Structural Optimization of Fuzzy Systems based on Determination of Linguistic Terms Number

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Abstract

This paper focuses of the development and study of the advanced approach for structural optimization of fuzzy control and decision making systems based on determination of linguistic terms (LTs) optimal number. The application of the proposed approach makes it possible to increase the fuzzy system (FS) performance and accuracy, to reduce the computational costs spent on the rule base (RB) composing and parameters optimization, as well as to simplify its further hardware and software implementation. The developed approach is tested in this paper at design and structural optimization of the fuzzy cruise control system for the electric vehicle. The obtained results of the conducted research confirm the high efficiency and feasibility of using the developed approach for synthesis and optimization of various fuzzy control and decision-making systems.

Keywords 1

Fuzzy system, linguistic terms number, structural optimization approach, fuzzy controller, cruise control, electric vehicle

1. Introduction

Artificial intelligence (AI) systems are currently implemented in almost all areas of human activity, starting from science, medicine and various industries, and ending with the spheres of management, logistics and security [1]. Breakthrough AI technologies allow analyzing huge arrays of complex information, recognizing graphic images and speech, performing various creative tasks that were previously possible only for humans, simulating little-studied natural phenomena, controlling complex robotic and space objects, etc. [2, 3]. Among the main technologies and approaches of AI, the fuzzy logic is one of the most widespread and promising for solving a wide range of tasks [4-6]. It gives the opportunity to use effectively expert information, to mimic mechanisms of human thinking and decision-making, as well as to create linguistic models of complex processes and plants. As practice shows, the application of fuzzy logic techniques is the most appropriate when building intelligent control and decision-making systems of different types [7-9]. In particular, fuzzy automatic control systems (FACS) of executive, tactical and strategic levels show impressive results in control of plants with randomly changing operating conditions, as well as nonlinear and non-stationary characteristics. Such plants are drones, underwater vehicles, spacecrafts and satellites, pyrolysis reactors and thermoacoustic units, marine floating docks, electric cars and others [10-13]. In the same way, fuzzy decision-making systems (FDMS) are successfully used as expert systems operating under conditions of uncertainty in medical diagnostics, transport logistics, stock market forecasting, financial management, etc. [14-16].

To use the full potential and maximize the efficiency of fuzzy systems, it is expedient to perform their design using progressive methods and information technologies, which are far from being limited to the use of only experts' assessments and recommendations. These approaches and techniques are

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based on specific procedures of structural and parametric optimization and can use experts' knowledge only as initial hypotheses [17-19]. The development, improvement and approbation of such approaches and software means for their implementation is currently one of the most promising areas of development of the advanced theory of fuzzy systems and soft computing [20-22].

To date, a sufficiently large number of works have already been published that are dedicated to development and successful application of efficient approaches and methods of FACSs and FDMSs design and optimization [23-25]. Among them are methods of parametric optimization of linguistic terms membership functions (LTMF) and consequents of rule base [26, 27], as well as technologies of structural optimization based on the optimal choice of defuzzification procedures, RB interpolation and reduction [28-30]. Moreover, in the most advanced studies, the synthesis and optimization of fuzzy systems is carried out not only with the help of classical optimization methods [31, 32], but also by means of bioinspired techniques of evolutionary and multi-agent optimization [33-35]. For instance, in recent works, the synthesis of RBs based on modified ant colony algorithms is performed [36], the optimization of rules weights based on particle swarm methods is carried out [37, 38], as well as the optimal choice of the membership functions types using genetic, immune and biogeography based algorithms is conducted [39].

When considering the problem of FS synthesis and optimization, as well as solving it using the above advanced intelligent techniques, the special attention should be paid to the tasks of structural optimization [39, 40]. This is due to the fact that the best variant of the FS structure obtained in the optimization process directly affects not only the performance and accuracy of the FS, but also the computational costs spent on the formation (composing) of the rule base and optimization of parameters, as well as the complexity of its further hardware and software realization. The FS optimal structure is such a variant of the structure, which ensures (a) the achievement of the optimal performance and accuracy, (b) the admissible computational costs for formation of the RB and parametric optimization, as well as (c) the acceptable complexity of the hardware and software implementation of the FS [29, 39, 40]. Thus, research aimed at the development and improvement of methods and information technologies for finding the optimal structure of the FS is undoubtedly relevant and important for the modern theory of FACSs and FDMSs.

