Visualization of pursuit differential game on a plane

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Abstract. This paper is dedicated to differential pursuit games. In the theory of dynamic games, a number of methods have been developed that provide a guaranteed result. The scheme of the Method of Resolving Functions is applied in the work. Sufficient conditions to end of the game have been found. For the task of simple pursuit, the visualization of the trajectory of the movements of the group of pursuers and the fugitive on the plane is realized. To do this, a software product was created in the Python programming language. This software product is a prototype of a "pursuer-evader" simulation system that can be used to select controls in pursuit tasks. Two cases of fugitive control selection were considered in the development. In the first one, the control of the fugitive is based on the following algorithm: the fugitive determines the nearest pursuer in terms of the Euclidean norm; the fugitive builds his control on the beam, which comes from the position of the nearest pursuer and crosses the position of the fugitive, in the direction opposite to the pursuer with maximum speed. In the second case, the control of the fugitive is set by the user. The visualization of the trajectory of one pursuer and one evader on the plane was realized for the Pontryagin's checking example of a game problem with simple motions. The results have a graphical presentation. The result showed the coincidence of the estimated time and the actual end time of the game.

Keywords: dynamic games, conflict-controlled processes, differential games, pursuit games, group pursuit games.

1 Statement of the problem. Scheme of the method

In the theory of dynamic games (conflict-controlled processes and differential games), along with Isaacs ideology [1], a number of methods have been developed that provide a guaranteed result. Such methods include, in particular, the first direct method of L.S. Pontryagin [2, 3], the method of extreme aiming of M.M. Krasovskii [4] and the method of resolving functions of A.O. Chikrii [5–15]. In this paper, the last method will be used, which gives a theoretical justification for the classical rule of parallel pursuit and the method of convergence by the beam. The method scheme for differential-difference games is developed in works [16–19]. In [20–22], the problem of rapprochement for a group of pursuers and a single evader is considered. The scheme of the method of resolving functions for differential-difference systems of neutral type was developed in

[23]. A modification of the method is proposed for objects with different inertia in [24]. Game problems of convergence in case of failure of control devices are considered in works [25-27].

Consider the motion of a controlled object, which is described by a system of differential equations:

$$\dot{z}_l = A_l z_l + \varphi_l(u_l, v),\ z_l \in R^{n_l},\ l = \overline{1, \vartheta},\ u_l \in U_l,\ v \in V \qquad (1)$$

where A_l are square constant matrices of order $n_l, n_1 + \cdots n_\vartheta = n$, U_l and V are nonempty compacts sets, $\varphi_l(u_l, v): U_l \times V \to R^{n_l}$, are jointly continuous in their variables.

Let $z(0) = z^\circ$ be the initial condition of the process (1). The terminal set has cylindrical form. i.e.

$$M^* = \cup_{l=1,\dots,\vartheta}\{M_l^\circ + M_l\} = \cup_{l=1,\dots,\vartheta} M_l^* \qquad (2)$$

where M_l° are linear spaces in R^{n_l} and M_l are compact sets from the orthogonal complement L_l of $M_l^\cdot$ in R^{n_l}.

Game (1), (2) is considered complete if for some $l = \overline{1, \vartheta}$ the condition $z_l \in M_l^\cdot$ fulfilled. The pursuers use quasi-strategies, and the evader uses software control..

Let π_l be the orthogonal projection mapping from R^{n_l} onto the subspace L_l. Consider the multivalued mappings

$$W_l(t, v) = \pi_l e^{A_l t} \varphi_l(U_l, v),$$

$$W_l(t) = \bigcap_{v \in V} W_l(t, v)\,, t \geq 0, v \in V.$$

<u>Pontryagin condition</u>. The mappings $W_l(t) \neq \emptyset$ for all $l = \overline{1, \vartheta}, t \geq 0$.

There exists at least one Borelian selector $\gamma_l(t) \in W_l(t)$ [28-32]. Let's fix it and set

$$\xi_l(t, z_l, \gamma_l(\cdot)) = \pi_l e^{A_l t} z_l + \int_0^t \gamma_l(\tau) d\tau.$$

Let's assign a resolving function to each pursuer:

$$\alpha_l(t, \tau, z_l, v, \gamma_l(\cdot)) =$$
$$= \sup\{\alpha_l \geq 0 : [W_l(t-\tau, v) - \gamma_l(t-\tau)] \cap \alpha_l \left(M_l - \xi_l(t, z_l, \gamma_l(\cdot))\right) \neq \emptyset\} \qquad (3)$$

