{\square}{R}$ .

As was noted, the relation  $\overset{\square}{R}$  is equivalent to  $R$ . Consequently, by definition of logical closure, we have  $\overset{\square}{R} \subseteq \xrightarrow{R}$ . Hence, since the relation  $\xrightarrow{R}$  is transitive, we obtain  $\overset{\square}{R} \subseteq \xrightarrow{R}$ . Thus, for an arbitrary relation  $R$  in the FS-system, the logical closure  $\xrightarrow{R}$  is the transitive closure  $\overset{\square}{R}$  of the corresponding relation  $R$ .

The next conclusion is that if  $R_1 \subseteq R_2$ , then  $\xrightarrow{R_1} \subseteq \xrightarrow{R_2}$ .

*Proof.* Firstly, from the description of the  $\overline{R}$  construction it is easy to see that if  $R_1 \subseteq R_2$ , then  $\overline{R_1} \subseteq \overline{R_2}$ . A similar statement holds for transitive closures of these relations, which means a proven fact. Next, for an arbitrary relation  $R$ , let us consider the relation  $\overline{R}$  built from this  $R$  by the consistent implementation of the steps inverse to the  $\overline{R}$  construction, namely:

- 1) to exclude from  $R$  the pairs of type contained in it  $A \supset_R B$  and to designate the new relation  $R_{-1}$ ;
  - 2) exclude from  $R_{-1}$  all pairs  $(A, B)$  with elements of the type  $A = \bigcup_i A_i$ ,  $B = \bigcup_i B_i$ , where all  $(A_i, B_i)$  ( $i = 1, \dots, n$ ) belong to  $R_{-1}$  and do not coincide with  $(A, B)$ ;
  - 3) to exclude all couples of reflections from the obtained relation.
- Conclusion: the relation  $\overline{R}$  is equivalent to  $R$ .

It should be noted that this approach to constructing a transitive reduction – "to remove all transitive pairs" – would be erroneous. The reason is that in some situations (the presence of cycles), the transitive reduction is not unique, and the simultaneous removal of all available transitive pairs can lead to loss of connections. However, since the FS system itself is acyclic, removing all of the above "coupled" pairs  $(A, B)$  leads to the same result, regardless of the order of deletion. For this reason, we can talk about the simultaneous removal of all such pairs.

Let  $R$  be a binary relation in the FS system. In order for  $R$  to be a logical reduction, it is necessary and sufficient that  $R$  does not contain any such pair  $(A, B)$  that the correlation  $A \xrightarrow{R \setminus \{(A, B)\}} B$  is executed.

This is proved as follows. Let the relation  $R$  be a logical reduction, that is, it is a minimal logical relation equivalent to itself. If there existed a pair  $(A, B) \in R$ , logically connected by the relation  $R \setminus \{(A, B)\}$ , then it could be excluded from  $R$ , with obtaining a lesser equivalent relation. Thus, in the presence of the specified pair the relation  $R$  cannot be a logical reduction.

To prove the opposite statement we assume that there is no pair  $(A, B) \in R$  for which  $A \xrightarrow{R \setminus \{(A, B)\}} B$  is true. It is necessary to prove that in this case  $R$  is a logical reduction. Suppose the inverse: let there be a relation  $R_0 \subset R$  equivalent to  $R$  and moreover,  $(A, B) \in R \setminus R_0$ . Then, since  $(A, B) \in R$ , because of the equivalence of these relations  $A \xrightarrow{R_0} B$  is valid. Since the relation  $R_0$  does not contain a pair  $(A, B)$ , then  $R_0 \subseteq R \setminus \{(A, B)\}$ , and the logical connection  $A \xrightarrow{R_0} B$  contradicts the assumption made - such pairs  $(A, B)$  are absent in  $R$ . The resulting contradiction proves the necessary statement.

Let us formulate a sufficient condition of existence and the method of constructing the logical reduction of this relation.