Among the main challenges of FSs structural optimization, the task of determination of the linguistic terms optimal number is the most complicated and important. Selection of different variants of the linguistic terms number gives the opportunity to implement various strategies of control and decision-making at composing FSs rule bases. Therefore, the number of LTs of input and output variables determines the initial sets of antecedents, the number of rules and possible consequents of the FS rule base, as well as the number of optimized parameters of the FS terms. Moreover, for each new solution found in the process of optimizing the number of LTs, it is necessary to compile a new RB for determination of the FS performance, which requires significant computational costs.

Thus, the main aim of this paper is development and research of an advanced approach for structural optimization of fuzzy control and decision making systems based on determination of linguistic terms optimal number.

2. Advanced Approach for Structural Optimization of FSs based on Determination of Linguistic Terms Optimal Number

The implementation of the proposed advanced approach will give the opportunity to increase the FS performance and accuracy, to reduce the computational costs spent on the RB composing and parameters optimization, as well as to simplify its further hardware and software realization. The given approach is based on the concept of sequential search of the best values of the LTs number for the fuzzy system, starting with the first input variable and ending with the last output variable. Moreover, it uses bioinspired method of ant colony optimization [36] and sequential search method [31] for automatic RB synthesis for each new obtained variant of the vector **S** that determines numbers of LTs. The main stages of the proposed advanced approach for determination of LTs optimal number are as follows.

Stage 1. Selection of input and output variables of the developed fuzzy system. At this stage n input variables and m output variables are selected for the developed fuzzy system, depending on the peculiarities of the task that this system will solve. In this case, the total number of the FS variables is equal to $n + m$.

Stage 2. Setting of the operating ranges of changing of input and output variables of the developed FS. At this stage the operating ranges are set for each i-th ($i = 1, 2, \ldots, n$) input and j-th ($j = 1, 2, \ldots, m$) output variables of FS, within which these variables can change. For example, if the input variables are fed to the FS input in relative units from their maximum value, then it is advisable to set their operating ranges from –1 to 1. If some variables can have only positive values (for example, "water consumption", "gas heating power", etc.), then their ranges of variation can be represented by the interval [0, 1].

Stage 3. Selection of the membership functions types of linguistic terms for input and output variables of the FS. At this stage the type of the LTs membership functions for each i-th ($i = 1, 2, \ldots, n$) input and j-th ($j = 1, 2, \ldots, m$) output FS variables is selected. In most cases, at the initial stage of design, it is advisable to select one type for membership functions of all FS variables. For example, triangular, trapezoidal, or Gaussian type I can be selected [4, 41]. As for the initial values of the parameters of the membership functions for all input and output variables, it is advisable to set them automatically in such a way that the linguistic terms, for any specified number, would be evenly distributed over their operating ranges.

*Stage 4. Formation of the structure of the vector **S** that determines numbers of linguistic terms of FS inputs and outputs.* At this stage the vector **S** is formed, depending on the selected at Stage 1 n input and m output variables, in the following way

$$\mathbf{S} = \{S_{x1}, S_{x2}, \ldots, S_{xi}, \ldots, S_{xn}, S_{y1}, S_{y2}, \ldots, S_{yj}, \ldots, S_{ym}\}. \tag{1}$$

This vector consists of variables $S_{xi,yj}$ that correspond to the number of linguistic terms for the FS variables, arranged in order, starting with the first input variable x_1 and ending with the last output variable y_m.

*Stage 5. Selection of constraints for vector **S**.* At this stage the constraints on the number of linguistic terms $\mathbf{S}_{min}$ and $\mathbf{S}_{max}$ are set for all input and output variables of the developed fuzzy system. For instance, for all input variables the minimum value of the number of LTs can be set equal to 2 ($S_{min} = 2$) and the maximum value – equal to 7 ($S_{max} = 7$). As for the output variables, the minimum value can be set equal to 3 ($S_{min} = 3$) and the maximum value – equal to 9 ($S_{max} = 9$). Also, additional constrains can be set at this stage. For example, for some variables of fuzzy control systems, which can take both positive and negative values in the symmetric operating ranges, it is advisable to set such additional constraints, that the number of LTs can be only odd. Thus, taking into account the main constraints ($\mathbf{S}_{min}$ and $\mathbf{S}_{max}$), in this case, the number of terms can take the following values: 3, 5, 7 – for input variables; 3, 5, 7, 9 – for output variables.

