Towards a Structural Characterization of Identity, Individuality and Sortality

João R. M. Nicola[1], Giancarlo Guizzardi[2,3]

[1]Ontology and Conceptual Modeling Research Group, PPGI, Federal University of Espírito Santo, Av. Fernando Ferrari, 514, 29075-910, Vitória, ES, Brazil

[3]Faculty of Computer Science, Free University of Bozen-Bolzano, piazza Domenicani, 3, 39100, Bolzano, Italy.

[4]Faculty of Electrical Engineering, Mathematics and Computer Science Zilverling, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands.

Abstract

Questions regarding the identity and individuality of objects and of sortality of concepts or universals are important questions often raised in the context of an upper ontology. In this paper we present a novel characterization of these concepts based on a structural approach that can be applied in the context of an upper ontology. It provides objective criteria for determining these concepts in function of the structural properties of particulars and universals in structures that model an upper ontology theory. This theory has important applications in the field of conceptual modeling and on inductive reasoning about ontologies.

Keywords

sortals, identity, individuality, formal ontology, upper ontology

1. Introduction

The identity and the identification of objects is a topic of interest in philosophy and in linguistics. In the former, one can find debates regarding whether identity is relative or absolute, how identity relates to time, or what is an identity criterion [1, 2]. In the later, the notions of identity and identification play a crucial role: unless we can presume that individuals in the domain of discourse are identifiable (and possess identity), it would be impossible to express, in a meaningful way, beliefs or facts regarding specific things, and our communication would, at best, be restricted to commonalities of classes of individuals or to general concepts.

For similar reasons, the concept of identity [1] is also relevant in the field of Information Systems, since the efficacy of the interaction of an Information System and its users depends on whether or not the user is capable of identifying the referents of the data that the Information

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joaoraf@gmail.com (J. R. M. Nicola); g.guizzardi@utwente.nl (G. Guizzardi)

https://people.utwente.nl/g.guizzardi (G. Guizzardi)

[1]We are interested in identity in an *ontic* sense, i.e. we are looking for the ontological elements provided by the foundational ontology that ground identity assessments. Although we also use the term *identification*, this only represents, in the context of a process of identification, these ontological elements and not, for example, logical or methodological aspects regarding the process of identification.

System collects, stores, processes and presents. Therefore, the investigation of the conditions that characterize the identification of the individuals in the domain is an important step in the development of an Information System.

This concern is particular evident in the design of the databases of Information Systems. The database designer usually has to specify, with the help of the documentation about the domain, which fields in each table comprise the identification keys of the table , i.e. which properties or relations determine the identity of the referents of each table row.

However, there are hypothetical and actual situations in which one might say that an object does not possess a unique determinate identity. For the actual case, an example is the lack of identity of boson particles that share the same state. For the hypothetical one, we can consider the twin spheres thought experiment, proposed by Max Black [3], that describes a universe that contains only two stationary spheres of the same material and size. In this example, it is not possible to determine the identity of any of the spheres, either by describing their properties or by specifying an algorithm to determine which one is which. Bosons and symmetric spheres, in these cases, are said to lack identity but still possess individuality, or, using Lowe's terms [4], are *quasi-objects*.

If we consider the way we perceive or represent real objects, we might say that even though the real objects might have a determinable identity, the representations or abstractions though which we perceive them might not: it might be irrelevant to a mechanical engineer whether or not each steel ball in a bearing is identifiable, but only how many of them are there. Another example could be a processed food factory, in whose viewpoint the identity of each raw item, e.g. fish, vegetable, egg, is irrelevant. In these cases, even though it might not be necessary for a user of a supporting information system to identify an individual referent, it might nevertheless be necessary to determine their count. Thus, the concept of individuality is also relevant in the development of information systems.

In other situations, the same itens, be they cattle, eggs, etc., might have to be tracked individually, at least until a certain phase of the production process. In these cases the representations of these individuals would need to have a determinate identity.

In the process of the development of an information system, the information regarding the identity and individuality of the objects of the domain is usually recorded in the document that describes the domain, e.g. in a conceptual model or in an ontology, and is commonly linked to one or more concepts, e.g. how to identify vehicles or books. Such concepts, and any specializations thereof, are said to be *sortal* concepts [5], or concepts that *carry* or *supply* a principle of identity for its instances. The distinction between sortal a non-sortal concepts plays an important role in ontology engineering methodologies such as OntoClean [6], the UFO upper ontology [7] and in the OntoUML conceptual modeling language, based on UFO, that contains syntactical rules that depend on the assessment of the sortality of the concepts represented in its diagrams.

In the UFO ontology original presentation [7] there is a stated assumption that the particulars in the domain possess both identity and individuality. The notion of sortality is characterized in UFO by distinguishing the domain of instantiation of *sortal types* and *non-sortal* types. This approach, described in [8], considers as a sortal those types that are instantiated by *individual concepts*, where an individual concept is represented by an *intensional function* that maps each possible world to an individual in that world. On the other hand, a non-sortal type is instantiated directly by possible world individuals. In this sense, we can say in the resulting models consider the distinction between sortals and non-sortals to be unalizable, i.e. a type is a sortal (or a non-sortal) **because it is implictly indicated as so** in the model. While this characterization allows us to consider the implications of having the distinction between sortals and non-sortals, i.e. sortals are associated with individual concepts, which can be considered to be representations of the identity of objects, they provide no means for questioning **why** a certain type is or is not a sortal. In other words, the model provides no means to *falsify* a statement regarding the sortality of a type, unless it does it in a trivial way, e.g. by saying that a type T is a sortal just because the model explictly says "T is a sortal". However, if facts regarding sortality are meaningful and informative, i.e. they are *synthetic* truths, whose determination is grounded in meaningful patterns in reality, then it would be interesting to provide a formal characterization of sortality that allows such analysis to be made, and that is the goal of the formalization proposed in this paper. In our proposed formalization, the sortality of a type is an *emergent* property, i.e. it is grounded in the structure of the world and in the principle of application of a type.