For the binary relation  $R$  given in the FS system, let there be a corresponding constructed relation  $\overline{R}$ . Then, if for  $\overline{R}$  there is a transitive reduction  $R^0$ , then according to it the relation  $R^0$  is a logical reduction of the initial relation  $R$ .

This is justified by the fact that from the construction of  $\overline{R}$  and  $\overline{R}$  it follows that the relation  $R^0$  is logical and equivalent to  $R$ . It remains to show that  $R^0$  is a logical reduction. Let  $(A, B)$  be an arbitrary pair of the relation  $R^0$ . It is necessary to show that a logical connection  $A \xrightarrow{R^0 \setminus \{(A, B)\}} B$  is impossible. Let's assume the opposite, namely, that this connection exists. Then the relation  $R^0 \setminus \{(A, B)\}$  is equivalent to  $R^0$ . Note that the use of the condition  $(A, B) \in R$  for inference  $A \xrightarrow{R^0 \setminus \{(A, B)\}} B$  is impossible, since the pair  $(A, B)$  is not contained in the set  $R^0 \setminus \{(A, B)\}$ .

Any logical connection can be constructed in such a way that all applications of the transitive rule will be made only in the final stage of its inference. This fact means that there is a chain of elements  $A = C_0, C_1, \dots, C_N = B$  such that the executed relations  $C_{k-1} \xrightarrow{R^0 \setminus \{(A, B)\}} C_k$ ,  $k = 1, \dots, N$ , when inferring each of them,

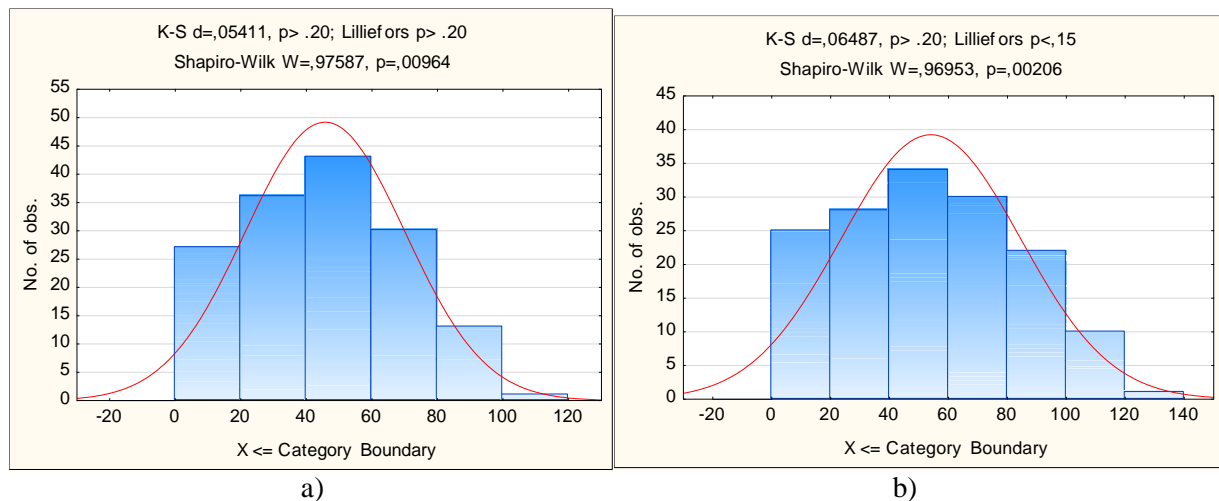
as a rule, there is an element  $C \in F$  such that  $A \xrightarrow{R} C$  and  $C \xrightarrow{R} B$  do not apply. Hence, according to Lemma 1, we have  $(C_{k-1}, C_k) \in R$ . Thus, with  $N > 1$ , the pair  $(A, B)$  is transitive in  $R$ . Consequently, it cannot be contained in  $R^0$ , that is a subset of a transitive reduction relation  $R$ . The result is a contradiction to the initial assumption  $(A, B) \in R^0$ .