Stage 6. Formation of the complex objective function J_C for evaluating the effectiveness of the developed fuzzy control or decision-making system. At this stage, the type, parameters and optimal value of the complex objective function J_C, used to find the optimal number of LTs, are determined. Since the initial sets of antecedents, the number of rules and possible consequents of the FS rule base depends on the selected number of LTs, then in the process of structural optimization of the vector **S** it is advisable to use both criterion J_1, which evaluates the efficiency of the problem solved by the FS (control, decision-making, etc.), and criterion J_2, which takes into account the complexity of the synthesized RB and, accordingly, further hardware and software implementation of the system being developed. Thus, the task of finding the optimal number of LTs is reduced to the task of multi-criteria optimization [39, 42, 43], for the solution of which it is necessary to find the optimal vector **S** taking into account the minimization of two criteria J_1 and J_2. When solving this task, it is advisable to use an a priori approach to solving multi-criteria search tasks based on the aggregation of objective functions [39, 42, 43], according to which it is necessary to search for the optimum of a single complex objective function (global criterion) J_C, formed on the basis of criteria J_1 and J_2 with preliminary assessment of their significance. In accordance with this approach, it is expedient to calculate the current value of the complex objective function J_C in the process of searching for the optimal LTs number of the FS based on the expression

$$J_C = J_1 + k_{J2} J_2, \tag{2}$$

where k_{J2} is the scaling factor at J_2, which determines the importance of taking into account this criterion in the process of computational search and provides scaling (normalization) of the values of J_2.

In turn, the values of the criterion J_2, which estimates the complexity of the synthesized RB and further hardware and software implementation of the FS, can be calculated based on the dependence

$$J_2 = s \cdot \left(\sum_{j=1}^{m} S_{yj} \right) = \left(\prod_{i=1}^{n} S_{xi} \right) \cdot \left(\sum_{j=1}^{m} S_{yj} \right), \quad (3)$$

where s is the total number of rules, which is determined by the number of all possible combinations of linguistic terms of the FS input variables.

In addition, at this stage, the optimal (boundary) value of the criterion $J_{1\mathrm{opt}}$ is preliminarily set, as well as the corresponding values J_{Copt} and k_{J2} are selected, based on the requirements and features of the FS design problem.

*Stage 7. Setting of the initial value of the vector **S**.* At this stage the initial values of LTs numbers for fuzzy system input and output variables (vector $\mathbf{S}_0$) are set, from which the iterative search procedure begins. It is advisable to set vector $\mathbf{S}_0$ equal to its minimum possible value $\mathbf{S}_{\min}$ according to constraints selected at Stage 5

$$\mathbf{S}_0 = \mathbf{S}_{\min} = \{S_{x1\min},...,S_{xi\min},...,S_{xn\min},S_{y1\min},...,S_{yj\min},...,S_{ym\min}\}. \quad (4)$$

At the further stages of the given advanced approach the sequential optimization of the LTs numbers for the fuzzy system is carried out, starting with the first input variable x_1 and ending with the last output variable y_{m}.

Stage 8. Transition to the 1st variable of the fuzzy system. The transition to the 1st FS (input) variable x_1 is carried out at this stage to initiate the iterative procedures of the sequential search for finding the optimal vector of the LTs number $\mathbf{S}_{\mathrm{opt}}$. In turn, the iterative search begins from the initial value of the vector of LTs numbers $\mathbf{S}_0$, which is preliminarily set at Stage 7.

Stage 9. Checking of the Checklist. All vectors of the LTs number **S**, for which the FS rule base has been already synthesized and the corresponding value of the complex objective function J_{C} has been calculated during the implementation of the given approach, are entered to the Checklist with their synthesized RB and values of the function J_{C}. In turn, the Checklist and its check at this stage are used to avoid repeated synthesis of RB and calculations of the complex objective function J_{C} for FS with the same vector of the LTs number **S**. It allows to get rid of extra iterations (each of which include RB synthesis and complex objective function calculation), the number of which is equal to $(n + m – 1)$. If the current vector **S** is already placed in the Checklist, then the transition is carried out to Stage 13, and in the opposite case the transition is performed to Stage 10.

