Approach to Solve the Problems of Filtration and Extrapolation in the Construction of Functionally Stable Stochastic Systems with Delay

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Abstract

Filtration and interpolation problems play an important role in the theory of complex technical object control. To improve the operating efficiency control of these objects, it is necessary to improve the mathematical models of the control objects. A large number of objects can be attributed to stochastic discrete objects with limited delay and, under a priori uncertainty, to stochastic discrete objects with unlimited delay as well. It is important to substantiate the application of stochastic analysis methods to solve problems of filtration and interpolation of systems with delay. The problem of conditionally optimal filter design is substantiated.

Keywords [1]

Filtration, interpolation, functional stability, recovery control, discrete stochastic systems with limited delay, discrete stochastic systems with unlimited delay, filtration estimation

1. Introduction

In the last quarter of the last century, due to the increase of the level of requirements for reliability and preciseness of the complex technical systems functioning, they became more complicated in their structure and more sophisticated in their control systems. That was the time of significant advances in aviation and cosmonautics. It is natural that exactly such complex systems headed the development of scientific theory. Exactly in aviation and cosmonautic systems, advanced scientific inventions were implemented. That was the time of arising new control theories mainly dedicated to spacecraft. That time refers to a range of crucial changes: the first instrument landing systems, the latest approaches to information exchange arrangement in the onboard hardware of aircraft, the new orbital space stations, the attempts of docking of spacecraft and orbital stations in space (the first attempts were not successful), reusable space systems, etc. All of them had a rather complicated structure and tight connections between their elements and had a significant impact on the environment. New specific approaches to the mathematical description of a scenario for such systems appeared as well as a theory of optimal control and then adjustment control, etc.

Problems of the mathematical description of complex technical system operating and their control are described in various sources, which offer the use of various approaches and theories [1–4]. As we can see, the range of the systems is quite diverse: the first steps were directed to the linear continuous dynamic systems, and today we move ahead with new fuzzy controllers for steel-smelting

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furnaces, unmanned aircraft control, and the up-to-date network technology. At present time, the theory of functional stability has become widespread [5–9]. The theory arose at the cross-section of many approaches to ensuring the stable functioning of complex technical systems: improvement of reliability of the system operation, improvement of repairability conditions, the possibility of recovery of some features, and self-adjustment. Such systems are supposed to be self-controlled and self-adjusted, redistribute tasks within a system to achieve the set tasks. The peculiarity of this theory application is a number of requirements to be met by the system for its application. It is important to take into account the costs both of hardware and software. In fact, on boards of aircraft and cosmic systems, the hardware redundancy was used, i.e. there were 3-4 sets of the onboard control unit blocks.

Concerning software of computing complexes, booting of processors and memory of onboard computers did not exceed a third of all capacities. Such an approach to arrangement allowed us to do maneuvers and to add the new features to the complexes. For the years of its existence, this theory has spread from space systems literally to all shears of industry. Especially, it has found application in intellectual control systems of network technology, including the control of individual or group unmanned aircraft. In [10–15], it was shown its usage possibility for complex technical systems with hardware or software operating at a loss. An important feature proved to be the ability of a system to self-check and find out failures in its operation. In this way, the theory of recovery control appeared. The essence of this theory consists in an ability of a system to change the program of its operation, and when necessary, to change the interconnections between the elements to attain the set goal [16]. For instance, when an airplane's elevator fails, it is possible, using the engine thrust and the wing mechanization, to cope with a change in flight altitude. With this, it is necessary to take into account that in such a case, the airplane response for the change of commands of control will be definitely different than in its ordinary operation. The main indicators of the transition processes will be significantly exceeded; therefore there is the expediency of additional investigations of the limits of application of such theories. Also, in [16–18], the study of the application of recovery control theory to functionally stable systems was carried out and positive outcomes of preliminary studies were revealed. An important element of any theory is the modeling apparatus and model parameter estimates.

First, the simple linearized differential equations were used as mathematical models, and then a stochastic component was added, which made it possible to partially take into account the peculiarities of real systems functioning. Here, it should be noted that the complication of mathematical models being used in the control systems necessitates the search for compromises [17]. That is, it is necessary to find a compromise between the accuracy of the system operation description, the time devoted to the operations, and the onboard capacities for the realization of the set tasks.

