Synthesis of Spherical Object Configurations: Models and Information Technologies

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Abstract

The problem of synthesis of optimal configurations of spherical objects is considered. The basis is a general approach to the formalization of the configuration space of geometric objects. The analysis of models is carried out, their typology and information technologies for structural synthesis of complex spherical objects are offered. Approaches to the development of new information technologies for solving problems based on their generalized information-analytical model are proposed. Data is transformed into an object-oriented structure for visualization of spherical configurations.

Keywords 1

Spherical object, configuration, analytical model, information technology

1. Introduction

The term configuration is understood as appearance, outline and also the relative positioning of objects and their parts. In mathematics, as a rule, the configuration means some placement of points in space. The problem of synthesis of spatial configurations will be considered as the definition of such a property of geometric objects that would satisfy the given relations (constraints, properties) and would deliver an extremum to a certain quality criterion.

In this article, we confine ourselves to a class of geometric objects of spherical form. However, the results obtained can be generalized to broader classes of spatial objects. A significant number of publications testify to the importance of the problems of spatial synthesis of spherical objects. The article [1] presents an overview of the works devoted to this topic, containing more than 60 references. In [2] - [10] a number of different approaches are considered for the search for optimal spherical configurations. Papers [11] - [16] describe practical applications of problems of placing spherical objects with additional constraints. It should be noted that when considering this class of problems, much attention is paid to mathematical modeling of systems with spherical objects and the development of special methods for nonlinear optimization to solve them. Issues related to information support, analysis of the data structure and ways to transform geometric information in the optimization process are not fully covered.

The purpose of this paper is to describe the possibilities of using modern information technologies when considering the investigated class of problems based on existing mathematical models and analysis of optimization methods.

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This paper proposes an information technology that connects into a single system modules for solving an optimization problem and solution visualizing. The decision maker is included in the system. Using the results of the decision, especially in its visual representation, the decision-maker can change the parameters of the decision, relying on his experience and intuition. Such transformed solutions are used in the next step as a starting point for finding a local extremum. This approach is because the problems under consideration are multi-extreme and have a large number of local extrema. The suggested method is one of the ways to improve the local solution.

2. Geometric information and configuration space of spatial objects

Investigations of configurations as mathematical objects are naturally associated with the notion of configuration space. The configuration space determines the configuration of the system, that is, the set of values of geometric variables called generalized coordinates and sets the location in the space of some system and its parts both relative to one another and with respect to a given fixed point.

In the papers [17] - [19] the concepts of geometric information and geometric objects configuration space are introduced. Geometric information $\boldsymbol{g} = \left(\{s\},\{\mu\},\{p\}\right)$ about an object $S \subset R^3$ includes a spatial form $\{s\}$ as an equivalence class on a set of point sets; metric parameters of the form $\{\mu\} = (\mu_1,...,\mu_k)$, specifying the dimensions of the object; placement parameters $\{p\} = (p_1,...,p_t)$, that determine the position of an object S in the space R^3.

In accordance with the general concept of constructing such spaces, we define their structure in the following way. The basis for specifying the components $\{s\}$ and $\{\mu\}$ of geometric information $\boldsymbol{g}$ is the equation of its boundary

$$f(\xi,\mu) = 0,\ \xi \in R^3, \tag{1}$$

where the variables (ξ,μ) have a domain of admissible values $D \subseteq R^k$, and the function $f(\xi,\mu)$ is defined and continuous everywhere on $R^3 \times D$.

Depending on the analytic type of function $f(\xi,\mu)$ various classes of objects can be formed. At the same time, from a practical point of view it is natural to distinguish classes of models of the most widespread material objects. These objects are called basic.

Different constraints may be imposed on the values of generalized variables in the configuration space $\Xi(S_B)$. These restrictions will form the domain of admissible configurations in the configuration space for the class of tasks under consideration. On the domain of admissible values of generalized variables may be imposed other additional restrictions that arise when solving specific classes of tasks, for example, when searching for optimal configurations in a certain sense. We divide the following restrictions into the following groups: restrictions associated with the fixation of some generalized variables, restrictions that arise on the mutual location of geometric objects, and the restrictions imposed by certain physical or mechanical properties of the synthesized system. If to give on a set of geometric objects some relations, binary in particular, we obtain different classes of placement configurations.

If for any pair of objects a non-overlapping binary relation must be performed, that is, these objects do not have common internal points, then the corresponding configuration of the placement is called the packing configuration of geometric objects. Let us note that in most practical packaging tasks an additional object is called a container. All geometric objects that are considered, must belong to the container. Note that the generalized variables in the configuration space may be subject to additional restrictions that generate special classes of configurations [19]. A distinctive feature of such configurations is the presence of restrictions on the minimum and the maximum permissible distances between objects.