*Stage 10. Synthesis of the RB and calculation of the complex objective function for the FS with the current vector **S**.* At this stage, the RB synthesis is carried out using bioinspired method based on ant colony optimization (ACO) or sequential search method, which are developed and successfully tested in works [36] and [31], respectively. After that, for the synthesized RB that corresponds to the current vector **S**, the complex objective function J_{C} is calculated. As the previous comprehensive studies of the RB synthesis methods [44] show, for an effective automatic RB synthesis, it is advisable to choose either the sequential search method or the bioinspired ACO-based method, depending on the complexity of the given rule base, which is determined by the criterion J_2 (3). In turn, if the criterion $J_2 \leq 600$, it is advisable to choose sequential search method, if the criterion $J_2 > 600$, the bioinspired ACO-based method is more appropriate [44]. Thus, first at this stage, the criterion J_2 (3) is calculated for the current vector **S**, then, based on its value, a suitable method for synthesizing the rule base is selected (sequential search or ACO-based method). Further, using the selected method, the synthesis of the RB is directly carried out with the determination of the the criterion J_1, and after that, based on criteria J_1 and J_2, the value of the complex objective function J_{C} is finally calculated. Herewith, the adjustable parameters of the sequential search and ACO-based methods (maximum number of iterations $N^*_{\max}$, number of agents in the population $Z_{\max}$, number of elite agents e, number of rounds of sequential search l, etc.) are previously selected based on the experiments and recommendations obtained in the previous studies [31, 36, 44]. The RB antecedents are generated automatically for the current vector **S** as all possible combinations of linguistic terms of FS input variables. In turn, the RB consequents are found in the synthesis process using the above ACO-based or sequential search methods.

Stage 11. Checking for the achievement of the optimal value of the complex objective function. At this stage, the checking of the achievement of the optimal value of the complex objective function J_{Copt}

is carried out for the current vector **S**. If this checking gave a positive result, then go to Stage 16. Otherwise, go to Stage 12.

*Stage 12. Recording the current vector **S**, the corresponding RB and the value of the complex objective function to the Checklist.* At this stage the current vector of LTs number **S**, the corresponding synthesized RB and the value of the complex objective function J_C are recorded to the Checklist.

Stage 13. Checking for completion of the optimization process of the current FS variable. Optimization calculations for the current FS variable are considered complete if the rule bases were synthesized and corresponding values of the complex objective function J_C were calculated for each possible variant of the LTs number $S_{xi,yj}$ within the constraints [$S_{xi,yj\min}$, $S_{xi,yj\max}$] for this certain variable. If this checking gave a positive result, then go to Stage 14. Otherwise, 1 is added to the current LTs number $S_{xi,yj}$ for this variable ($S_{xi,yj}$ + 1), and the transition to Stage 9 is carried out.

Stage 14. Selection of the best variant of the LTs number for the current FS variable. At this stage, the selection of the LTs number $S_{xi,yj\text{best}}$ is carried out, for which the value of the complex objective function J_C is the smallest ($J_{C\text{best}}$) among all obtained in the optimization calculations for the given variable. This value of the number of linguistic terms $S_{xi,yj\text{best}}$ is set for this variable.

Stage 15. Checking for completion of the optimization process of all FS variables. At this stage structural optimization calculations are considered complete if the LTs numbers were optimized (with selection and setting best variants $\mathbf{S}_{\text{best}}$) for all $n + m$ variables (from the first input variable x_1 to the last output variable y_m) of the developed FS. If this checking gave a positive result, then go to Stage 16. Otherwise, the transition to the next variable (i, j + 1) is carried out, and further transition to Stage 9 is performed.

Stage 16. Completion of the LTs number optimization process. After that, additional structural-parametric optimization of the membership functions of the fuzzy system and its software and hardware implementation for further use in control and decision-making processes can be carried out. In this case, the hardware and software implementation will be simplified due to the optimal structure of FS and number of rules of its RB.

At the implementation of the given advanced approach the maximum number of iterations $N_{\max}$ is defined on the basis of the following equation

$$N_{\max} = \sum_{i=1}^{n}\left(\Delta S_{xi} + 1\right) + \sum_{j=1}^{m}\left(\Delta S_{yj} + 1\right) - \left(n + m - 1\right), \tag{5}$$

where

$$\Delta S_{xi} = S_{xi\max} - S_{xi\min}, i = 1, 2, ..., n; \tag{6}$$

$$\Delta S_{yj} = S_{yj\max} - S_{yj\min}, j = 1, 2, ..., m. \tag{7}$$

In turn, at each N-th iteration of the given approach, $N^*_{\max}$ iterations of the sequential search method or ACO-based method are performed at implementation of the Stage 10.