If $\xi_l(t, z, \gamma_l(\cdot)) \in M_l$, then put $\alpha_l(t, \tau, z, v, \gamma_l(\cdot)) = +\infty, 0 \leq \tau \leq t, v \in V$. In other cases the function $\alpha_l(t, \tau, z, v, \gamma_l(\cdot))$ assumes finite values for each $\tau \in [0, t],\ v \in V$. Let's take $\gamma(\cdot) = column(\gamma_1(\cdot), \dots, \gamma_\vartheta(\cdot)$ and let

$$\Gamma_\vartheta = \{\gamma(\cdot) : \gamma_l(t) \in W_l(t), t \geq 0, l = \overline{1, \vartheta}\}.$$

Introduce the function

$$T_\vartheta\big(z,\gamma(\cdot)\big) = \min\{t \geq 0\colon \inf_{v(\cdot)\in\Omega_V} \max_{l=1,\ldots,\vartheta} \int_0^t \alpha_l\big(t,\tau,z_l,v(\tau),\gamma_l(\cdot)\big)\,d\tau \geq 1\}.$$

Theorem 1 [5, 33]. Assume that the Pontryagin condition holds for the conflict-control process (1), (2), $M_l = coM_l, l = 1,\ldots,\vartheta$, for the initial state z° and a certain selector $\gamma^\circ(t) \in \Gamma_\vartheta$ we have $T_\vartheta(z^\circ,\gamma^\circ(\cdot)) < +\infty$.

Then at least for one l, the corresponding trajectory of process (1) can be steered from the initial state z° to the set M_l^* at the moment $T_\vartheta\big(z^\circ,\gamma^\circ(\cdot)\big)$.

Theorem 2 [5]. Assume that the control process (1), (2) is linear (i.e., $\varphi_l(u_l,v) = u_l - v$), the Pontryagin condition holds, the multivalued mappings $\pi_l e^{A_l t} U_l = r_l(t) S_l$, and the sets $M_l = \epsilon_l S_l, l = \overline{1,\vartheta}$, where $r_l(t)$ are continuous nonnegative numerical functions, $\epsilon_l = const \geq 0$, and S_l is the unit ball centered at zero in the space L_l.

Then the resolving functions $\alpha_l\big(t,\tau,z_l,v(\tau),\gamma_l(\cdot)\big)$ for $\xi_l\big(t,z_l,\gamma_l(\cdot)\big) \notin M_l$ are large roots of the quadratic equations

$$\left\|\pi_l e^{A_l(t-\tau)} v + \gamma_l(t-\tau) - \alpha_l \xi_l\big(t,z_l,\gamma_l(\cdot)\big)\right\| = r_l(t-\tau) + \alpha_l \epsilon_l,$$

$$0 \leq \tau \leq t,\ l = \overline{1,\vartheta},\ v \in V,\ \gamma_l(t) \in W_l(t)$$

with respect to $\alpha_l, \alpha_l \geq 0$.

2 Example of problem for a process with simple matrices

Consider the problem of group pursuit with ϑ pursuers and one evader [5, 34]:

$$\dot{z}_l = a_l z_l + u_l - v, \qquad a_l < 0, z_l \in R^k, \|u_l\| \leq 1, \|v\| \leq 1, l = \overline{1,\vartheta} \tag{4}$$

The game is considered over if for some l $x_l = y$.

The terminal set

$$M^* = \cup_{l=1,\ldots,\vartheta}\{M_l^\circ\} = \cup_{l=1,\ldots,\vartheta}\{z_l\colon z_l = 0\}.$$

Thus we have $W_l(t) = \{0\}, l = \overline{1,\vartheta}$. The selectors of the multivalued mappings $W_l(t)$ are clearly defined as $\gamma_l(t) = 0$. The functions $\xi_l\big(t,z_l,\gamma_l(\cdot)\big) = e^{a_l t} z_l, l = \overline{1,\vartheta}$. The resolving functions of the pursuers

$$\alpha_l(t,\tau,z_l,v,0) = \max\big\{\alpha_l \geq 0\colon -\alpha_l e^{a_l t} z_l \in e^{a_l(t-\tau)}(S - v)\big\} = e^{-a_l\tau}\alpha_l(z_l,v),$$

where $\alpha_l(z_l,v)$ has the form

$$\alpha_l(z_l, v) = \frac{(z_l,v)+((z_l,v)^2+\|z_l\|^2(1-\|v\|^2))^{1/2}}{\| z_l\|^2}, l = \overline{1,\vartheta}. \quad (5)$$

Hence we obtain the time to the end the group pursuit:

$$T_\vartheta(z) = \min\left\{ t \geq 0: \inf_{v(\cdot)\in\Omega_V} \max_{l=1,\dots,\vartheta} \int_0^t e^{-a_l\tau}\alpha_l\big(z_l, v(\tau)\big)\, d\tau = 1 \right\}.$$

Letting $\delta(z) = \min_{\| v\|\leq 1} \max_{l=1,\dots,\vartheta} \alpha_l(z_l, v)$. Finally, we deduce the upper bound

$$T_\vartheta(z) \leq \frac{1}{a_*}\ln(1 + \frac{a_*\vartheta}{\delta(z)}), \quad (6)$$

where $a_* = -\max_{l=1,\dots,\vartheta} a_l$ [1].