Thus, potentially possible situations for the intended logical conclusion  $A \xrightarrow{R^0 \setminus \{(A, B)\}} B$  are investigated. As a result, it was found that in each such case, the presence of the connection  $A \xrightarrow{R^0 \setminus \{(A, B)\}} B$  contradicts the fact  $(A, B) \in R^0$ . Consequently, the relation  $R^0$  is a logical reduction.

## 5. Simulation results

In order to substantiate the effectiveness of the proposed models used in the corresponding algorithms of the so-called F-inference, 2100 automated tests were conducted with the knowledge base of DSS, control and management of the Zabbix communication system. Test tasks are generated randomly with the ability to control the depth of inference, the number of generated rules and objects.

The described experiments allow to determine the number of requests to external sources of information, depending on the volume of initial facts and rules. Obtained results are compared by the number of external requests with the procedures of normal reverse inference, F-inference and its modifications. Experiment results were processed in the Statistica 13.3 package. The treatment of the results of the experiment is carried out by checking statistical hypotheses (Figure 1).

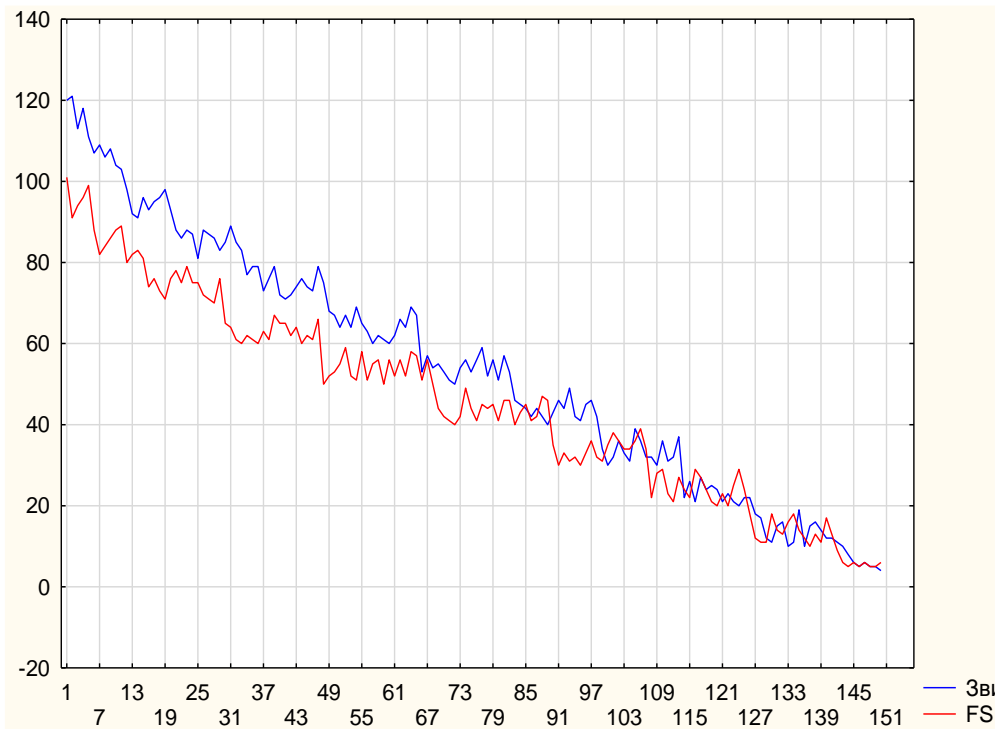


**Figure 1:** Checking the normalcy of distribution while substantiating the construction of a telecommunication network: a) usual inference; b) F-inference

A quantitative estimate of the increase in efficiency based on the ratio of the average number of external requests indicates their decrease (increase in efficiency) from 10.6% when substantiating the dynamic routing option to 23.8% in substantiating the construction of the core of the reference network (Figure 2). On average, by using the F-inference method, you can state the efficiency increase by 13.5%.