The effectiveness study of the proposed advanced approach is conducted in this paper at optimization of the LTs number for a FACS of the electric car velocity (cruise control system) [45-47].

3. Optimization of LTs Number for Fuzzy Control System of the Electric Car

The task of the cruise control system is to keep the car at a set velocity while driving on the highway under conditions of existing disturbances (change in wind speed, slope of the road, type of road surface, etc.) [48-50]. To simulate the processes of movement of an electric vehicle in this work, a simplified mathematical model is used, which consists of the following basic equations [49]:

$$m_{\text{c}} \frac{dv_{\text{c}}}{dt} = P_{\text{T}} - P_{\text{n}} - P_{\text{f}} - P_{\text{w}}; \tag{8}$$

$$P_{\text{T}} = \frac{M_{\text{M}} U_0 \eta}{r_{\text{w}}}; \tag{9}$$

$$P_{\text{n}} = m_{\text{c}} g \sin\gamma; \tag{10}$$

$$P_{\text{f}} = m_{\text{c}} g f \cos\gamma \,\text{sgn}\, v_{\text{c}}; \tag{11}$$

$$P_{\mathrm{w}} = k_{\mathrm{c}} F_{\mathrm{c}} v_{\mathrm{c}}^{2} \operatorname{sgn} v_{\mathrm{c}}; \tag{12}$$

$$M_{\mathrm{M}} = C_{\mathrm{mM}} I_{\mathrm{M}} - J_{\Sigma \mathrm{M}} \frac{d\omega_{\mathrm{M}}}{dt}; \tag{13}$$

$$\omega_{\mathrm{M}} = \frac{v_{\mathrm{c}} U_{0}}{r_{\mathrm{w}}}; \tag{14}$$

$$L_{\mathrm{M}} \frac{dI_{\mathrm{M}}}{dt} + R_{\mathrm{M}} I_{\mathrm{M}} = u_{\mathrm{M}} - C_{\mathrm{m}\omega} \omega_{\mathrm{M}}; \tag{15}$$

$$T_{\mathrm{PC}} \frac{du_{\mathrm{M}}}{dt} + u_{\mathrm{M}} = k_{\mathrm{PC}} u_{\mathrm{F}}, \tag{16}$$

where m_{c} is the electric car total mass; v_{c} is the electric car velocity (controlled variable); P_{T} is the traction force of the drive motor; P_{n} is the lift resistance force caused by movement of the car on an inclined plane; P_{f} is the rolling resistance force of the car; P_{w} is the air resistance force of the car; M_{M} is the motor electromagnetic torque; U_0 is the main gear of transmission of the electric car; η is the electric drive efficiency; r_{w} is the wheel radius of the car; g is the acceleration of gravity; γ is the angle of inclination of the plane at which the car is moving; f is the rolling friction coefficient that depends on the type of road surface on which the car is moving; k_{c} is the car air resistance coefficient; F_{c} is the drag area of the car; I_{M} is the current of the electric motor; C_{mM} is the electromagnetic torque coefficient, which is determined by the parameters of the motor anchor and the value of its magnetic flux; $J_{\Sigma\mathrm{M}}$ is the total reduced moment of inertia of the electric motor, transmission and driving wheel; ω_{M} is the angular speed of rotation of the motor anchor; L_{M} and R_{M} are the inductance and resistance of the electric motor winding; u_{M} is the supply voltage of the electric motor; $C_{\mathrm{m}\omega}$ is the electromotive force coefficient, which is determined by the parameters of the motor anchor and the value of its magnetic flux; T_{PC} and k_{PC} are the time constant and gain of the power converter; u_{F} is the control signal of the cruise control system.