In the case when geometric objects are solids with given masses, a balanced system of such bodies gives the balanced packing configuration. Note that the configuration spaces both layout and balanced packing configurations are the same.

We indicate also the covering configurations [20], [21] and partitioning configurations [22], [23]. A special class of configurations are Euclidean combinatorial configurations that are formed in the case when the placement parameters and metric parameters of the objects take discrete values. Such configurations have a number of important properties, the implementation of which allows us to propose new approaches to modeling and solving optimization problems [24], [25].

3. Synthesis of spherical object configurations

In this paper, we will examine the class of geometric objects of a spherical shape. Three groups of spherical objects will be consider: two-dimensional (2D), three-dimensional (3D) and multidimensional (nD) objects. Each of the basic objects is given by the equation of its boundary. Then the generalized variables of the configuration space of spherical objects are their radii and the coordinates of the centers.

Consider the spherical object $S \subset R^n, n = 2,3,\ldots$ of the radius $r > 0$. We associate with S the moving coordinate system by choosing its beginning (the pole of the sphera) in the center of its symmetry. We define the spatial shape $\{s\}$ of the sphere by the equation of its boundary

$$f(\xi,r)=0, \xi \in R^n, \tag{2}$$

which, in accordance with the dimension of space, has the form:

$$f(\xi,r)=-x^2-y^2+r^2, \xi=(x,y), \tag{3}$$

$$f(\xi,r)=-x^2-y^2-z^2+r^2, \xi=(x,y,z), \tag{4}$$

$$f(\xi,r)=-\sum_{i=1}^{n}\xi_i^2+r^2, \xi=(\xi_1,\ldots,\xi_n). \tag{5}$$

If $S \subset R^3$, then $\mu = r$ is a metric parameter, and we denote $p=(v_1,v_2,v_3)$ location parameter. The configuration space of a sphere is given by generalized coordinates v_1,v_2,v_3,r, and the equation of the general position of the sphere has the form

$$F(x,y,z,v_1,v_2,v_3,r)=r^2-(x-v_1)^2-(y-v_2)^2-(z-v_3)^2=0. \tag{6}$$

Suppose $\Omega=\{S_1,\ldots,S_n\}$ is a set of basic spherical objects. We assign the structure of a complex object in the form

$$S_B = B(S_1,\ldots,S_n)=\bigcup_{i=1}^{n} S_i . \tag{7}$$

An object S_B whose structure is given by the expression (7), we call a composite object.

Based on the general approach and the proposed general classification, we propose the *typology* of spherical objects configuration.

In the configuration spaces of spherical objects S_i and S_j with corresponding generalized variables $g^i=(x_i,y_i,z_i,r_i)$ and $g^j=(x_j,y_j,z_j,r_j)$ the constrains of non-overlap have the form

$$(r_i+r_j)^2-(x_i-x_j)^2-(y_i-y_j)^2-(z_i-z_j)^2 \geq 0. \tag{8}$$

The condition for the contain of a sphere $S_i(g^i)$ into a spherical container $S_0(g^0)$ with generalized variables $g^0=(x_0,y_0,z_0,r_0)$ will be written as

$$-(x_0-x_i)^2-(y_0-y_i)^2-(z_0-z_i)^2+(r_0-r_i)^2 \geq 0, \tag{9}$$

where $x_0 = y_0 = z_0 = 0$.

Note that the generalized variables $g^0, g^1, \ldots, g^n$ in the configuration space $\Xi(S_0) \times \Xi(S_B)$ may be subject to additional constraints that generate special classes of packing configurations. First of all, it is about layout configuration and balanced packing configuration.

A distinctive feature of such configurations is the constraints on the minimum and the maximum admissible distances between objects. In this case, the conditions will be

$$d_{ij}^{min} \le (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 - (r_i + r_j)^2 \le d_{ij}^{max}, \tag{10}$$

where $d_{ij}^{min}, d_{ij}^{max}$ are corresponding distances between objects.

In the case when geometric objects $S_i, i \in \boldsymbol{J}_n$ are solids and have masses $m_i, i \in \boldsymbol{J}_n$, a balanced system of such bodies defines the balanced packing configuration. If the poles of the objects $S_i, i \in \boldsymbol{J}_n$ coincide with the centers of their masses, balanced packing takes place under condition

$$\sum_{i=1}^{n} x_i m_i = 0, \sum_{i=1}^{n} y_i m_i = 0, \sum_{i=1}^{n} z_i m_i = 0. \tag{11}$$

Note that the configuration spaces $\Xi(S_0) \times \Xi(S_B)$ both layout and balanced packing configuration are the same.