The practical application of the decision support system based on the use of a model example allowed to form a number of recommendations for building a functionally stable communication system. These recommendations are based on the thesis that the main direction of improving the field reference network of the communication system is to create a single information and telecommunications environment using the latest digital communications based on the information and telecommunications system. The obtained results emphasize the prospects of the chosen direction of research for the creation of information technologies for the synthesis of intelligent systems based on products, as well as in the development and improvement of other types of complex information systems designed to ensure their functional stability in a wide range of applications.





**Figure 2:** Number of requests to external sources when substantiating the construction of a telecommunication network

## 6. Summary

Thus, the further development of the conceptual apparatus of the artificial intelligence theory with respect to the intellectualized telecommunications management system, including a set of mathematical models of the formal description of production-logical inference of knowledge in the automation of decision support process, has received further development. The theoretical and mathematical basis for information technology of DSS in the telecommunication system are the original algebraic systems where the possibilities of production-logical systems for the emergence of new knowledge for managerial decisions are implemented. The proposed conceptual apparatus allows to mathematically formalize the technology of logical programming based on the use of algebraic structures in computer algorithms, namely, the ratio of partial order with the production-logical content, which results in reduction in the number of operations and a boost in efficiency.

## 7. References

- [1] S. Russell, P. Norvig, Artificial Intelligence, A Modern Approach, Prentice Hall, 2003.
- [2] O. Barabash, N. Shevchenko, N. Dakhno, Y. Kravchenko, O. Leshchenko, Effectiveness of Targeting Informational Technology Application, IEEE International Conference on System Analysis & Intelligent Computing, SAIC'2020, Proceedings, pp. 193 – 196. doi: 10.1109/SAIC51296.2020.9239154
- [3] Y. Kravchenko, V. Vialkova, The problem of providing functional stability properties of information security systems, Modern Problems of Radio Engineering, Telecommunications and Computer Science, Proceedings of the 13th International Conference on TCSET 2016. pp. 526 – 530. doi: 10.1109/TCSET.2016.7452105
- [4] A. Sobchuk, Y. Kravchenko, Y. Tyshchenko, M. Gawliczek, O. Afanasyeva, Analytical aspects of providing a feature of the functional stability according to the choice of technology for construction of wireless sensor networks, in: Proceedings of IEEE International Conference on Advanced Trends in Information Theory, ATIT'2019, Kyiv, 2019, pp. 102-106. doi: 10.1109/ATIT49449.2019.9030474.
- [5] O.A. Mashkov, L.M. Artiushyn, Optimization of digital automatic systems, fault tolerance, 1991.