In this paper the development and study of the fuzzy cruise control system is carried out for the electric car with the following main parameters: car mass $m_{\mathrm{c}} = 1200$ kg; angle of road inclination $\gamma = 0°$; rolling friction coefficient $f = 0.02$; car drag area $F_{\mathrm{c}} = 1.86$ m^2; car air resistance coefficient $k_{\mathrm{c}} = 0.29$; main gear of transmission $U_0 = 3.875$; wheel radius of the car $r_{\mathrm{w}} = 0.263$ m; nominal electric power of the electric motor of the car $N_{\mathrm{M}} = 90$ kW; efficiency of the electric drive $\eta = 0.9$; electromagnetic torque coefficient $C_{\mathrm{mM}} = 1.93$; electromotive force coefficient $C_{\mathrm{m}\omega} = 1.022$; electric motor total resistance $R_{\mathrm{M}} = 1.72$ ohm; electric motor total inductance $L_{\mathrm{M}} = 0.03$ H.

The fuzzy cruise control system includes main velocity feedback, fuzzy controller (FC), power converter and other auxiliary elements. Further, the design and structural optimization with determination of linguistic terms optimal number of the fuzzy controller is carried out using the proposed advanced approach.

At the Stage 1, three input and one output variables are selected for the developed FC: ε_V, $\frac{d\varepsilon_V}{dt}$, $\int \varepsilon_V dt$, u_{F}. So, the total number of the FC variables is equal to 4. At the second stage, the operating ranges of changing of FC input and output variables are set from –1 to 1 for all variables. In turn, at the Stage 3, triangular types of the membership functions are selected for all the linguistic terms of the FC input and output variables. Also, the parameters values of the LTMF for all input and output variables are set automatically in such a way that at the further optimization stages the linguistic terms, for any specified number, would be evenly distributed over their operating ranges. At the fourth stage, the structure of the vector **S**, that determines numbers of linguistic terms for FC inputs and output, is formed in the following way

$$\mathbf{S} = \{S_{x1}, S_{x2}, S_{x3}, S_{y1}\}. \tag{17}$$

At the fifth stage, the constraints for the vector **S** on the number of linguistic terms are set in the following way

$$\begin{gathered} S_{xi} \in \{3,5,7\},\ i = 1,2,3; \\ S_{y1} \in \{3,5,7,9\}. \end{gathered} \tag{18}$$

Moreover, for each input and output variable of the velocity FC when using 3, 5, 7, and 9 linguistic terms, the following sets of LTs of the triangular type are used, respectively:

$$\begin{gathered}\{\mathrm{N,Z,P}\};\\ \{\mathrm{BN,SN,Z,SP,BP}\};\\ \{\mathrm{BN,N,SN,Z,SP,P,BP}\};\\ \{\mathrm{VBN,BN,N,SN,Z,SP,P,BP,VBP}\},\end{gathered} \tag{19}$$

where VBN is very big negative; BN is big negative; N is negative; SN is small negative; Z is zero; SP is small positive; P is positive; BP is big positive; VBP is very big positive.

At the Stage 6, the complex objective function J_C for evaluating the effectiveness of the developed fuzzy cruise control system is formed, that is calculated according to equation (2). In turn, the criterion J_1 is presented as the mean integral quadratic error of velocity control

$$J_1(t,\mathbf{S}) = \frac{1}{t_{\max}} \int_0^{t_{\max}} \varepsilon_V^2 dt, \tag{20}$$

where $t_{\max}$ is the total transient time of the FACS for the electric car.

In turn, the criterion J_2 is represented by the expression (3). As the optimal values of the functions J_C and J_1 the following values are selected: $J_{Copt} = 20$; $J_{1opt} = 10$. The scaling factor k_{J2}, in this case, is equal to 0.035.

Further, the Stage 7 of the proposed approach is implemented, at which the initial value of the vector **S** for the FC is set. In this case, the initial value of the vector **S** is equal to its minimum possible value $\mathbf{S}_{\min}$ according to constraints selected at Stage 5

$$\mathbf{S}_0 = \mathbf{S}_{\min} = \{3,3,3,3\}. \tag{21}$$

Then, the iterative procedure of sequential optimization is conducted from the first input variable ε_z and up to the output variable u_F, in accordance with the remaining stages of the given approach (from 8^{th} to 16^{th}). Herewith, the adjustable parameters of the sequential search and ACO-based methods for RB synthesis at the Stage 10 are previously set as follows. The number of rounds of sequential search l is equal to 3 for each possible vector of LTs number **S** at implementing the sequential search method. For ACO-based method the number of agents in the population $Z_{\max} = 30$, the number of elite agents in the population $e = 10$, the maximum number of iterations $N^*_{\max} = 100$, the other adjustable parameters are: $\alpha = 2$; $\beta = 1$; $Q = 0.1$; $\rho = 0.5$.

