Dialetheic Truth Theory: Inconsistency, Non-Triviality, Soundness, Incompleteness

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Abstract. The best-known application of dialetheism is to semantic paradoxes such as the Liar. In particular, Graham Priest has advocated the adoption of an axiomatic truth theory in which contradictions arising from the Liar paradox can be accepted as theorems, while the Liar sentence itself is evaluated as being both true and false. Such eccentricities might be tolerated, in exchange for great rewards. But in this note I show that it is not possible in Priest's truth theory to express certain semantic facts about that very theory, and thus that it enjoys no definite advantage over more orthodox approaches to semantic paradox.

Key words: Paraconsistent logic, dialetheism, truth theory.

1   Naive axiomatic truth theory

Let $\mathcal{L}$ be a first-order language with equality containing semantic and arithmetic vocabulary. I will say that an axiomatic truth theory T for $\mathcal{L}$ is *naive* if it has the following characteristics:

(i) T is expressed in $\mathcal{L}$

(ii) T contains a significant subset of the standard axioms of Peano arithmetic (e.g., T contains the axioms of Robinson arithmetic)

(iii)   Every instance of the Truth schema is provable in T. That is to say, for every closed sentence α of $\mathcal{L}$,

$$\vdash_T \mathit{True}(\langle\alpha\rangle) \leftrightarrow \alpha.^1$$

A theory satisfying (i)–(iii) is said to be naive because, if its underlying logic is such that

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[1] Angle brackets are used as a name-forming device, to be further specified shortly.

(iv)   equivalent sentences are intersubstitutable

(v) $\{\alpha \leftrightarrow \neg\alpha\} \vdash \alpha \& \neg\alpha$, for all sentences α

then the theory is inconsistent. This may be demonstrated as follows. The arithmetic axioms of T are such that, for every formula α of $\mathcal{L}$, one can construct a term $\langle\alpha\rangle$ of $\mathcal{L}$, namely the numeral denoting the Gödel number of α, which serves as α's name. The arithmetic axioms of T are also sufficient for proving a fixed-point theorem stating that, if $\gamma(v)$ is any formula of $\mathcal{L}$ with one free variable, v, then there is a sentence β such that

$$\vdash_T \beta \leftrightarrow \gamma(\langle\beta\rangle).^2$$

Let $\gamma(x)$ be the open sentence $\neg True(x)$. Then, by the fixed-point theorem, there is a sentence B such that

$$\vdash_T B \leftrightarrow \neg True(\langle B\rangle).$$

By (iii), we also have that

$$\vdash_T True(\langle B\rangle) \leftrightarrow B.$$

From the last two, by (iv), we get that

$$\vdash_T True(\langle B\rangle) \leftrightarrow \neg True(\langle B\rangle)$$

from which the explicit contradiction

$$\vdash_T True(\langle B\rangle) \& \neg True(\langle B\rangle)$$

follows by (v). It will be noted that (iv) and (v) hold in classical logic. Indeed, if the logic underlying T is classical, then T is not only inconsistent, but trivial, since any sentence can be classically deduced from a contradiction.

    What is an axiomatic truth theorist to do in response to the problem just outlined, that is to say, to the formalized version of the Liar paradox? With (i) and (ii) beyond reproach, the traditional line has been to reject (iii), i.e. to give up on having a truth theory that yields every instance of the Truth schema as a theorem. Alternatively, a truth theorist may leave the truth theory itself alone, and adopt an underlying logic that arrests inconsistency by invalidating either (iv) or (v) — where invalidating (v) seems by far the easier course to justify.[3] Finally, an axiomatic truth theorist may insist on keeping all of

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[2] For a proof of the fixed-point theorem (a.k.a. the diagonal lemma), see, e.g., [1] p. 222.

[3] This strategy has been explored in Field's [2], [3], [4]. More specifically, the logic Field proposes does not license the inference from $\alpha \leftrightarrow \sim\alpha$ to $\alpha \& \sim\alpha$ when α contains semantic vocabulary.

(i)–(v), but use as underlying logic a paraconsistent logic that prevents the theory's inconsistency from degenerating into triviality. Priest's view, which he has defended with remarkable acumen over almost three decades, is that this third alternative is preferable to the other two.[4] My goal is to contest this view.