The generalization of the spherical objects placement problem is the layout and packing of composite spheres. For each component of composite sphere (object of the first order) whose structure has the form (1), we fix the distances between the poles of the base spheres $S_i, S_j, i, j \in \boldsymbol{J}_n, i < j$ (objects of zero order). This will create an additional rigid system of restrictions on the placement parameters $p^i = (x_i, y_i, z_i), i \in \boldsymbol{J}_n$ of the base spheres

$$(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 = \rho_{ij}^2. \tag{12}$$

Depending on the possible affine transformations of complex objects (congruence, translation, rotation, etc.) the placement options may be subject to additional restrictions.

To formalize of non-overlap conditions of the components of spherical objects, the decomposition of such conditions is naturally performed on non-overlap of base spheres for various composite objects.

Let a symmetric matrices $B = \left[b_{ij}\right]_{n \times n}$ and $B^0 = \left[b_{ij}^0\right]_{n \times n}$ be given with elements

$$b_{ij} = \begin{cases} 1, & if\ S_i * S_j; \\ 0, & otherwize, \end{cases} \qquad b_{ij}^0 = \begin{cases} 1, & if\ S_i \in S_j \\ 0, & otherwize \end{cases}, \tag{13}$$

where $S_i * S_j$ means non-overlapping S_i and S_j.

The configuration of the spheres $S_i, i \in \boldsymbol{J}_n$ is said to be admissible if the inequalities (8) holds for any $i, j \in \boldsymbol{J}_n, i < j$ such that $b_{ij} = 1$. The configuration of the spheres $S_i, i \in \boldsymbol{J}_n$ is said to be valid if it is included, if the inequality (9) holds for any $i, j \in \boldsymbol{J}_n, i < j$ such that $b_{ij}^0 = 1$.

The vast majority of modern publications are devoted to the tasks of the layout and packaging of objects, the metric parameters are fixed, with the exception of the container. Introduction of the configuration spaces of geometric objects allows us to investigate fundamentally new, practically important problems from the general positions, one of the important classes of which is the problem of synthesis of optimal configurations of spherical objects.

Suppose there is a certain function on the set of permissible configurations of the spheres, which we call the quality criterion of the configuration. Then the problem of synthesis of optimal configurations of spherical objects is to determine the generalized variables of the configuration space, which has a lot of geometric interpretations and practical applications.

Select a set of numbers $\boldsymbol{L}' \subset \boldsymbol{J}_n$. Spheres $S_i, i \in \boldsymbol{L}'$ are called prohibition zones and fix their position in R^3, putting

$$x_i = \hat{x}_i,\ y_i = \hat{y}_i,\ z_i = \hat{z}_i,\ \ i \in \boldsymbol{L}'. \tag{14}$$

We will form a matrix $B = \left[b_{ij} \right]_{n \times n}$ with elements

$$b_{ij} = \begin{cases} 0, \forall\ i, j \in \boldsymbol{L}'; \\ 1,\ otherwize. \end{cases} \tag{15}$$

A set of admissible configurations will be given by a system of inequalities

$$(r_i\ + r_j\ )^2 - (x_i - x_j\ )^2 - (\ y_i - y_j\ )^2 - (\ z_i - z_j\ )^2 \geq 0, \tag{16}$$

where $i \in \boldsymbol{J}_n \setminus \boldsymbol{L}',\ j \in \boldsymbol{J}_n$.

Let a quality criterion , which need to be minimum, is

$$\varphi_1\left(x_1, y_1, z_1, r_1, \ldots, x_n, y_n, z_n, r_n \right) = \max_{i \in \boldsymbol{J}_n \setminus \boldsymbol{L}'} \left\{ x_i + r_i, y_i + r_i, z_i + r_i \right\}. \tag{17}$$

If $\boldsymbol{L}' = \varnothing$, we have a classic problem of placing unequal spheres in a sphere of minimal radius. In this case, we can consider a container S_0 as a sphere of variable radius r_0 with the placement parameters $x_0 = y_0 = z_0 = 0$. Then

$$\varphi_1\left(x_0, y_0, z_0, r_0, x_1, y_1, z_1, r_1, \ldots, x_n, y_n, z_n, r_n \right) = r_0. \tag{18}$$

New classes of problems of synthesis of optimal configurations of spherical objects are considered, generalizing the classical problems of packing and arrangement in the case of variables of metric parameters of objects (radii of spheres).