In turn, at the implementation of the proposed advanced approach the maximum number of iterations $N_{\max}$ is equal to 10, that is defined on the basis of the equation (5) and accepted constraints.

The obtained results during the optimization of the vector **S** for the cruise control FC by means of the given approach are given in Table 1.

Table 1
Optimization results obtained by means of the developed approach

Iteration N	Vector **S**	Criterion J_1	Criterion J_2	Complex objective function J_C
1	{3, 3, 3, 3}	19.256	81	22.091
2	{5, 3, 3, 3}	15.647	135	20.372
3	{7, 3, 3, 3}	14.79	189	21.405
4	{5, 5, 3, 3}	13.297	225	21.172
5	{5, 7, 3, 3}	12.562	315	23.587
6	{5, 3, 5, 3}	14.982	225	22.857
7	{5, 3, 7, 3}	12.829	315	23.854
8	{5, 3, 3, 5}	8.494	225	16.369

As can be seen from the Table 1, the optimal value of the complex objective function has been achieved ($J_C \le J_{Copt}$) at the 8^{th} iteration of the given approach and at this the optimization process was stopped. In turn, the obtained at the 8^{th} iteration optimal vector of the LTs number is as follows

$$\mathbf{S}_{opt} = \{5,3,3,5\}. \tag{22}$$

A particular feature of the optimization calculations, carried out in this case, is that at all iterations the FC rule base was synthesized using the sequential search method, without using the ACO-based method, since for all obtained variants of the vector **S**, the value of criterion J_2 was less than 600.

The rule base fragment synthesized in the optimization process for the obtained optimal vector $\mathbf{S}_{opt}$ is given in Table 2. The obtained RB consists of 45 rules.

Table 2
Fragment of RB for the vector $\mathbf{S}_{opt}$ obtained by the proposed approach

Rule number	Input and output variables of the FC			
	ε_V	$\frac{d\varepsilon_V}{dt}$	$\int \varepsilon_V dt$	u_F
1	BN	N	N	BN
5	BN	Z	Z	BN
12	SN	N	P	SN
18	SN	P	P	SN
23	Z	Z	Z	Z
28	SP	N	N	SP
30	SP	N	P	SP
36	SP	P	P	BP
41	BP	Z	Z	BP
45	BP	P	P	BP

Furthermore, the full vector of consequents **R** for the developed RB corresponding to the optimal vector $\mathbf{S}_{opt}$ has the following form:

R = (BN, BN, BN, BN, BN, BN, BN, BN, BN, BN, BN, SN, SN, SN, SN, SN, SN, SN, SN, SN, SN, SN, Z, Z, SP, SP, SP, SP, SP, SP, SP, SP, SP, SP, BP, BP, BP, BP, BP, BP, BP, BP, BP, BP, BP).

To evaluate the effectiveness of the designed FACS, as well as developed advanced approach of structural optimization with the determination of the LTs number, the transient graphs of the electric car velocity control are presented in Fig. 1 for two cases: 1 – for cruise control system with designed and optimized FC; 2 – for cruise control system with conventional optimally tuned PID controller.