In order to do so, I must provide additional background. As Priest and others have long observed, not just any paraconsistent logic is suitable for serving as the underlying logic of a naive axiomatic truth theory. In particular, the Curry paradox shows that a naive truth theory is trivial if its underlying positive logic satisfies, in addition to (iv) and (v), the following mild requirements:[5]

    (vi)   it contains *modus ponens* as a deductive rule

    (vii)   it contains &-elimination as a deductive rule

    (viii)   its conjunction connective is idempotent

    (ix)   it contains all instances of the so-called *Assertion* theorem schema:

$$(\alpha \& (\alpha \rightarrow \beta)) \rightarrow \beta.$$

Since the Curry paradox is central to my argument, I will spell it out in more detail. Consider again the language $\mathcal{L}$ and the truth theory T discussed above. Let $\gamma(x)$ be the open sentence $True(x) \rightarrow \xi$, where ξ could be any closed sentence at all. Then, by the fixed-point theorem for T, there is a sentence C such that

$$\vdash_T C \leftrightarrow (True(\langle C \rangle) \rightarrow \xi).$$

By (iii), we also have

$$\vdash_T True(\langle C \rangle) \leftrightarrow C$$

and by (iv), these two together give

$$\vdash_T True(\langle C \rangle) \leftrightarrow (True(\langle C \rangle) \rightarrow \xi).$$

However, on the assumption that the biconditional involved in the above theorem schema is defined in the usual way, i.e. as a conjunction of conditionals, one can derive the arbitrary ξ, with the help of (vi)–(ix). To better display the reasoning involved, I continue the derivation in proof format:

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[4] See, e.g., [5].

[5] See [6]. There are a number of variants of Curry's argument, all of which a naive truth theorist must take into account (as Priest does). However, I will for simplicity restrict my attention to just one version of the paradox.

1. $True(\langle C\rangle) \leftrightarrow (True(\langle C\rangle) \rightarrow \xi)$

2. $(True(\langle C\rangle) \,\&\, (True(\langle C\rangle) \rightarrow \xi)) \rightarrow \xi$ Instance of Assertion

3. $(True(\langle C\rangle) \,\&\, True(\langle C\rangle)) \rightarrow \xi$ 1, 2, substitution of equivalents

4. $True(\langle C\rangle) \rightarrow \xi$ 3, idempotence of conjunction

5. $(True(\langle C\rangle) \rightarrow \xi) \rightarrow True(\langle C\rangle)$ 1, definition of $\leftrightarrow$, &-elimination

6. $True(\langle C\rangle)$ 4, 5, *modus ponens*

7. ξ 4, 6, *modus ponens*.

At first blush one might think that the Curry paradox is just a more roundabout way of reaching a conclusion that had already been reached by means of the Liar. But note that the assumptions appealed to in the present argument are weaker than those appealed to in the earlier one: the Curry paradox generates triviality without the full resources of classical logic. In particular, the argument to triviality does not involve deducing an arbitrary conclusion from a contradiction — indeed, the argument of the Curry paradox does not involve negation at all. The upshot is that one who insists on having a theory satisfying (i)–(iii), and an underlying logic in which (iv) and (v) hold, must not only use paraconsistent reasoning in order to avoid trivialization by the Liar, but also give up one or more of (vi) to (ix) in order to avoid trivialization by the Curry.

Which of (vi)–(ix) is Priest prepared to give up? Discarding (vi) is not an option: the availability of *modus ponens* would certainly be taken for granted by anyone wanting to reason in an axiomatic truth theory. The principles involved in (vii) and (viii), conjunction elimination and idempotence of conjunction, are hardly objectionable. Priest therefore rejects (ix): he formulates a paraconsistent logic, which I will call *LPC*, containing a detachable intensional conditional connective whose semantics are such that the instances of Assertion do not come out as logical truths. The semantics of *LPC* are messy (and questionable); fortunately it is not necessary to go over them here. It suffices for our purposes to state the following facts about *LPC*:

- the semantic consequence relation of *LPC* is paraconsistent

- the instances of Assertion are not logical truths of *LPC*

- a deductive system that is sound and complete with respect to the *LPC* semantics can be formulated

- *LPC* contains an intensional conditional operator, →, and the biconditional defined in terms of it in the usual way. The conditional → is detachable — thus satisfying (vi) — as well as transitive and contraposible.[6]