We fix the generalized variables

$$x_0 = y_0 = z_0 = 0, r_0 = \hat{r}_0,\ x_i = \hat{x}_i,\ y_i = \hat{y}_i,\ z_i = \hat{z}_i,\ \ i \in \boldsymbol{L}', \tag{19}$$

in the configuration space $\Xi(S_B) = \Xi(S_0) \times \Xi(S_1) \times \ldots \times \Xi(S_n)$.

Then, depending on the choice of specific constraints, we describe the appropriate set of admissible configurations. In turn, the variety of quality criteria of the configurations expands. For example, we will consider such a criterion as the maximum of occupied part of a container with variable radii of spheres:

$$\varphi\left(r_1, \ldots, r_n \right) = \frac{4}{3}\pi \sum_{i \in \boldsymbol{J}_n \setminus \boldsymbol{L}'} r_i^3. \tag{20}$$

In the configuration space $\Xi(S_B) = \Xi(S_1) \times \ldots \times \Xi(S_n)$ we fix the placement parameters $x_i = \hat{x}_i,\ y_i = \hat{y}_i,\ z_i = \hat{z}_i$ of spheres $S_i, i \in \boldsymbol{J}_n$.

Let $\varphi\left(r_1, r_2, \ldots, r_n \right)$ - arbitrary convex function on R_+^n. Then we have the problem of convex programming

$$\varphi\left(r_1, r_2, \ldots, r_n \right) \to \min \tag{21}$$

with linear constraints

$$r_i\ + r_j\ - \rho_{ij} \geq 0,\ \ i, j \in \boldsymbol{J}_n,\ \ i < j\ , \tag{22}$$

where

$$\rho_{ij} = \sqrt{(\hat{x}_i - \hat{x}_j)^2 + (\hat{y}_i - \hat{y}_j)^2 + (\hat{z}_i - \hat{z}_j)^2}, \ i, j \in \boldsymbol{J}_n, \ i < j. \tag{23}$$

Let a given matrix $\boldsymbol{C} = \left[c_{ij} \right]_{n \times n}$ whose elements determine the number of connections between the spheres S_i and S_j, $i, j \in \boldsymbol{J}_n$, $i < j$. Then the total length of the network connecting the objects is given by the function

$$\varphi(x_1, y_1, z_1, r_1, \dots, x_n, y_n, z_n, r_n) = \sum_{i \in \boldsymbol{J}_n} \sum_{\substack{j \in \boldsymbol{J}_n \\ j > i}} c_{ij} \ \rho(S_i, S_j), \tag{24}$$

where

$$\rho(S_i, S_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} - r_i - r_j. \tag{25}$$

Let the spheres have fixed radii and represent a solid density body. We minimize the function that characterizes the deviation of the center of the masses of the combination of bullets from a given point. We have

$$\varphi(x_1, y_1, z_1, r_1, \dots, x_n, y_n, z_n, r_n) = = \sum_{i \in \boldsymbol{J}_n} \left[\left(x_0 - \frac{m_i x_i}{m_\Sigma} \right)^2 + \left(y_0 - \frac{m_i y_i}{m_\Sigma} \right)^2 + \left(z_0 - \frac{m_i z_i}{m_\Sigma} \right)^2 \right], \tag{26}$$

where $m_i = 4/3 \ \pi v r_i^3$, $i \in \boldsymbol{J}_n$, $m_\Sigma = \sum_{i \in \boldsymbol{J}_n} m_i$.

Thus, the formalization of problems of synthesis of optimal configurations of spherical objects allows us to conclude that depending on the quality criteria and the choice of specific constraints that form the set of admissible configurations, such tasks can be attributed to the corresponding class of mathematical programming problems.

4. Informational technology of synthesis of spherical object configurations

The paper proposes a unified approach for solving all types of problems of synthesis of spherical objects under consideration. This approach is based on information technology, which includes the interaction of several interconnected units. Each of these units solves its own specific tasks. In our work, 4 such units can be distinguished:

- optimization;
- visualization;
- consolidated data warehouse;
- modeling and control.

The optimization unit as a basis contains a solver that allows you to effectively solve nonlinear optimization problems to obtain local extrema. In our work, we use the IpOpt software package for this purpose. It uses interior point techniques for high-dimensional NLP problems. Freely distributed, has open source code and does not require obligations when creating commercial software.