- [6] O.A. Mashkov, V.A. Chumakevich, Y.V. Mamchur, V.R. Kosenko, The method of inverse problems of dynamics for the synthesis of a system of stabilization of the movement of a dynamic object on operatively programmable trajectories, *Mathematical Modeling and Computing* (2020) Vol. 7, No. 1, pp. 29–38. doi: 10.23939/mmc2020.01.029.
- [7] E. Mizraji, Vector logics: The matrix-vector representation of logical calculus, *Fuzzy Sets and Systems* (1992) pp. 179–185. doi: 10.1016/0165-0114(92)90216-Q.
- [8] J.F. Luger, *Artificial Intelligence. Strategies and methods for solving complex problems*, 2003.
- [9] A. Teise, P. Gribomon, *A logical approach to artificial intelligence: from classical logic to logical programming*, 1998.
- [10] V. Mashkov, J. Barilla, P. Simr, Applying Petri Nets to Modeling of Many-Core Processor Self-Testing when Tests are Performed Randomly, *Journal of Electronic Testing Theory and Applications* (2013) pp. 25-34. doi:10.1007/s10836-012-5346-8.
- [11] R.Yager, D. Filev, Generation of Fuzzy Rules by Mountain Clustering, *Journal of Intelligent & Fuzzy Systems* (1994) pp. 209-219.
- [12] N. Nilsson, *Principles of Artificial Intelligence*, 1985.
- [13] K.Park, K.Lee, S.Park, H.Lee, Telecommunication node clustering with node compatibility and network survivability requirements, *Management Science*, vol. 46(3), 2000, pp.363-374.
- [14] H. Hnatiienko, Choice Manipulation in Multicriteria Optimization Problems, *Selected Papers of the XIX International Scientific and Practical Conference Information Technologies and Security, ITS'2019, Kyiv, 2019*, pp. 234–245.
- [15] H. Hnatiienko, N. Tmienova, A. Kruglov, Methods for Determining the Group Ranking of Alternatives for Incomplete Expert Rankings, in: Shkarlet S., Morozov A., Palagin A. (eds) *Mathematical Modeling and Simulation of Systems, MODS 2020. Advances in Intelligent Systems and Computing*, vol 1265. Springer, Cham. doi:10.1007/978-3-030-58124-4\_21.
- [16] G. Luger, W. Stubblefield, *Artificial Intelligence: Structures and Strategies for Complex Problem Solving*, 2004.
- [17] D. Poole, A. Mackworth, R. Goebel, *Computational Intelligence: A Logical Approach*, 1998.
- [18] S.D. Makhortov, *Mathematical foundations of artificial intelligence: the theory of LP-structures for the construction and study of production-type knowledge models*, 2009.
- [19] S.D. Makhortov, A.A. Nogikh, LP Structures Theory Application to Building Intelligent Refactoring Systems, in *Proceedings of the Fourth International Scientific Conference Intelligent Information Technologies for Industry, IITI'19. Advances in Intelligent Systems and Computing*. Cham: Springer, 2020, pp. 403–411. doi: 10.1007/978-3-030-50097-9\_41.
- [20] S.D. Makhortov, A.A. Nogikh, Application of LP Structures Theory to Intelligent Attribute Merger Refactoring , in *Proceedings of the 18th Russian Conference on Artificial Intelligence, RCAI 2020. Lecture Notes in Artificial Intelligence*. Cham: Springer, 2020, pp. 437–447. doi: 10.1007/978-3-030-59535-7\_32.
- [21] Y. Kravchenko, O. Leshchenko, N. Dakhno, O. Trush, O. Makhovych, Evaluating the effectiveness of cloud services, in: *Proceedings of IEEE International Conference on Advanced Trends in Information Theory, ATIT'2019, Kyiv, 2019*, pp.120–124. doi: 10.1109/ATIT49449.2019.9030430.
- [22] C. Wang, L. Ming, J. Zhao, D. Wang, “General Framework for Network Survivability Testing and Evaluation”, *Journal of Networks*, vol. 6, №6, 2011, pp. 831-841.
- [23] O. Korostil, J. Korostil, Human Factor in the Tasks of Ensuring Functioning Safety of Complex Technical Objects, *Journal of Konbin*, (2017), 43(1), pp. 313–320. doi:10.1515/jok-2017-0052.
- [24] M.Yu. Rakushev, Computational scheme of ordinary differential equations integration on the basis of differential taylor transformation with automatic step and order selection. *Journal of Automation and Information Sciences*, 2012, 44 (12) pp. 12-22. doi: 10.1615/JAutomatInfScien.v44.i12.20.
- [25] M. Rakushev, O. Permiakov, S. Tarasenko, S. Kovbasiuk, Y. Kravchenko, O. Lavrinchuk, Numerical Method of Integration on the Basis of Multidimensional Differential-Taylor Transformations, *International Scientific-Practical Conference Problems of Infocommunications Science and Technology, PIC S&T'2019, Proceedings*. pp.675–678.