- (iv)–(viii) hold in *LPC.*

Once the logic *LPC* is in place, Priest can proceed to formulate a naive axiomatic truth theory. To my knowledge, he has done this in two alternative ways. In one formulation, he postulates arithmetic axioms, as well as axioms governing denotation and satisfaction, gives recursive satisfaction conditions, and defines a truth predicate in terms of satisfaction. Then, by means of a standard technique, he shows that for every closed sentence of the language of the theory, every instance of the Truth schema is provable.[7] Priest's alternative formulation of an axiomatic truth theory is more direct, and simply takes the Truth schema itself as an axiom.[8] Either way, the resulting theory will be inconsistent, as is shown by the formalized version of the Liar paradox presented above. Since the truth theory's underlying logic *LPC* is paraconsistent, this inconsistency does not lead to triviality. In fact, Priest has shown, in a classical metalanguage, and for the simpler formulation of the truth theory in which the Truth schema is taken as an axiom, that it is possible to construct a model that verifies all of the theorems of the theory, but not all the sentences in its language (he also asserts that the same holds of the more roundabout formulation).[9] Hence these theories are non-trivial.

2   Naive truth theory and semantic closure

For the sake of definiteness, let us focus our attention on the version of Priest's axiomatic naive truth theory that includes the Truth schema as an axiom, as well as the standard first-order axiomatization of Peano arithmetic (*PA*). I will call this arithmetic-*cum*-truth theory '*PA**', and its language *Ł*.

I will henceforth be assuming that everything that is classically provable in *PA* is provable in *PA** when reasoning in the latter in *LPC*.[10] Although this is a substantive

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[6] *LPC* also contains a second intensional conditional operator that does not contrapose. The biconditional defined from this second conditional is the one that Priest uses inside the Truth schema (and nowhere else). Priest introduces this complication for philosophical reasons that are tangential to our present concerns; since no argument made in the course of this paper hinges on the distinction between the two conditionals of *LPC*, I will ignore it.

[7] For the details, see [5] ch. 9.

[8] See [7] §8.2.

[9] See the appendix to [8], [7] §§8.2 and 8.5, as well as [5] p. 277.

[10] In formulating this assumption, and in much else besides, I have followed Stewart Shapriro's illuminating [9]. There, he argues that *PA** contains contradictions among the purely arithmetic sentences, and not only among sentences featuring semantic vocabulary. The argument fails, because Shapiro assumes that *PA**'s soundness can be proved within *PA**, which it cannot, on pain of triviality, as I will argue

assumption, it is also a natural one to make: if Priest's chosen logic were not suitable for proving the theorems of arithmetic, then that in itself would be, by his own lights, a definite shortcoming of his position, since he has repeatedly maintained that his aim is to formulate a logic that does not sacrifice any part of classical mathematics, but allows to extend mathematical investigation into inconsistent domains.[11] Let us therefore proceed with the assumption, and note that, because $PA*$ has a recursive proof relation ($PA*$-derivations form a recursive set of sequences of strings on the alphabet of $\mathcal{L}$), and because all recursive relations can be represented in PA (and therefore, by assumption, also in $PA*$), it is possible to formulate a proof predicate $Prf_{PA*}(x, y)$ for $PA*$. In particular, if m is the code of a $PA*$-proof of the sentence with code n, then $Prf_{PA*}(m, n)$ is provable in PA, and in consequence in $PA*$, and if m is not the code of a $PA*$-proof of the sentence with code n, then $\neg Prf_{PA}(m, n)$ is provable in PA, as well as in $PA*$.

What advantages does $PA*$ have over consistent alternatives? For one thing, $PA*$ contains as theorems all instances of the Truth schema, and is therefore appealing to deflationists about truth.[12] However, a non-trivial naive axiomatic truth theory preserving an unrestricted Truth schema can be obtained in other ways, e.g., by reasoning not with LPC but with a logic that invalidates the inference from $\alpha \leftrightarrow \sim\alpha$ to $\alpha \& \sim\alpha$.[13] Moreover, and in any case, Priest rejects deflationism about truth.[14] No, what really sets $PA*$ apart from consistent alternatives, and the reason why, despite many difficulties, its underlying logic LPC merits a close look, is the alluring possibility of its being semantically closed. Briefly put, Priest aims with $PA*$ to provide a truth theory that, unlike any other theory so far proposed, can express all semantic facts about the sentences of its own language.[15]

Does he succeed? One reason to doubt this is that the soundness and completeness proofs for the logic LPC are carried out in a classical metatheory, which is subject to the usual expressive limitations, and this may be taken as showing that $PA*$ is not genuinely semantically closed. This is not an issue that has been emphasized much in Priest's writings, and which deserves more attention.[16] But I will not go into it here, and concentrate on a second reason to doubt that $PA*$ should be considered a semantically closed theory. This concerns the soundness of $PA*$ itself.