Theorem 3 [5]. Let z° be the initial state of the process (4). Then if $0 \in int\ co\{z_l^\circ\}$ then the problem of group pursuit is solvable at finite time $T_\vartheta(z^\circ)$ for which the estimate (6) is true.

If $t_* = t_*\big(v(\cdot)\big), t_* \leq T_\vartheta(z^\circ)$, is the instant of switching, at which the test function

$$1 - \max_{l=1,\dots,\vartheta} \int_0^t e^{-a_l\tau}\alpha_l\big(z_l, v(\tau)\big)\, d\tau$$

vanishes, then the controls of the pursuers, ensuring of the game at the time $T_\vartheta(z^\circ)$, on the interval $[0, t_*]$ have the form

$$u_l(\tau) = v(\tau) - \alpha_l\Big(z_l^\circ, v(\tau)\Big)z_l^\circ, l = \overline{1,\vartheta},$$

and on the interval $(t_*, T_\vartheta(z^\circ)]$ –

$$u_l(\tau) = v(\tau)$$

for the indices l, satisfying the equality

$$\int_0^t e^{-a_l\tau}\alpha_l\Big(z_l^\circ, v(\tau)\Big)\, d\tau = \max_{l=1,\dots,\vartheta} \int_0^t e^{-a_l\tau}\alpha_l\Big(z_l^\circ, v(\tau)\Big)\, d\tau,$$

the form $u_l(\tau) = v(\tau)$ allowing the controls of the remaining pursuers to be arbitrary.

If $0 \notin int\ co\{z_l^\circ\}$ then the problem is solvable by means of any constant evader's control which furnished minimum to function $\max_{l=1,\dots,\vartheta} \alpha_l(z^\circ, v)$.

3 Visualization of a simple pursuit on the plane

There are ϑ pursuers and one evader. The positions of the participants are geometric points, that is we do not take into account their size. Motion of pursuers have the form

$$\dot{x}_l = u_l, \quad x_l \in R^k, k \geq 1, ||\, u_l|| \leq a_l, l = \overline{1, \vartheta}.$$

The law of motion of the evader $\dot{y} = v,\ y \in R^k, ||\, v|| \leq b$. Players move at limited speeds. The numbers a_l and b show the maximum velocity of the players. the game is over if for some $l = \overline{1, \vartheta}, x_l = y$.

There is a certain sampling of time. Players choose their motion every time of sampling and move to the directions indicated by them, considering their speed limits. The evader knows the position of the pursuers now, and the pursuers know the position of the evader and the motion he has chosen now. Strategy of pursuers is determined by the following algorithm:

1. Calculate $\alpha_l(z_l, v)$ according to the formula
$$\alpha_l(z_l, v) = \frac{(z_l, v) + ((z_l, v)^2 + \|z_l\|^2(a_l - \|v\|^2))^{1/2}}{||\, z_l||^2}, l = \overline{1, \vartheta}$$
2. Check $\alpha_l(z_l, v) < 1,\ l = \overline{1, \vartheta}$. If it is true, the motion is calculated by formula $u_l = v - \alpha_l(z_l, v)z_l, l = \overline{1, \vartheta}$.
3. If some $\alpha_l(z_l, v) > 1,\ l = \overline{1, \vartheta}$, then we find the pursuer with the greatest value $\alpha_{l_{max}} = \max \alpha_l(z_l, v)$ and determine his motion $u_{l_{max}} = v - z_{l_{max}}$, and the motion of others is set to zero.

To represent the trajectories of the pursuers and the evader in the game for the plane case, a software product was written in the Python programming language. For the program, the input data is the following information: the number of pursuers, initial coordinates of all participants; maximum speed values for all participants. For the algorithm to work, you need to choose the evader motion. Two cases of evader movement were used in the development. In the first, the motion of the evader is based on the following algorithm: 1) the evader determines the nearest pursuer in terms of the Euclidean norm; 2) the evader builds his control on the beam, which comes from the position of the nearest pursuer and crosses the position of the evader, in the direction opposite to the pursuer with maximum velocity. In the second case, the motion of the evader is set by the user.

In the figures, the initial positions of the pursuers are marked with stars, the initial position of the evader is marked with a rhombus, and the point of capture is marked with a pentagon.

In the Fig. 1-8, we can see several examples of pursuit game. In the Fig. 1-4, the evader uses first algorithm. The evader does not choose his movement optimally because he pays attention only to the nearest pursuer. Such actions can be seen in real life, when of all the threats, the object notices only the nearest and does not pay attention to others. In the Fig. 5-8, the evader`s motion is set by user.