The visualization unit is an interactive graphical interface for working with 3D graphics. For this purpose, the work uses software from Autodesk 3ds MAX, which is provided under a free license. To automate the display of data, programs are used - scripts written in the built-in 3ds MAX Scripts language. They are executed in the environment of the internal Listener code interpreter, which renders the objects. Thanks to this unit, you can display the obtained solutions, interactively change their parameters or create a new problem using a graphical interface.

The need for a consolidated data warehouse is caused by the following. When using traditional tools, there is redundancy in data volumes, and when processing significant resources, they are converted. Thus, the data obtained from a variety of sources and different types of information resources integrated into the system must be consolidated into an adequate information storage model. For this we used ORM (Object Relational Mapping), and to transform ADO.NET Entity Framework Code First into a repository strategy, and to save the results, we used a portable SQLite database.

The modeling and control unit is the core of the entire system. It is implemented in C ++ language. In it, using polymorphic classes, information models of both spherical objects of varying degrees of complexity and problems of synthesis of configurations of all considered types are built, stored and transformed. In addition, this unit provides interaction between all units of the system and automatically converts the internal representation of the problem into the form required for an optimization, visualization or data store units.

The model of information technology for synthesis of spatial configurations of spherical objects is shown in Fig. 1.

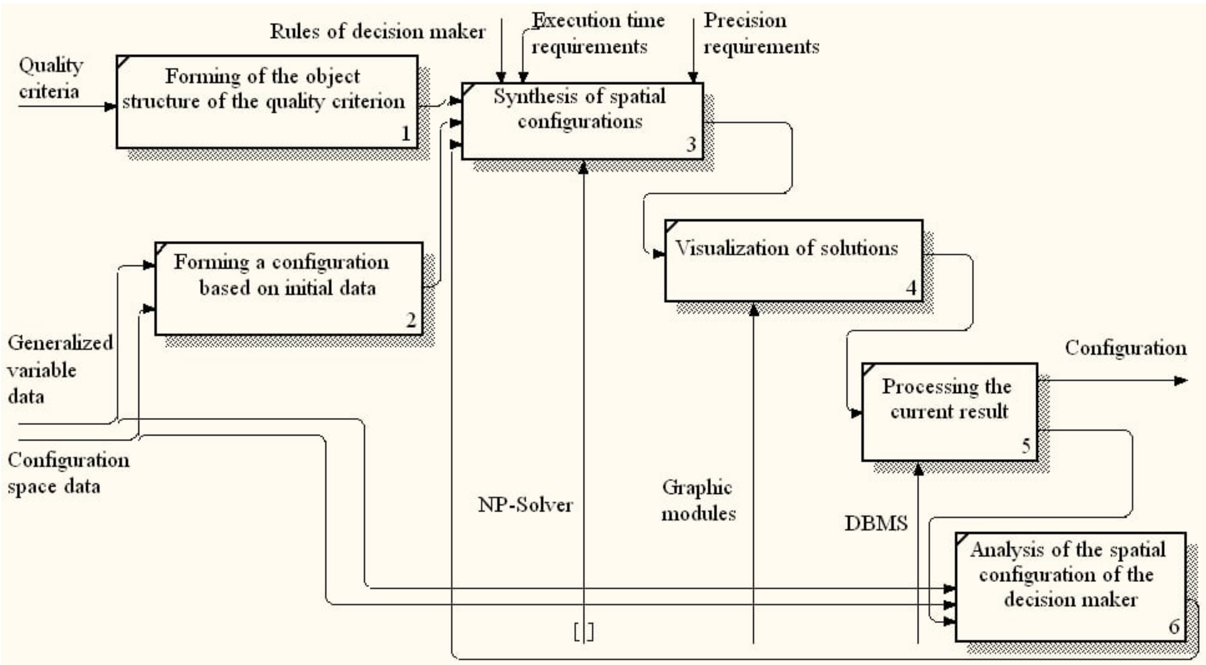

Figure 1: The model of information technology for synthesis of spatial configurations

The expediency of this approach is due to the following. All the problems of the considered classes of synthesis of spatial configurations are are NP-complex complicated ones, which have a high dimension (in real systems with more than 1000 variables), are multiextreme (with a non-linear number of local extrema). For many of them it is possible to find the local extremum using standard classical methods. One of the ways to improve a local solution, based on the proposed information model, consists in a dialogue between a solver and a decision maker. The dialog is based on a graphical display of the solver result. The decision maker interactively modifies the received locally optimal configuration and sends it as an initial placement to the solver. This dialogue can continue until the decision-maker stops the process. All obtained locally optimal solutions are stored in the database and are the basis for the final selection. This process does not preclude the use of automatic global search methods, for example, an genetic algorithms [26, 27].