In the general case, stochastic differential systems with a state vector Z, characteristic functions $a(Z, t)$, $b(Z, t)$ during a stochastic process W can be described by the equation:

$$dZ = a(Z,t)dt + b(Z,t)dW \,. \tag{1}$$

In previous studies [17], there was obtained the equation for the conditionally optimal extrapolator based on expansion in terms of Hermite polynomials:

$$\begin{aligned} \dot{C}_\nu &= M\{q_\nu(\partial / i\partial\lambda)[i\lambda^T a(z,t) + X(b(z,t)^T;t)\exp(i\lambda^T z)\} + \\ &+ [q_\nu^m(\partial / i\partial\lambda)^T g_1(\lambda,t)]_{\lambda=0}\dot{m} + tr\{[q_\nu^k(\partial / i\partial\lambda)g_1(\lambda,t)]_{\lambda=0}\dot{K}. \end{aligned} \tag{2}$$

It has been also concluded that it is possible to evaluate the accuracy of filtering extrapolation processes and comparing different filters based on the proposed mathematical apparatus.

With the widespread use of computerized control systems, stochastic discrete systems have developed significantly. On the sampling interval T at the instant of time $t(k) = kT$, the state vector values Z_k and the random vector values V_k with the function $\varphi_k(Z, V)$, it is advisable to describe using the difference equation:

$$Z_{k+1} = \phi_k(Z_k, V_k) \qquad (k = 0,1,\cdots)\,. \tag{3}$$

The state vector Z_k has an initial value Z_0 and does not depend on the sequence that describes the domain of states of the system $\{V_k\}$.

In the research [18], there were considered the peculiarities of these processes for simple discrete and discrete-continuous systems. Using the method of quasi-moments and orthogonal expansions, it is also possible to solve the problems of filtering and extrapolation for such systems. By their notation, the following solutions correlate with previous studies

$$\dot{C}_\nu = M\{q_\nu([\partial^T / i\lambda' Z''(t) Z'''(t)^T]^T)[i\lambda'^T a(Z,t)+$$
$$+\chi(b(Z,t)^T \lambda'; t)]\exp(i\lambda'^T Z^T)\}_{\lambda'=0} + Mq_\nu^m(\bar{Z})^T \dot{m} + t_r\{Mq_\nu^m(\bar{Z})\dot{K}\},$$
$$C_\nu(t^{(l+1)}) = Mq_\nu([Z_{l+1}'^T \omega_l(Z_l, V_l)^T Z_{l+1}'^T]^T, \quad l = 0,1,\dots.$$

Barbashin E. A. and Galiullin A. S. [19–21] have worked out the solution for such systems.

Modern complex dynamic systems, such as aircraft systems, electromechanical systems, etc., have a more complex structure. They include setters, calculators, and actuators, thus it is expedient to describe them using stochastic discrete systems with delay (limited or unlimited under uncertain operating conditions).

The application of such models to determine the control effects was shown in [22–24].

2. Description of the mathematical model

2.1. Models of stochastic systems with delay

The randomness of the complex system functioning processes complicates significantly the choice of the mathematical body for processes and interconnections description within the system as well as between the system and the environment. Let us first consider **models of stochastic systems with unlimited delay**. All the above models are defined by the Markov random processes. In the case of the Markov process being unable to serve as a corresponding mathematical model of the system with a consequence, often a more general stochastic differential equation is an adequate mathematical model of the system.

$$dZ = a(Z_{t_0}^t, t)dt + b(Z_{t_0}^t, t)dW. \tag{4}$$

In contrast to (1), a and b are functionals of $Z(\tau)$, $t_0 \le \tau \le t$, i.e, functions of an elementary event ω, measurable for each $t \ge t_0$ with respect to σ-algebra induced by the values of a random process $Z(\tau)$ (or, what is the same, by the values of a random process $W(\tau)$) corresponding to all $\tau \in [t_0, t]$. By $Z_{t_0}^t$, in equation (4) the value set Z_τ of the $Z(\tau)$ process for $\tau \in [t_0, t]$ is denoted, $Z_{t_0}^t = \{Z_\tau : t_0 \le \tau < t\}$.

These models are called stochastic systems with unlimited delay.