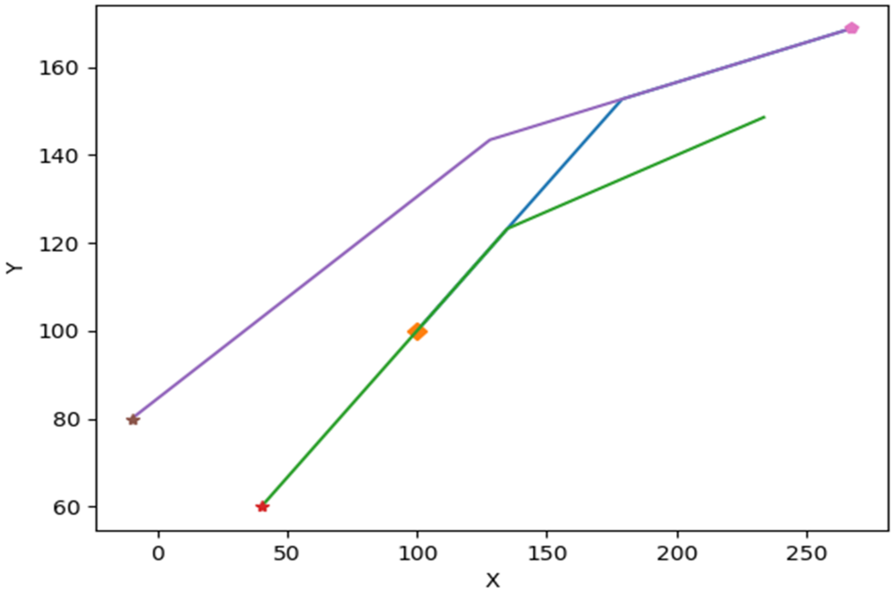

Fig.1. Two pursuers.

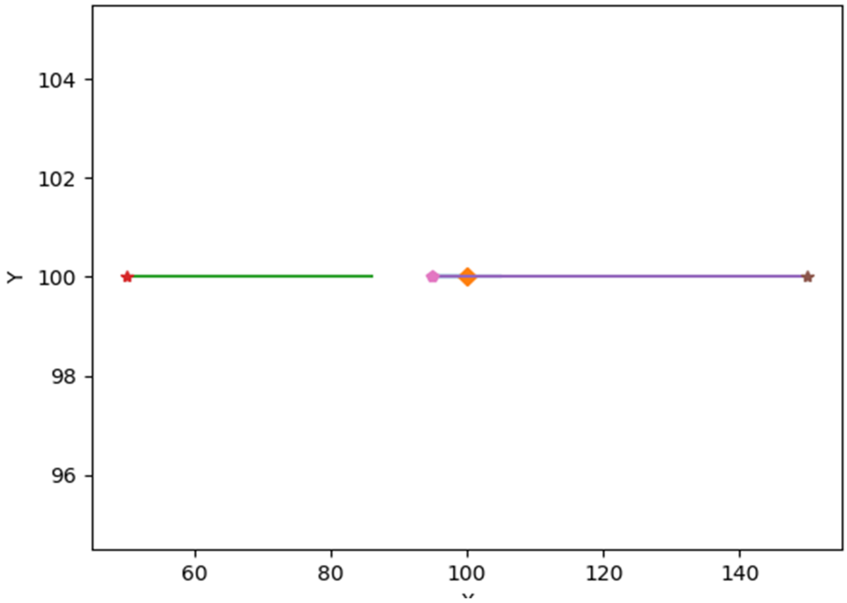

Fig. 2. Two pursuers and surrounded evader.

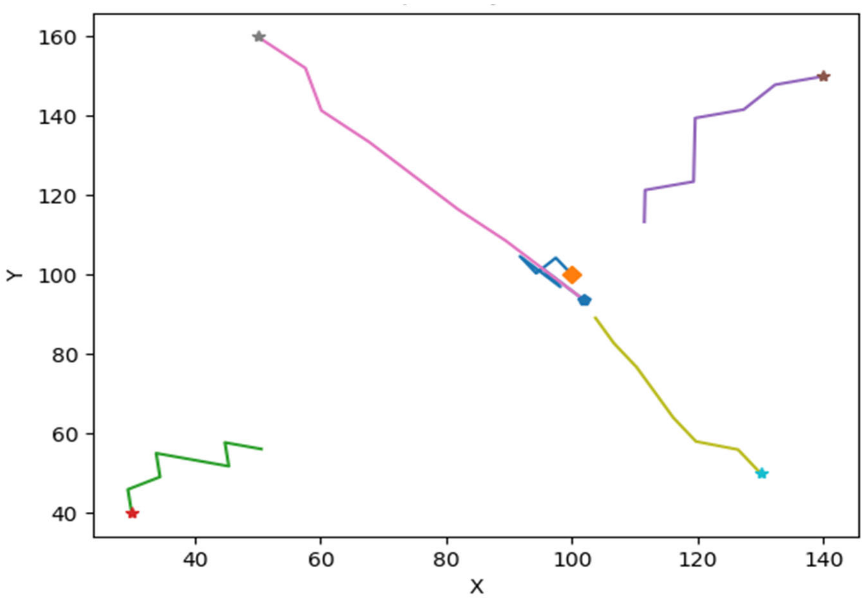

Fig. 3. Four pursuers and surrounded evader.