Process of synthesis of optimal spherical configurations including the participation of a decision maker is shown on Fig. 2.

The inclusion of a decision maker in the process will also allow solving problems of the types under consideration with additional constraints that are difficult to formalize. In such cases, only the decision maker can choose the final outcome. Consider as an example the spherical objects placement problem. It is necessary to find the minimum spherical shell for a set of spheres of different radii. Starting point is shown on Fig. 3 and a locally optimal solution obtained by the solver given on Fig. 4.

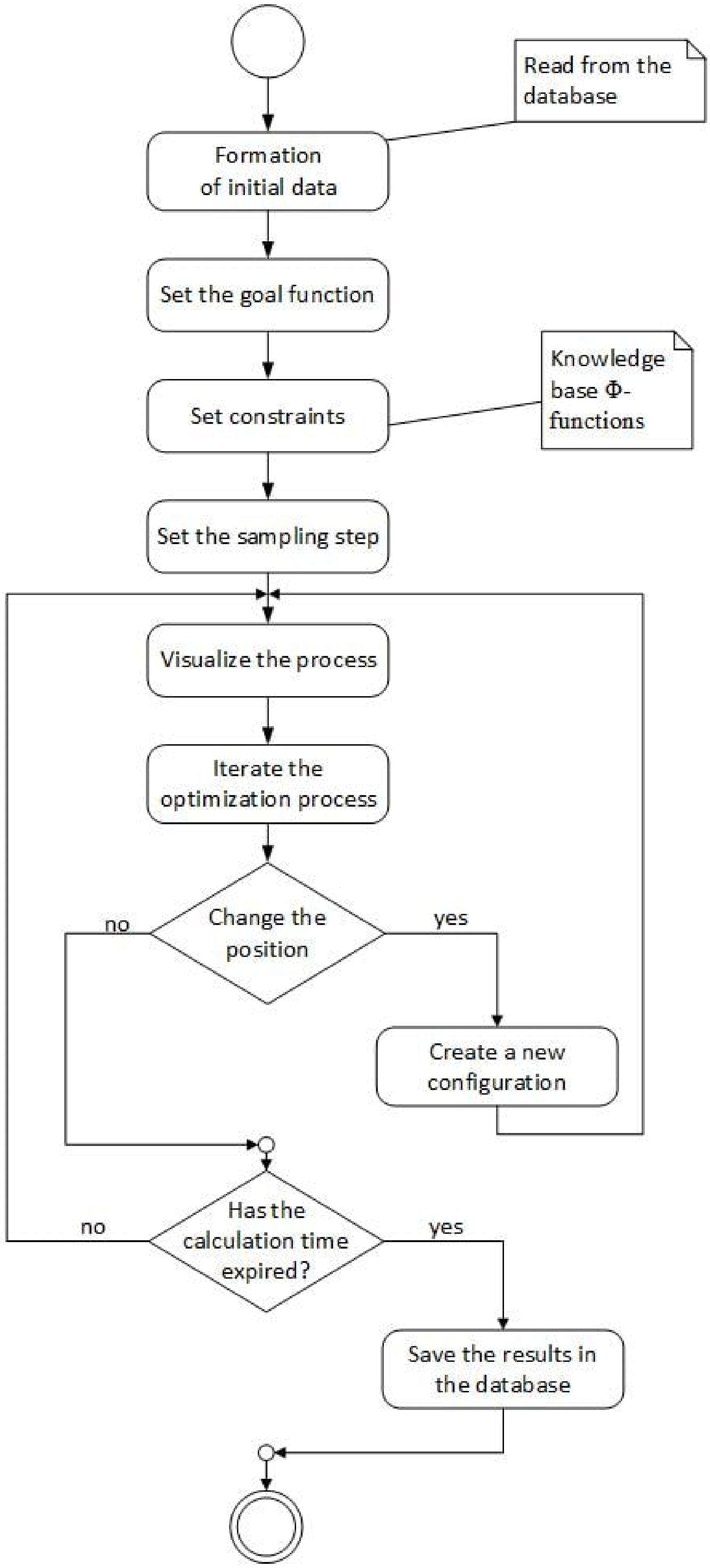

Figure 2: UML diagram of a process of synthesis of optimal spherical configurations including a decision maker