As an example of the simplest such model, equation (4) can serve with

$$a(Z_{t_0}^t, t) = a(Z(t), \vartheta(t), t),$$
$$b(Z_{t_0}^t, t) = b(Z(t), \vartheta(t), t), \tag{5}$$

were $\vartheta(t)$ is determined by the following integral equation:

$$\vartheta(t) = \int_{t_0}^{t} A(t, \tau, Z(\tau), \vartheta(\tau))dt + \int_{t_0}^{t} B(t, \tau, Z(\tau), \vartheta(\tau))dW(\tau). \tag{6}$$

In (5) and (6), $a(Z, U, t)$, $b(Z, U, t)$ are functions mapping $R^p \times R^r \times R$ to R^p and R^{pq}, respectively; $A(\mathrm{t}, \tau, z, u)$ is a function mapping $R^p \times R^r \times R$ to R^p, $B(\mathrm{t}, \tau, z, u)$ is a function mapping $R^p \times R^r \times R$ to R^{pq}; the second integral in (6) is stochastic integral.

Equations (4) and (6) are a stochastic integro-differential system of equations that determines the extended state vector of the system $[Z^T \ \vartheta^T]$. In the special case, when the functions $A(\mathrm{t}, \tau, z, u)$ and $B(\mathrm{t}, \tau, z, u)$ do not depend on u, equation (4) is a stochastic integro-differential equation.

In a number of cases, we have a necessity not a possibility to limit the processes in time. For example, during the taking off, landing, docking, etc., the time for these processes is strictly limited. Therefore, it is possible to differentiate the stochastic systems with limited delay.

In modeling **stochastic systems with limited delay**, there are a number of peculiarities. A stochastic system with limited delay is a special case of the stochastic systems with unlimited delay and is described by stochastic differential equations of the following form:

$$dZ = a(Z_t, Z_{t-\tau_1}, \dots, Z_{t-\tau_m}, t)dt + b(Z_t, Z_{t-\tau_1}, \dots, Z_{t-\tau_m}, t)dW\ , \tag{7}$$

where $Z_t = Z(t)$ for $t > t_0$, $Z_t = 0$ and $t < t_0$; $\tau_1, \dots, \tau_n$ are deterministic or random variables.

This class of stochastic systems requires special methods of study.

It is assumed that stochastic equations (4)-(7) with corresponding initial conditions have solutions that correspond to stable dynamical systems.

2.2. The solution of the analysis problem of stochastic systems with delay

Let us begin the consideration with stochastic systems with unlimited delay.

It is not possible to derive in general the equation for finite-dimensional distributions of the state vector of the system (4). However, for a wide class of systems, equations (4)-(6) can be reduced to stochastic differential equations.

Let us first consider the stochastic systems (4)-(6) for the case when the functions A(t, τ, z, u) and B(t, τ, z, u) have the following form:

$$\begin{aligned} A(t,\tau,z,u) &= G(t,\tau)\phi(z,u,\tau); \\ B(t,\tau,z,u) &= \Gamma(t,\tau)\psi(z,u,\tau), \end{aligned} \tag{8}$$

where $G(t, \tau)$ and $\Gamma(t, \tau)$ – are matrix functions called the memory kernels; $\varphi(z, u, t)$, r – is a measuring function, $\psi(z, u, t)$ $r \times q$ is a matrix function.

In practice, the kernels $G(t, \tau)$ and $\Gamma(t, \tau)$ usually satisfy the following conditions (physical feasibility):

$$G(t, \tau) = 0; \quad \Gamma(t, \tau) = 0; \quad t < \tau, \tag{9}$$

$$\begin{aligned} &\int_{-\infty}^{\infty} |G_{ij}(t,\tau)|\,d\tau < \infty\,; \\ &\int_{-\infty}^{\infty} |\Gamma_{ij}(t,\tau)|\,d\tau < \infty\,, \end{aligned} \tag{10}$$

where $G_{ij}(t, \tau)$ and $\Gamma_{ij}(t, \tau)$ are matrix elements of $G(t, \tau)$ and $\Gamma(t, \tau)$.

For stationary kernels $G(t,\tau) = \bar{G}(\xi)$, $\Gamma(t,\tau) = \bar{\Gamma}(\xi)$, $\xi = t - \tau$. The Laplace transforms in this case represent rational functions of a complex variable S, i.e the following representation holds:

$$\begin{aligned} &\int_0^{\infty} \tilde{G}(\xi)\exp(-s_{\xi})d\xi = F(s)^{-1}H(s); \\ &\int_0^{\infty} \tilde{\Gamma}(\xi)\exp(-s_{\xi})d\xi = Q(s)^{-1}P(s). \end{aligned} \tag{11}$$

The initial stochastic system (4)-(6), and (8) can be reduced to the following stochastic differential system:

$$\begin{aligned} dZ &= a(z,v,t)dt + b(z,v,t)dw; \\ v &= v' + v''; \\ FU' &= H\phi(z,v,t); \quad QU'' = P\psi(z,v,t)\dot{w}. \end{aligned} \tag{12}$$

Here F, H, Q, P are the $r \times r$ matrix differential operators of the n-th orders, m ($n > m$) and K, l ($K > l$), respectively. Applying the known methods of the theory of linear systems, the last two equations of equation (12) can be reduced to the Cauchy form by expansion of the state vector with their subsequent presenting in the form of stochastic differential equations.