In at least one place, Priest has suggested that establishing the soundness of a theory such as $PA*$ in $PA*$ itself represents no significant complication.[17] The proof would

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below. Even if the argument went through, moreover, it would not convince Priest that $PA*$ is untenable, since he thinks that arithmetic itself may be inconsistent. See, e.g., [10], [11].

[11] See, e.g., [5] §8.5.

[12] See [12], [13].

[13] See n. 3, above.

[14] See [5] ch. 4.

[15] See, e.g., [5] ch. 1, [14].

[16] See, however, [5] §18.5.

[17] [5] pp. 24, 49–50.

proceed by induction on the length of derivations. One would begin by arguing that each axiom of *PA** is true in its intended interpretation, move on to show that the *LPC* rules of inference preserve truth, and conclude with an inductive argument establishing that all theorems of *PA** are true. Such a proof would be extraordinary, in that normally (i.e., classically) the soundness of a theory cannot established within that very theory, but it would otherwise proceed along the usual lines for a soundness proof.[18]

As Hartry Field has recently pointed out, however, the soundness of *PA** cannot be established along the lines just described, on pain of triviality.[19] The reason is simple. As we know from Section 1, *modus ponens* is one of the rules of inference of the logic *LPC*. Among other things, then, a proof of the soundness of *PA** that proceeds along the "usual" lines would establish that *modus ponens* preserves truth when applied to the axioms and theorems of *PA** — it would establish, that is, that for all closed sentences α of the language of *PA**

$$\vdash_{PA*} (True(\langle\alpha\rangle) \mathbin{\&} True(\langle\alpha\rightarrow\beta\rangle)) \rightarrow True(\langle\beta\rangle).$$

By means of *PA**'s Truth schema, the above can be turned into

$$\vdash_{PA*} (\alpha\mathbin{\&}(\alpha\rightarrow\beta)) \rightarrow \beta.\text{[20]}$$

But this is just the Assertion schema, and we have seen that the instances of this schema cannot be theorems of *PA**, on pain of triviality, because of the Curry paradox. (The logic *LPC* is designed to bar instances of the Assertion schema from the set of logical truths; the problem is that a proof of the soundness of *PA** proceeding along standard lines would re-instate this schema as a theorem.)

Field has shown, then, that a proof of the soundness of *PA** in *PA** cannot proceed along standard lines. His argument does not preclude, however, the possibility that soundness could be established in some non-standard, unforeseen way. But I will now show that there is no such possibility. I take it that, independently of the type of argument used, any proof of the soundness of *PA** in *PA** would establish the global reflection principle

$$\vdash_{PA*} \forall x(\exists y Prf_{PA*}(y, x) \rightarrow True(x))$$

an instance of which is

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[18] Classically, the soundness of a theory implies its consistency, and a theory cannot prove its own consistency by Gödel's second incompleteness theorem.

[19] [15] pp. 30, 34.

[20] I mentioned in n. 7 above that Priest uses a special biconditional inside the Truth schema. One effect of this is that, on his view, $True(\langle\alpha\rangle)$ and α are not always intersubstitutable. However, Priest's special biconditional does not block the intersubstitutivity of $True(\langle\alpha\rangle)$ and α in any instance concerned in the present argument.

$$\vdash_{PA*} \exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow True(\langle 0=1 \rangle).$$

By the Truth schema for *PA**, this can be turned into

$$\vdash_{PA*} \exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow (0=1).$$

However, the above sentence cannot be proved in *PA**, on pain of triviality. For in [16] Löb has proved that, if *Prv(x)* is a provability predicate for any theory *T* that is an extension of minimal arithmetic, then, for every sentence α,

$$\text{if } \vdash_T Prv(\langle \alpha \rangle) \rightarrow \alpha, \text{ then } \vdash_T \alpha.$$

Hence, if it could be established that

$$\vdash_{PA*} \exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow 0=1$$

then it would follow by Löb's theorem that

$$\vdash_{PA*} 0=1.$$

One concludes that if the soundness of *PA** were provable in *PA**, then so would be every other arithmetical sentence.