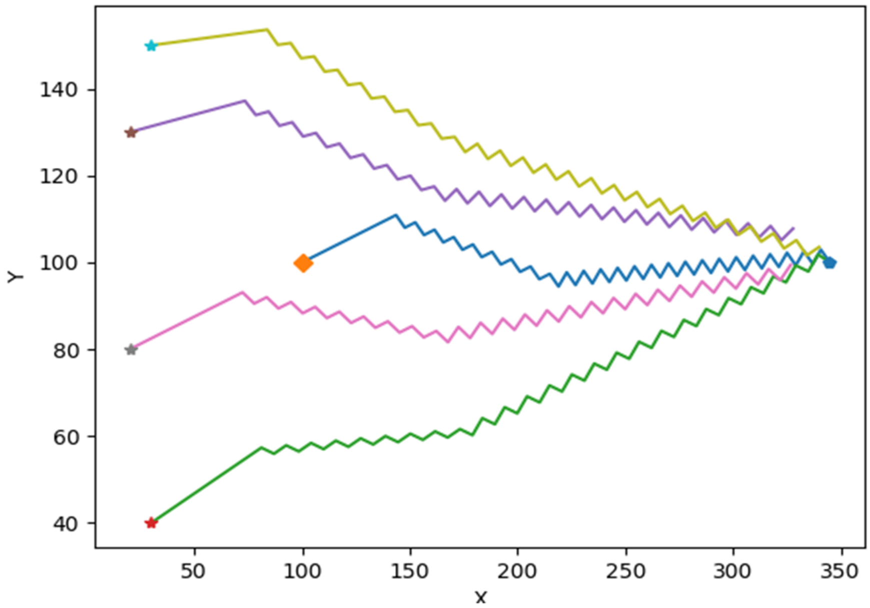

Fig. 4. Four pursuers with equal velocities.

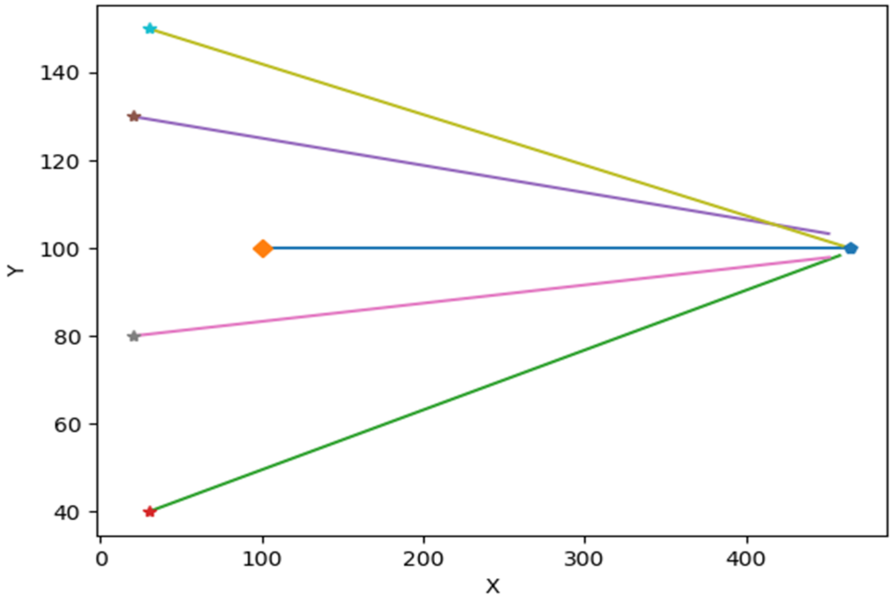

Fig. 5. Four pursuers with equal velocities and user`s movement.