Let the nonstationary kernels $G(t, \tau)$ and $\Gamma(t, \tau)$ with fixed τ be solutions of the linear differential equations:

$$\begin{aligned} F_t\, G(t,\tau) &= H_t \delta(t-\tau) G(t,\tau) = 0, \quad t < \tau; \\ Q_t\, \Gamma(t,\tau) &= P_t \delta(t-\tau) \Gamma(t,\tau) = 0, \quad t < \tau, \end{aligned} \tag{13}$$

and at a fixed t are determined as follows:

$$G(t,\tau)=H_{\tau}^{*}G'(t,\tau); \qquad \Gamma(t,\tau)=P_{\tau}^{*}\Gamma'(t,\tau), \tag{14}$$

where the functions $G'(t, \tau)$ and $\Gamma'(t, \tau)$ at a fixed t are the solutions of linear differential equations:

$$F_{\tau}^{*}G'(t,\tau)=I_{r}\delta(t\cdot\tau); \qquad Q_{\tau}^{*}\Gamma'(t,\tau)=I_{r}(t\cdot\tau). \tag{15}$$

Here $F_t = F_t(t, D)$, $H_t = H_t(t, D)$, $Q_t = Q_t(t, D)$, $P_t = P_t(t, D)$, are the $r \times r$ matrix linear differential operators of the n-th orders, m ($n > m$) and K, l ($K > l$), respectively, the index of the operator denotes that the operator acts on a function considered as the function t at a fixed ε, with an asterisked bound operator, and I_r is a unity matrix of the r-th order. In this case, equation (6) can be substituted by the following equations:

$$U=U'+U'';$$
$$F_{t}U'=H_{t}\phi(z,v,t); \quad .$$
$$Q_{t}U''=P_{t}\psi(z,v,t)w.$$

As a result, the stochastic system (4)-(6) will be reduced to a stochastic differential system (12) with nonstationary operators F_t, H_t, Q_t, P_t.

For a wide class of more complex functions $A(\mathrm{t}, \tau, z, u)$ and $B(\mathrm{t}, \tau, z, u)$, in (6), the approximation of the form is often useful

$$\begin{aligned} A(t,\tau,z,u)&=\sum_{h=1}^{N}G_{h}(t,\tau)\phi_{h}(z,u,t); \\ B(t,\tau,z,u)&=\sum_{h=1}^{N}\Gamma_{h}(t,\tau)\psi_{h}(z,u,t), \end{aligned} \tag{16}$$

where $G_h(t, \tau)$ and $\Gamma_h(t, \tau)$ are kernels falling into the types considered.

In practice, due to insufficient a priori information, the functions $A(\mathrm{t}, \tau, z, u)$ and $B(\mathrm{t}, \tau, z, u)$ are usually known approximately. Therefore, they can almost always be approximated by the formula (8) or (16) with functions $G(t, \tau)$ and $\Gamma(t, \tau)$ of one of the two above types. After that, by means of the suggested above method, equations (4)-(6) are reduced to stochastic differential equations of the form (12).

Another special case, when the stochastic system (4)-(6) can be reduced to a stochastic differential system of the form (1) by expansion of the state vector, is the case when the functions $A(\mathrm{t}, \tau, z, u)$ and $B(\mathrm{t}, \tau, z, u)$ allow the following representation

$$A(t,\tau,z,u)=A^{+}(t)\cdot A^{-}(\tau,z,u); \;\; B(t,\tau,z,u)=B^{+}(t)\cdot B^{-}(\tau,z,u), \quad t>\tau. \tag{17}$$

Assuming that

$$Y'=\int_{t_0}^{t}A^{-}(\tau,z,u)d\tau; \qquad Y''=\int_{t_0}^{t}B^{-}(\tau,z,u)dw(\tau). \tag{18}$$