The foregoing argument would be blocked, obviously, if Löb's theorem did not hold in *PA**. But *PA** satisfies the conditions for the theorem: it is an extension of minimal arithmetic in which a provability predicate can be represented. Furthermore, the theorem is proved by an appeal to the fixed point theorem (which is provable in *PA**), and the only logical steps required for its proof are *modus ponens* and the transitivity of the conditional, which are valid steps in *LPC*.[21]

One consequence of the fact that the soundness of *PA** cannot be proved in *PA** is that if the soundness of *PA** is to be established at all, then this could only be done in some stronger theory. But ascending to a metatheory to establish the soundness of *PA** is incompatible with the principal motivation one might have had for adopting *PA** in the first place. For to resort to metatheoretical ascent in order to express semantic facts about *PA** is to admit that *PA** is not semantically closed.

It remains for Priest to refuse to avail himself to metatheoretical ascent, and stand by *PA** without any assurance that the theory is sound. This would be to acknowledge the possibility that some sentences are provable in *PA**, and yet are not true. The problem is that it is no longer clear what role the notion of proof is playing for one who acknowledges such a possibility.

Worrisome as the last option seems, Priest might want to argue that it is not as bad as it looks. The situation with *PA**, he might argue, is no worse than that with *PA*: it cannot be proved from within *PA* that this theory is consistent, and thus the possibility that any sentence whatever is provable in *PA*, and hence that proofs are worthless, cannot be ruled

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[21] See Section 1.

out. The difference, however, is that one who accepts *PA* has no problem with accepting a proof of the consistency of *PA* in a stronger theory. But is the defender of *PA* not thereby led to a regress, in which the proof of the consistency of *PA* in a stronger theory is not secure until a proof of the consistency of the stronger theory is produced, and so on? I do not think so. The reasoning involved in the consistency proof of the vast majority of mathematical theories can be carried out in set theory, say in *ZF*. The consistency of *ZF* itself cannot be established in *ZF*, of course, but there are strong independent grounds to think that *ZF* is consistent: namely, that its axioms are postulated precisely with an eye on restricting the formation of sets so as to avoid inconsistency, and that long experience with this theory leaves its practitioners in no practical doubt of its consistency. This does not give final assurance of the consistency of *ZF* — but it does give incomparably better assurance than we have about the soundness of *PA**.

The fact that *PA** cannot prove its own soundness carries additional problems in its wake. Either one can prove in *PA** the negation of the global reflection principle, i.e.

$$\vdash_{PA*} \neg \forall x (\exists y Prf_{PA*}(y, x) \rightarrow True(x))$$

or one cannot. If one can, then *PA** is provably unsound; if one cannot, then *PA** is incomplete, since the global reflection principle itself is not provable in *PA**, on pain of triviality. But the former option disqualifies *PA**, as proofs carried out within this theory no longer carry any conviction, and the latter takes away another reason one might have given in support of *PA**, i.e., that it is deductively complete.

I will mention one last problem that arises for *PA**. We have seen that, if *PA** is non-trivial, then '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow 0=1$' cannot be provable in it. But one can argue, by appeal to a semantic argument that should be acceptable to Priest, that this sentence must be true. To do so, begin by considering the sentence '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle)$', which says, in the language of *PA**, that there is a proof in *PA** of 0=1. On the assumption that *PA** is non-trivial, then this sentence is false.[22] Now it is a fact that *LPC*, the underlying logic of *PA**, permits for some sentences to be both true and false. Nonetheless, '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle)$' must be simply false: if it were both true and false, then it would be at least true, and this means that there would be a *PA** proof that 0=1, which is untenable. But according to Priest's semantics for *LPC*, a sentence with the conditional $\rightarrow$ as its main connective is true when its antecedent is simply false.[23] Hence, since '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle)$' is simply false, '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow 0=1$' is true — even though, as we have seen, it is not provable in *PA**. And of course this situation obtains not only for '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle) \rightarrow 0=1$', but for any number of sentences like it.

The arithmetic-*cum*-truth theory *PA**, which Priest proposes as a solution to the Liar and kindred semantic paradoxes, and as a model for the workings of natural language, is not only inconsistent, but cannot prove its own soundness — indeed, it is either unsound, or incomplete. In addition, there are countless obviously true sentences expressed in its

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[22] Except if *PA** is not sound, in which case '$\exists y Prf_{PA*}(y, \langle 0=1 \rangle)$' does not imply the triviality of the theory — but in which case the game is up anyway.

[23] See [5] §19.8.

language that it cannot prove. It is not on its account, I conclude, that dialetheism is to be thought of as an appealing position.

Acknowledgments. In writing this note I have benefited from the suggestions and comments of Hartry Field, Achille Varzi, Greg Restall, Peter Milne, and, especially, Haim Gaifman.

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