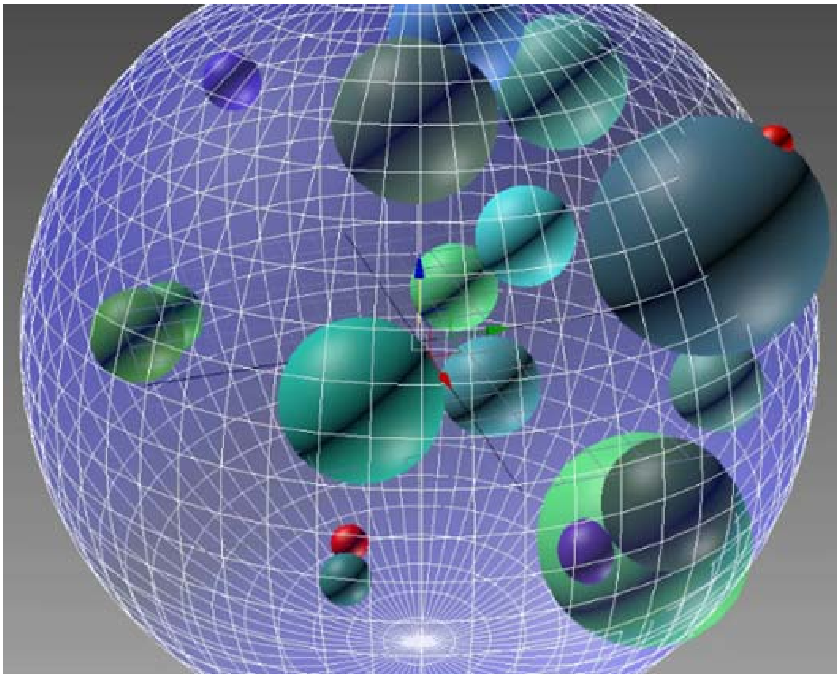

Figure 3: Starting point

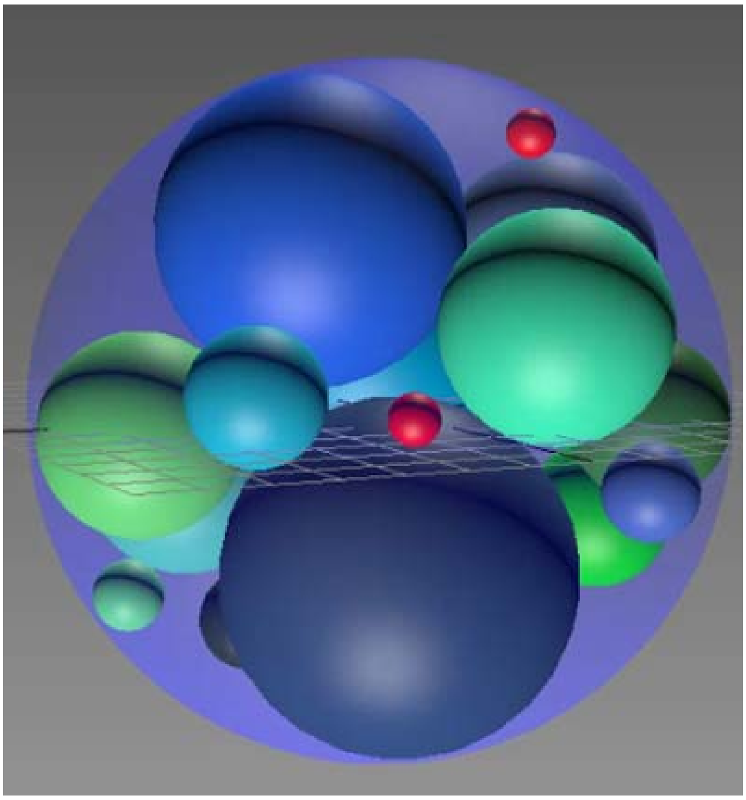

Figure 4: Locally optimal solution obtained by the Solver

The decision maker changed the received decision and sent it back to the Solver (Fig. 5). A new locally optimal solution is shown on Fig. 6.