It is possible to reduce the initial stochastic system to the following stochastic differential one:

$$\begin{gathered} dZ=a(z,u,t)+b(z,u,t)dw; \;\; u=A^{+}(t)Y'+B^{+}(t)Y''; \\ dY'=A^{-}(t,z,u)dt; \;\; dY''=B^{-}(t,z,u). \end{gathered} \tag{19}$$

It should be noted that usually the functions $A(\mathrm{t}, \tau, z, u)$ and $B(\mathrm{t}, \tau, z, u)$ in (6) can always be approximated by expressions of the form (17) or by more general expressions:

$$\begin{aligned} A(t,\tau,Z(\tau),U(\tau)&=\sum_{h=1}^{N}A_{h}^{+}(t)A_{h}^{-}(\tau,Z(\tau),U(\tau)); \\ B(t,\tau,Z(\tau),U(\tau)&=\sum_{h=1}^{N}B_{h}^{+}(t)B_{h}^{-}(\tau,Z(\tau),U(\tau)). \end{aligned} \tag{20}$$

Therefore, equations (4)-(6) can almost always be reduced to the stochastic differential equation (1) by expanding the state vector of the system.

Thus, the stochastic system of the form (4)-(6) can almost always be approximated by a further stochastic differential system of the form (1), after which any of the described methods of stochastic analysis can be applied.

Let us consider **stochastic systems with limited delay**.

Any stochastic system with delay of the form (7) is a case of the system (4)-(6), where the function $A(\mathrm{t}, \tau, z, u)$ of (6) is a linear function z, and its coefficients are delta functions of the form $\delta(t - \tau_k - \tau)$ and $B(\mathrm{t}, \tau, z, u)$. It is clear that none of the above considered approximations is appropriate to these systems. Special methods should be developed here.

In practical problems, for the analysis of systems with delay, the transfer function of the link with pure delay is often approximated.

$$\exp(s\tau) = 1 + s\tau + \cdots + s^n\tau^n / n! \tag{21}$$

Using this approximation, introduce new variables Y_1, ..., Y_m that satisfy the following differential equation

$$\tau_k^n D^n / n! + \cdots + \tau_k D + 1) Y_k = Z \qquad (k = 1, \ldots, m). \tag{22}$$

Then, equation (7) is replaced by equation (22)

$$dZ = a(Z, Y_1 \ldots, Y_m, t) + b(Z, Y_1 \ldots, Y_m, t) dw. \tag{23}$$

Equations (22), (23) with initial conditions

$$Y_k = 0 \text{ for } t_0 < t \le t_0 + \tau_k \tag{24}$$

describe stochastic differential systems that approximate the system with delay (7).

Substituting (7) by equations (22), (23), any of the above methods for the approximate determination of finite-dimensional distributions of the system state vector can be used.

3. Results and Discussions

3.1. The peculiarities of solving filtration and extrapolation problems in the construction of functionally stable discrete systems with delay

The issues of filtering have not been enough theoretically substantiated for our class of systems and could be applied with significant limitations for prompt estimation. Requirements for the simplicity of computational estimates lead to the idea of conditionally optimal estimation.

To operate in real time with limited computing power, there is recommended to use conditionally optimal filtering [16–18, 22–24]. We will use simple filters (for example, to evaluate the solutions of difference equations), which can serve as an example of filters that meet the requirements of simplicity of calculations.

For our case, we use the following approach:

$$\Sigma = AU, \tag{25}$$

$$U_{k+1} = \delta_k \xi_k(y_k, u_k) + \gamma_k, \tag{26}$$

where A is a constant $p \times N$ matrix, $\xi_k(y,u)$ are functions mapping $R^m \times R^N$ to R^r; δ_k, γ_k are arbitrary matrices of $N \times r$, $N \times 1$ sizes, respectively.

The choice of sequences $\{\delta_k\}$, $\{\gamma_k\}$ determines the allowed filter. The selection (selection of parameters) of the matrix A in equation (25), the function $\xi_k(y,u)$ (26), the numbers N, r is decisive for the selection of filters. For optimal filtering, the determination of the coefficients δ, γ, ξ, η, ζ, in (26) can be performed according to the methods described in the papers [18, 22–26].