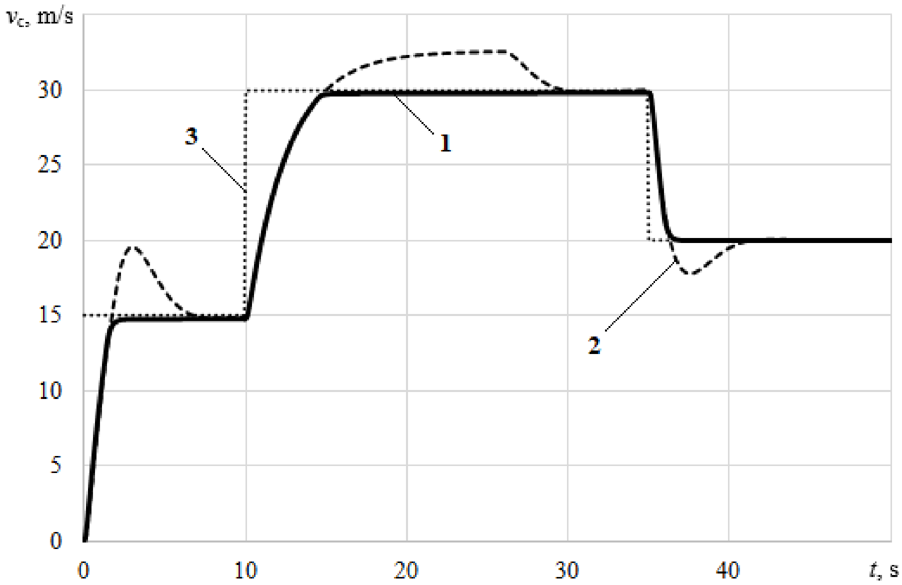

Figure 1: Transients for the cruise control system of the electric car

In turn, for both cases, the set value of the car velocity v_{set} (line 3) changed 3 times during the transient process in the following way: $v_{set1}(t = 0\text{ s}) = 15$ m/s; $v_{set2}(t = 10\text{ s}) = 30$ m/s; $v_{set3}(t = 35\text{ s}) = 20$ m/s.

As can be seen from Fig. 1, the fuzzy cruise control system for the electric car with optimized LTs number of the FC has significantly higher quality indicators in comparison with the conventional cruise control system, that uses conventional optimally tuned PID controller. Specifically, it has zero overshoot for all steps (σ = 0%), whereas the system with conventional PID controller has sufficiently larger values (σ = 29.5% – for the first step; σ = 8.33% – for the second step; σ = 10.12% – for the third step). Also, the system with FC has lower transient time t_t for all steps (t_t = 1.92 s – for the first step; t_t = 4.8 s – for the second step; t_t = 1.64 s – for the third step) compared to the same one with conventional PID controller (t_t = 6.18 s – for the first step; t_t = 18.64 s – for the second step; t_t = 4.93 s – for the third step). In addition, as for the value of the criterion J_1, it is significantly higher for a system with a conventional optimally tuned PID controller (J_1 = 11.53), than for a system with a developed and optimized fuzzy controller (J_1 = 8.494).

Moreover, the fuzzy controller developed and optimized by means of the proposed advanced approach has enough simple structure and the minimum size of the rule base, which will significantly simplify its further software and hardware implementation. In addition, if it is necessary to carry out further parametric optimization of membership functions for additional improvement of the given FC, this procedure will also be simplified, since the developed controller has a minimum number of linguistic terms for its input and output variables. And, finally, fairly small computational costs were spent during the FC development and structural optimization (8 iterations), which in general confirms the high efficiency of the proposed advanced approach for determination of the LTs optimal number.

4. Conclusions

This paper presents the development and research of the advanced approach for structural optimization of fuzzy control and decision making systems based on determination of linguistic terms optimal number. The implementation of the proposed approach gives the opportunity to increase the FS performance and accuracy, to reduce the computational costs spent on the RB composing and parameters optimization, as well as to simplify its further hardware and software realization. The given approach consists of 16 main stages, is based on the concept of sequential search of the best values of the LTs number for the FS, as well as uses the bioinspired method of ant colony optimization and sequential optimization method for automatic RB synthesis for each new obtained variant of the vector of LTs numbers.

The effectiveness study of the presented approach is conducted in this paper at design and structural optimization of the fuzzy cruise control system for the electric car. In this case, the iterative procedure of sequential optimization is performed for the FACS of the electric car, and at the 8th iteration of the approach the optimal vector of the LTs number $\mathbf{S}_{opt}$ is found, which provided the achievement of the optimal value of the complex objective function J_{Copt}. In turn, the obtained fuzzy cruise control system for the electric car with optimized LTs number of the FC has significantly higher quality indicators compared to the conventional cruise control system, that uses classic optimally tuned PID controller. Furthermore, the developed and optimized FC by means of the proposed advanced approach has enough simple structure, minimum number of LTs and the minimum size of the RB, which will significantly simplify its further software and hardware implementation. Thus, the research results obtained in this paper fully confirm the high efficiency of the developed advanced approach for FS structural optimization by means of the determination of LTs optimal number. Moreover, minor computational costs make this approach quite promising and attractive for use in the development and structural optimization of various fuzzy control and decision-making systems.

5. References

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