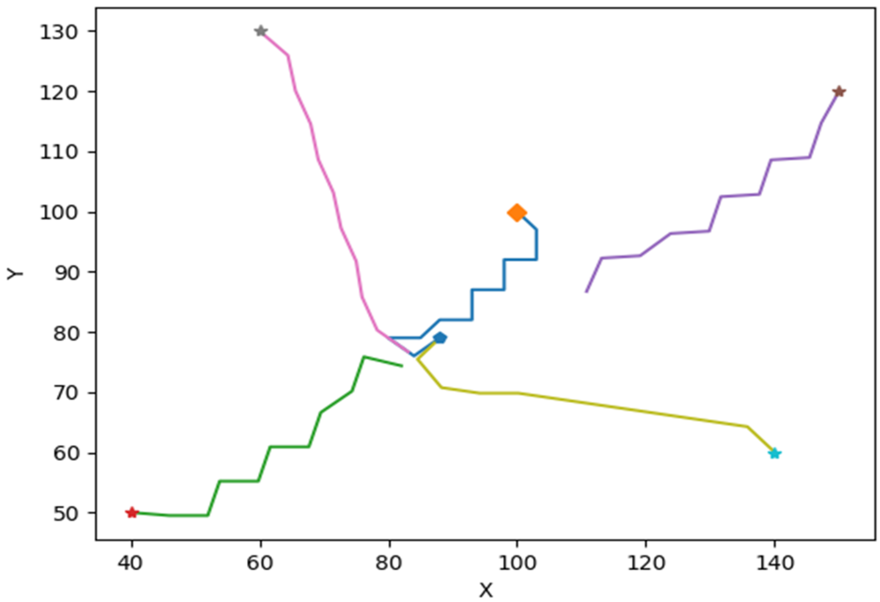

Fig. 6. Four pursuers, surrounded evader and user`s movement.

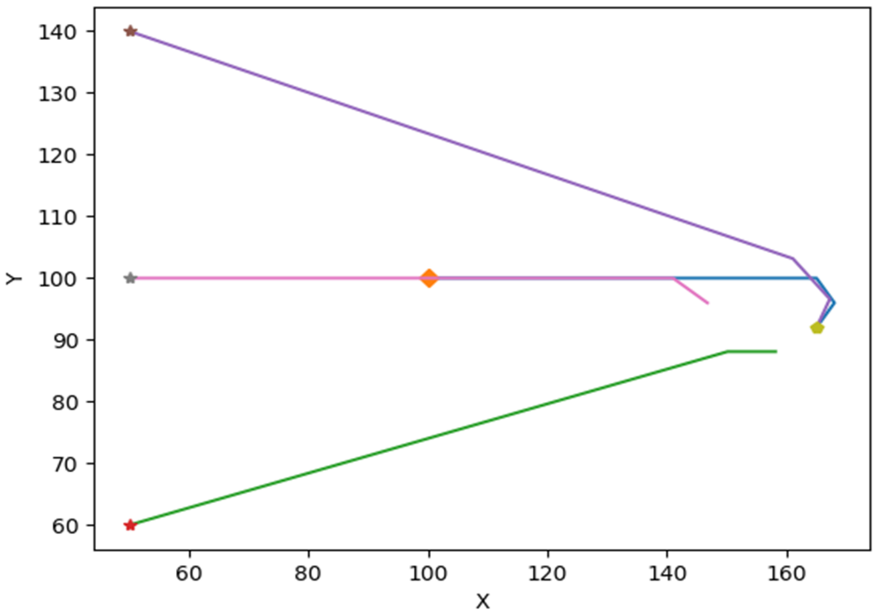

Fig. 7. Three pursuers and user`s movement.

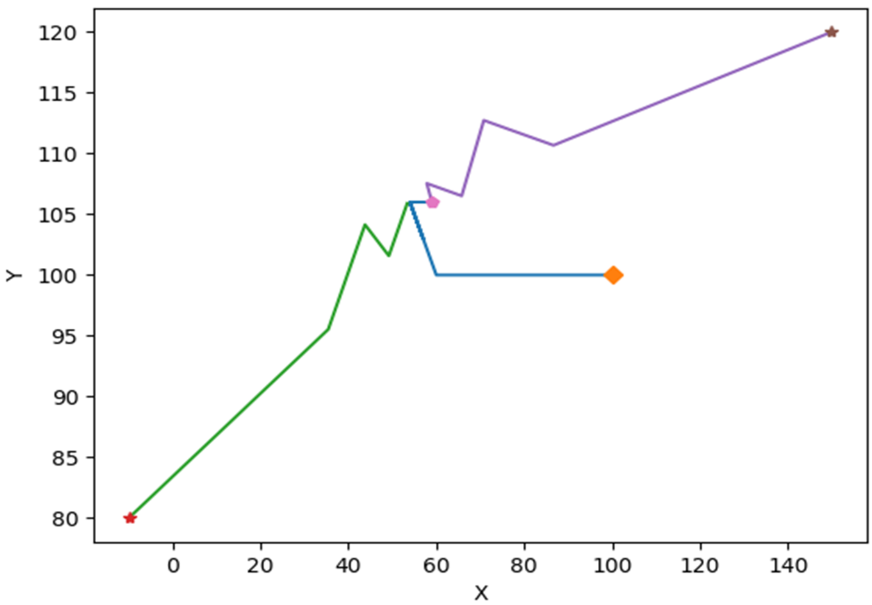

Fig. 8. Two pursuers, surrounded evader and user`s movement.