An important issue for filtering is the issue of optimality criteria. Minimizing the mean square of the error $M|\hat{Z}_t - Z_t|^2 \, (M|\hat{Z}_{k+1} - Z_{k+1}|^2)$ in the case of the filtering problem and $M|\hat{Z}_t - Z_{t+\tau}|^2 \, (M|\hat{Z}_{k+1} - Z_{k+j+1}|^2)$ in the case of the extrapolation problem at any time instant $t(t^{(k+1)})$ may have no solutions. The use of the theory of suboptimal filtration for discrete systems is shown in the papers [18, 22–26].

3.2. Conditionally optimal filters for discrete stochastic systems

The complex system functioning (Fig. 1) implies the programmed trajectory, which can be designed under strict conditions or be adaptive. One more important is the prompt assessment of self-position in space. All above requires powerful computing capacities. As it has been shown above, there exists a mathematical body allowing significant simplification of the working models and reduce the requirements for computing capacities.

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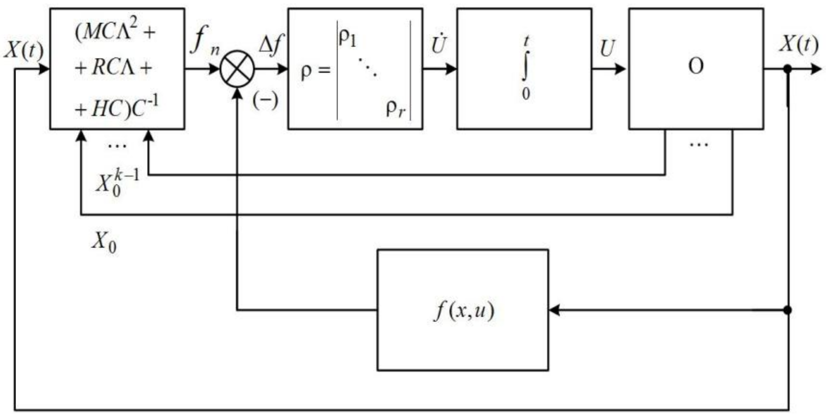

Figure 1: Model of functionally stable complex dynamic object

For optimization of δ_k, γ_k in equation (26) the linear root mean square regression of the value equation or Z_{k+j+1} on a random vector $\xi_k(X_k, U_k)$ is found. The standard methods lead to the following equations:

$$A\delta_k P_k = L_k;\ A\gamma_k = m_k - A\delta_k L_k, \qquad (27)$$

where

$$m_k = MZ_{k+j+1};\ l_k = M\xi_k(X_k, U_k); \qquad (28)$$
$$P_k = M[\xi(X_k, U_k) - l_k]\xi_k(X_k, U_k)^T;$$
$$L_k = M(Z_{k+j+1} - m_k)\xi_k(X_k, U_k)^T.$$

To calculate the mathematical expectations in these formulae in the case of the filtering problem ($j = 0$), let us apply equation (3) and use the equation:

$$g_{k1},\dots k_{n+1}(\lambda_1,\dots,\lambda_n) = M\exp\left\{ i\sum_{i=1}^{n-1}\lambda_l^T Z_{kl} + +i\lambda_n^T\omega_{k_n}(Z_{k_n}, V_{k_n}),\ (n=1,2,\dots)\right. \qquad (29)$$

with the known function $g_1(\lambda)$ and initial conditions [25–27]:

$$g_{k1,\dots,k_{n-1},k_{n-1}}(\lambda_1,\dots,\lambda_n) = M\, g_{k1,\dots,k_{n-1}}(\lambda_1,\dots,\lambda_{n-1}+\lambda_n),\ (n=2,3,\dots) \qquad (30)$$

for a one-dimensional characteristic function of a random sequence $\{[X_k^T Z_k^T U_k^T]^T\}$ determined by the difference equations (3) and (26). The joint solution of equations (27), (28), (29) completely solves the problem of designing conditionally optimal filters. To find the mathematical expectations in (28) in the case of the extrapolation problem, equations (29), (30) are added to the previous equations of the two-dimensional characteristic function of the sequence $\{[X_k^T Z_k^T U_k^T]^T\}$.

3.3. Algorithms of identification and adjustment for complex discrete dynamic functionally stable systems

In the works [16, 22–24], the theorem on the existence of optimal control of a complex dynamic object Figure 1 by means of a special device Figure 2 using a specialized filter Figure 3 has been proved. Consider the features of such systems in conditions of limited computing resources and in real time, which is inherent in the operation of a large number of mobile systems.