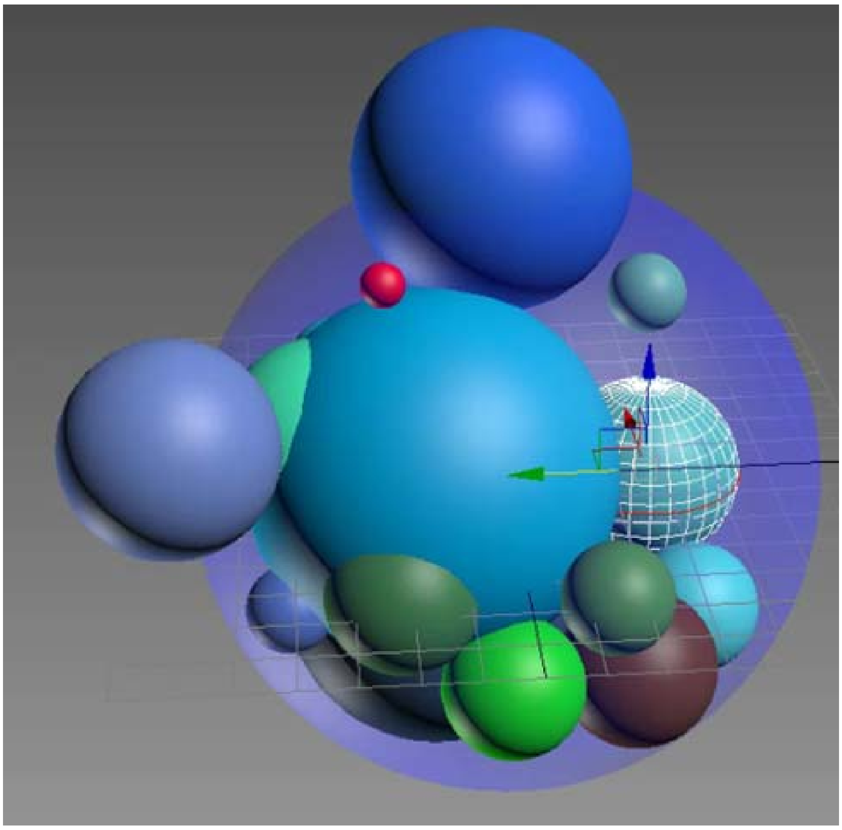
Figure 5: New point proposed by the Decision maker

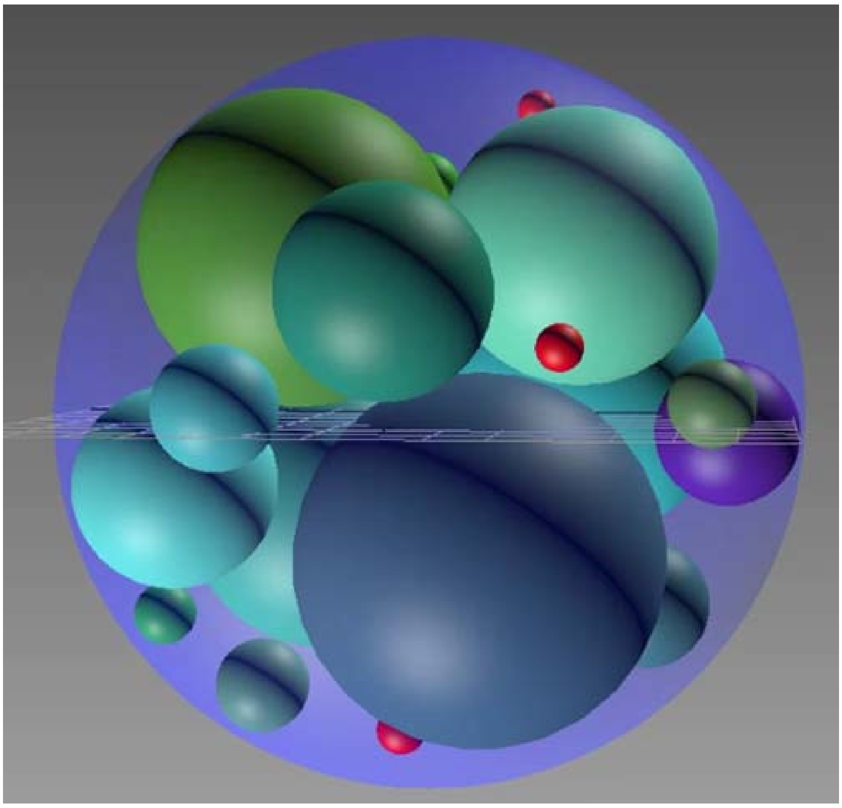
Figure 6: Locally optimal solution obtained by the Solver

5. Conclusions

The paper provides a general formulation of problems for the synthesis of spatial configurations and the classification of these problems. Analytical models of various types are given for the case of spherical objects. To solve the problems of synthesis of spherical configurations, an information technology has been proposed, including modules for optimization, visualization and the participation of a decision maker. This approach made it possible to use the experience and intuition of a specialist for a global search for a solution, i.e. attempts to improve local extrema. An example of using the proposed technology is given.

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