4 Pontryagin's checking example

The motions of the one pursuer and the one evader are described by the equations [5]:

$$\begin{cases} \ddot{x} + \dot{x} = 2u, \ x \in R, \quad \|u\| \le 1, \ \alpha, \rho > 0, \\ \ \ddot{y} + 2\dot{y} = v, \ y \in R, \ \|v\| \le 1, \ \beta, \sigma > 0\,. \end{cases} \tag{7}$$

Letting

$$\alpha = 1, \ \beta = 2, \ \rho = 2, \ \sigma = 1, \ \begin{cases} x(0) = 6, \ \dot{x}(0) = 1, \\ y(0) = 2, \ \dot{y}(0) = 1. \end{cases}$$

Substitute the values of the parameters in (7):

$$\begin{cases} \ddot{x} + \dot{x} = 2u, \quad x \in R, \quad \|u\| \le 1, \quad \alpha, \rho > 0, \\ \ddot{y} + 2\dot{y} = v, \quad y \in R, \quad \|v\| \le 1, \quad \beta, \sigma > 0\,. \end{cases}$$

The pursuit is completed when $x = y$. Let's move on to the system of first-order equations. To do this, we'll use new variables

$$z_1, z_2, z_3, column(z_1, z_2, z_3) = z,$$
$$z_1 = x - y, \qquad z_2 = \dot{x}, \qquad z_3 = \dot{y} \tag{8}$$

Differentiating over the time (8) , we obtain an equivalent system

$$\begin{cases} \dot{z_1} = z_2 - z_3, \\ \dot{z_2} = -\alpha z_2 + \rho u, \\ \dot{z}_3 = -\beta z_3 + \sigma v. \end{cases}$$

Therefore

$$\begin{cases} \dot{z_1} = z_2 - z_3, \\ \dot{z_2} = -z_2 + 2u, \\ \dot{z_3} = -2z_3 + \ v \end{cases}$$

The terminal set $M^{\cdot} = \{z: z_1 = 0\}$, and $M^{\circ} = \{z: z_1 = 0\}$, $M = \{z: z_1 = z_2 = z_3 = 0\}$.

Then $L = \{z: z_2 = z_3 = 0\} = \{R, 0, 0\}$. The operator of orthogonal projecrion $\pi: R^s \to L$ is given by the matrix

$$\pi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here matrix A and the control domains are

$$U = \left\{ \begin{pmatrix} 0 \\ 2u \\ 0 \end{pmatrix} : \|u\| \le 1 \right\}, V = \left\{ \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} : \|v\| \le 1 \right\}.$$

The fundamental matrix of a homogeneous system is

$$e^{At} = \begin{pmatrix} 1 & \frac{1-e^{-t}}{1} & -\frac{1-e^{-2t}}{2} \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{-2t} \end{pmatrix},$$

$$W(t) = \frac{1-e^{-t}}{1}2S \;\underline{*}\; \frac{1-e^{-2t}}{2}S = \left(\frac{1-e^{-t}}{1}2 - \frac{1-e^{-2t}}{2}\right)S = w(t)S.$$

We set $\gamma(t) \equiv 0,\ z(0) = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$. Then $\xi(t,z,0) = 4 + \frac{1-e^{-t}}{1}1 - \frac{1-e^{-2t}}{2}1.$

By Theorem 2, we obtain the resolving function $\alpha(t,\tau,z,v,0)$ as a large positive root of the quadratic equation

$$\left\| \frac{1-e^{-2(t-\tau)}}{2}v - \alpha\xi(t,z(0),0) \right\| = \frac{1-e^{-(t-\tau)}}{1}2.$$

Next, by virtue of the capture time $T(z(0),0)$ is defined as $\min\{t \geq 0: \int_0^t \frac{w(t-\tau)}{||\xi(t,z,0)||} d\tau = 1\}$, we have $T = 5$.

Let's find the control of the pursuer. Since the motion occurs on the plane and the set *M* consists of one point, we conclude that

$$u = v(\tau) - \frac{\alpha(T,\tau,z^{\circ},v(\tau),0)[\,\xi(T,z^{\circ},0)]}{\pi e^{A(T-\tau)}}, T = 5,$$

and therefore

$$u1 = v(\tau) - \frac{\alpha(5,\tau,z^{\circ},v(\tau),0)[\,\xi(5,z^{\circ},0)]}{\pi e^{A(5-\tau)}}, u = lex \min u1.$$

5 Visualisation of Pontryagin's checking example

To represent the trajectories of the pursuers and fugitives in the game described in the previous paragraph and for the case of the plane, was created a software in the Python programming language. This is a prototype of a modeling system "fugitive-pursuers", which can be used to select control in control tasks.

The inputing data for the program is the following information:

1. The values of the parameters α, β, σ, ρ;
2. Initial coordinates of all participants.

Consider the algorithm by which the software works:

1. Check the conditions $\rho \geq \sigma, \frac{\rho}{\alpha} \geq \frac{\sigma}{\beta}$
2. Calculate the fundamental matrix e^{At}.
3. Check the condition $W(t) \neq \emptyset$, and find $w(t)$.