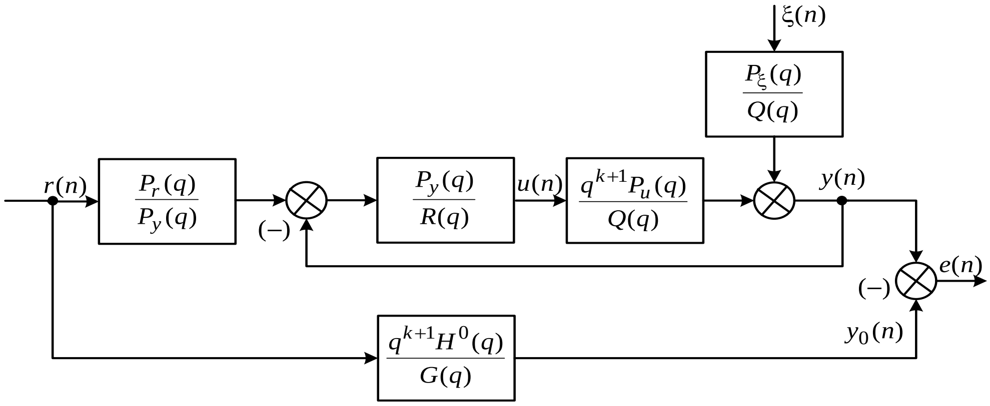

Figure 2: Block diagram

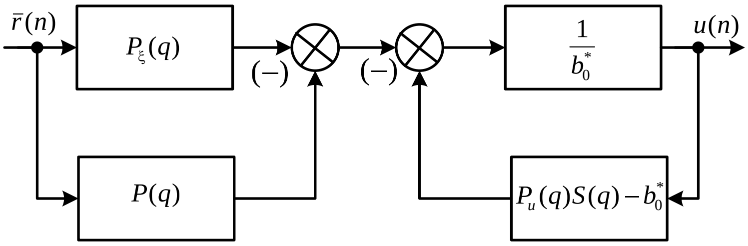

Figure 3: Block diagram of the optimal recovering filter

The algorithms of identification include the residuals of the form:

$$\varepsilon(n/n-1) = y(n) - \hat{y}(n/n-1), \tag{31}$$

$$\varepsilon(n/n-1) = y(n) - \theta^T Z_1(n-1)\varepsilon(n,\theta). \tag{32}$$

The recurrent identification algorithms can be written as:

$$\theta(n) = \theta(n-1) - \Gamma_1(n)\vartheta(n)\varepsilon(n,\theta(n-1)). \tag{33}$$

Specific identification algorithms

$$\varepsilon(n/n-1) = y(n) - \theta^T Z_1(n-1)\varepsilon(n,\theta),$$

$$\theta(n) = \theta(n-1) - \Gamma_1(n)\vartheta(n)\varepsilon(n,\theta(n-1)),$$

differ from each other by the gain matrix $\Gamma_1(n)$, the direction vector $\vartheta(n)$, and by the observation vector $Z_1(n-1)$. References to the literature in which the specific algorithms are given and investigated are indicated there.

In the adjustment algorithms, there are residuals of one of the form

$$\varepsilon(n/n-k-1) = y(n) - \hat{y}(n/n-k-1), \tag{34}$$

$$\varepsilon(n/n-k-1) = y(n) - \alpha^T Z_2(n-k-1) = \varepsilon(n,\alpha). \tag{35}$$

The recurrent algorithms of adjustment can be written as:

$$\alpha(n) = \alpha(n) - \Gamma_2(n)u(n)\varepsilon(n,\alpha(n-1)). \tag{36}$$

The specific adjustment algorithms (35), (36) differ from each other in the gain matrix $\Gamma_2(n)$ and the direction vector $U(n)$, possibly, by the observation vector $Z_2(n-k-1)$. Examples of adjustment algorithms are given in Table 1.

Along with the most common algorithms of the form (33) or (36), sometimes algorithms with filtered residual occur:

$$\tilde{\varepsilon}(n,\theta) = \frac{P_f(q)}{Q_f(q)}\varepsilon(n,\theta); \tag{37}$$

$$\tilde{\varepsilon}(n,\alpha) = \frac{P_f(q)}{Q_f(q)}\varepsilon(n,\alpha),$$

where $P_f(q)$ and $\theta(q)$ specially similar stable polynomials.