4. Calculate $\xi(t,z,0) = z_1 + \frac{1-e^{-\alpha t}}{\alpha} z_2 - \frac{1-e^{-\beta t}}{\beta} z_3$
5. Calculate fugitive delay time as $\min\{t \geq 0: \int_0^t \frac{w(t-\tau)}{||\xi(t,z,0)||} d\tau = 1\}$
6. Construct the pursuer control as

$$u = v(\tau) - \frac{\alpha(T,\tau,z^{\circ},v(\tau),0)[\,\xi(T,z^{\circ},0)]}{\pi e^{A(T-\tau)}}$$

$$\alpha(t,\tau,z,v,0) = \frac{\frac{1-e^{-\beta(t-\tau)}}{\beta}\sigma\left(v,\xi(t,z,0)\right)}{\left||\xi(t,z,0)|\right|^2} +$$

$$+\frac{\left(\left(\frac{1-e^{-\beta(t-\tau)}}{\beta}\sigma\right)^2 \left(v,\xi(t,z,0)\right)^2 + \left||\xi(t,z,0)|\right|^2 \left(\left(\frac{1-e^{-\alpha(t-\tau)}}{\alpha}\right)^2 \rho^2 - \left(\frac{1-e^{-\beta(t-\tau)}}{\beta}\right)^2 \sigma^2 ||v||^2\right)\right)^{1/2}}{||\xi(t,z,0)||^2}$$

7. Using the Runge-Kutta method, we construct the solutions of differential equations at time t.

Let's use this control example to check the program.

Letting

$$\alpha = 1,\ \ \beta = 2,\ \ \rho = 2,\ \ \sigma = 1, \begin{cases} x(0) = 6, \dot{x}(0) = 1 \\ y(0) = 2, \dot{y}(0) = 1 \end{cases}.$$

We get the capture time T that coincides with the time obtained in our calculations (see Fig. 9).

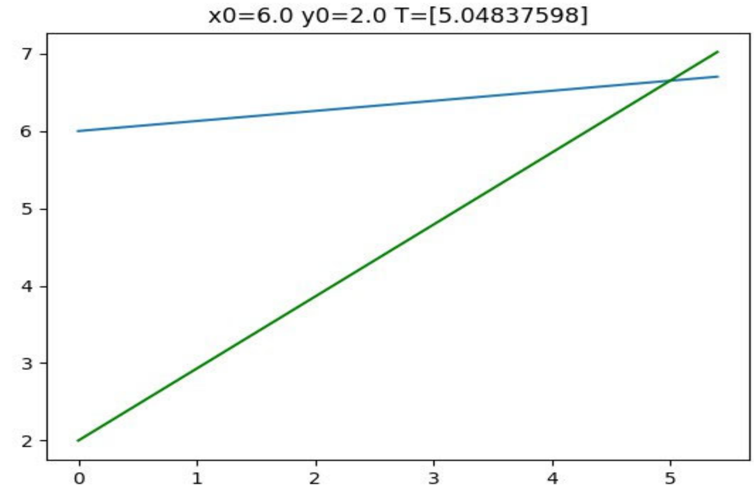

Fig. 9. The results of the program.

Letting

$$\alpha = 1,\ \beta = 2,\ \rho = 2,\ \sigma = 1, \begin{cases} x(0) = 10, \dot{x}(0) = 1 \\ y(0) = -3, \dot{y}(0) = 1 \end{cases}$$

We obtain the capture time T (see Fig. 10).

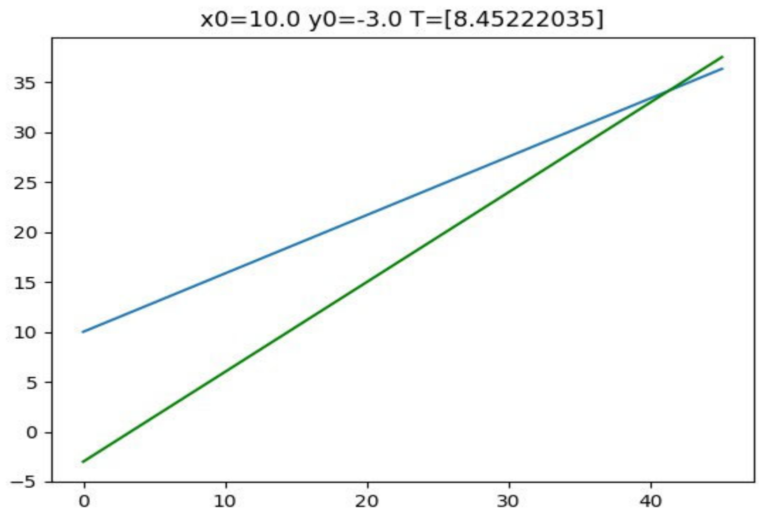

Fig. 10. The results of the program

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