Table 1

Examples of identification algorithms

Methods	Inverse gain matrix	Direction vector	Estimate vector
Stochastic approximation method	$\gamma(n)=\gamma(n-1)+$ $+Z^T(n-1)Z(n-1)$	$z(n-1)=[-y(n-1),\dots,$ $y(n-N),u(n-k-1),\dots,$ $u(n-k-1-N_1)]^T$	$[a_1(n),\dots,a_N(n),$ $b_0(n),b_1(n),\dots,b_N(n)]$
Least squares method	$\Gamma^{-1}(n-1)+$ $+Z^T(n-1)Z(n-1)$	–"–	–"–
Stochastic approximation method	$\gamma(n)=\gamma(n-1)+$ $+Z_1^T(n-1)Z_1(n-1)$	$z_1(n-1)=[-y(n-1),\dots,$ $y(n-N),u(n-k-1),\dots,$ $u(n-k-1-N_1),\varepsilon(n-1),\dots,$ $\varepsilon(n-N_2)]^T$	$[a_1(n),\dots,a_N(n),$ $b_0(n),b_1(n),\dots,b_{N_1}(n),$ $c_1(n),\dots,c_{N_2}(n)]$
Advanced least squares method	$\Gamma^{-1}(n)+$ $+Z_1(n-1)Z_1^T(n-1)$	–"–	–"–
Maximum plausibility method	$\Gamma^{-1}(n)+$ $+\nu_1(n)\nu_1^T(n)$	$\nu_1(n)=\hat{P}_\xi^{-1}Z_1(n-1)$	–"–
Stochastic approximation method $P_\xi(q)\equiv 1$	$\gamma(n)=\gamma(n-1)+$ $+Z^T(n-k-1)x$ $xZ(n-k-1)$	$z(n-k-1)=[u(n-k-1),\dots,$ $u(n-2k-1-N_1),y(n-k-1),\dots,$ $y(n-k-1-N_7)]^T$	$[b_0(n),b_1(n),\dots,b_{N_7+k}(n),$ $g_0(n),g_1(n),\dots,g_{N_7}(n)]$
	$\Gamma^{-1}(n-1)+$ $+Z(n-k-1)\times$ $\times Z^T(n-k-1)$	–"–	–"–
	$\Gamma^{-1}(n-1)+$ $+Z_2(n-k-1)\times$ $\times Z_2^T(n-k-1)$	$z_z(n-k-1)=[u(n-k-1),\dots,$ $u(n-2k-1-N_7),y(n-k-1),\dots,$ $y(n-k-1-N_7),-\hat{y}(n-1/n-k-2),$ $\dots,-\hat{y}(n-z/n-k-1-N_7)]^T$	$[b_0(n),b_1(n),\dots,b_{N_1+k}(n)$ $g_0(n),g_1(n),\dots,g_{N_7}(n),$ $c_1(n),\dots,c_{N_2}(n)]$
	$\Gamma^{-1}(n-1)+$ $+v(n)v^T(n)$	$v(n)=\dfrac{z_z(n-k-1)}{\hat{P}_\xi(q)}$	–"–

Until a certain time, the choice of one or another identification algorithm or setting was not sufficiently argued. In the works [18, 24, 26, 27], the necessity to use not arbitrarily taken algorithms, but optimal algorithms, which have the maximum possible rate of convergence, has been proved. The formation of such algorithms is based on taking into account a priori information about the structure of dynamic objects and the characteristics of the noise that affect the object. For this purpose, we use a generalization of the optimality condition that follows from the criterion of quadratic residual and depends on the nonlinear transformation of residual. This condition generates the algorithm which is optimal with respect to the gain matrix. The choice of the nonlinear transformation of residual allows obtaining the optimal and nonlinear transformation algorithms that reach the maximum and limit rate of the convergence.

4. Conclusions

Let us summarize.

The joint solving of the obtained equations (27), (28), (29), and (30) determines the conditionally optimal extrapolator. Systems of equations can be solved using approximate methods described in [18, 23, 24, 26, 27]. Any of the above can be used for these purposes.

The identification algorithms are generated by minimizing the corresponding quadratic and weighted quadratic criteria of residual, and, finally, the optimal identification and adjustment algorithms correspond to the minimum of the asymptotic error covariance matrix, which is the criterion of convergence rate.

Thus, in functionally stable systems, optimization should be carried out according to several optimality criteria. The basic structure of the system has to meet the minimum of the quadratic error criterion. The predictor structures have to meet a minimum of residual criteria.

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