Another approach is described in [9], in which a sortal is characterized as a type that specializes a *kind*, i.e. a rigid type that *provides an identity criterion* to its instances. However, similarly to the approach described in [8], the characterization does not provide an explanation as to **how** a type provides (or not) an identity criterion. The characterization proposed in this paper aims to provide a clear and falsifiable criteria for evaluating whether or not a type is a sortal based on the properties of the particulars in the domain and in the property that characterizes all types, their principle of application.

This paper presents a formal characterization of the concepts mentioned in the title: *individuality*, *identity* and *sortality*. The intuition on which the formalization is based is an idealized game, a thought experiment proposed in section 2 through which we defend a desiderata for the formalization of these concepts. The formalization, described in section 3, is based on a simplified theory of substantials, moments and universals derived from the UFO ontology. In section 4, we briefly discuss possible pratical implications of this theory. Section 5 then presents final considerations of the paper.

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2. The Identification Game

Consider the following game, called *identification game*, played cooperatively by two players, which we shall call Alice and Bob. In the first phase of the game, Alice is placed in the midst of a large, round and symmetrical room, in which there can be found various objects of diverse sizes, shapes and colors, placed arbitrary in the room's floor. Alice then proceeds by choosing one of this objects and writing down, in a notepad, a description or an instruction that can be used to identify the chosen object. These notes can contain, for example, the color of the chosen object, its distance to the center of the room, the relative distance to other objects in its surrounding, etc. After taking these notes, Alice is removed from the room and Bob is placed on the room, in a arbitrary position. With Alice's notes in hand, Bob has to identify correctly which object Alice has chosen before.

Note that Alice's chosen object might be exactly similar to another object in the room.

Furthermore, it might be positioned symmetrically with regards to another similar object. The questions we can present to this thought experiment are these: Is it possible to describe any object in the room with absolute certainty? Is there an objective way to determine which objects can or cannot be identified, irrespective of the strategy used when writing the notes?

Let's consider a hypothetical case based on Max Black's twin spheres: suppose the room contains just two spheres which are completely stationary, are placed in symmetrical positions with respect to the center of the room, and that are perfectly similar. In this case, no matter which sphere Alice chooses and what Alice writes down in the notepad, Bob will have at most a 50% chance of guessing the object correctly. Note that, in this case, Alice can ground an assertion that none of the spheres are identifiable by pointing out the symmetry of the configuration, without even considering whether or not which strategy should be used in taking the notes.

On the other hand, if we break the symmetry of the configuration by adding some other element, such as a distinct size or color for each sphere or a single extra object in any place but the center of the room, then both spheres would be identifiable. In this case Alice could ground the assertion that a sphere is identifiable just by pointing out the lack of symmetry in the room, even before she decides what shall be described in the notes or which language and strategy shall be used to write them down.

Therefore, the existence of a description that determines the identity of an object is unnecessary to ground its identifiability or non-identifiability. On the other hand, the intrinsic properties of an object are insufficient to determine whether or not it is identifiable. Instead, it is necessary to consider the *whole structure*, i.e. identity should be considered a *structural property*. This notion of identity can be characterized formally through an analysis of the various homomorphisms of the structure that represents the world configuration, e.g. that represent the room in the game.

2.1. Individuality

While its conceivable that a configuration with symmetrical objects described in the identification game could exist in reality, the same cannot be done with objects that lack also individuality, since if there were two distinct objects in reality that lacked individuality, then we would not be able to perceive them as distinct at all. However, if we consider the abstract representations of reality, this distinction becomes pertinent.

For example, suppose there is a representation of a configuration that contains two pieces of information: (1) there is a sphere, let's call it A, with a diameter of 10 meters; (2) there is a sphere, let's all it B, with a diameter of 10m. Now let's consider the following question: How many spheres are there in that universe? Are the descriptions (of A and B) describing the same sphere? Or are they describing distinct spheres? In case the problem is not just whether or not one of the spheres are identifiable. We cannot prove they are distinct, in the first place. In this case we say that the representations of the particulars contained in the representation of the configuration lack individuality.

Similarly to the notion of identity, we can determine the individuality (or lack thereof) by examining the homomorphisms of the representing structure, as we shall describe formally in the next section.

2.2. Sortality

In the OntoClean methodology [10], the concept of a sortal (substantial) universal is described as being a universal that provides a principle of identity for its instances. The evidence that a universal is a sortal is the existence of a binary identity predicate $P(x, y)$ that satisfies Leibniz's Principle of the Identity of the Indiscernibles, i.e. $P(x, y)$ holds if and only if x and y have the same properties.

This definition suffers from a few issues: (1) it reduces a fundamental ontological notion, sortality, to the existence of an element (a predicate) which could be considered dependent upon the existence of a human being and of some language understood by human beings; (2) providing a predicate as an evidence implies also the arbitrary choices of language and of the expression of the predicate in that language, that cannot be explained in terms of the configuration itself; and (3) the existence of the binary predicate of distinguishability does not guarantee the identifiability of the instances, but only their distinguishability.

However, we can avoid this issue by considering the relation between sortality and identity from a different perspective: instead of characterizing sortals as universals that provide (or carry or supply) identity for its instances, we characterize non-sortals as universals that preclude the identification of its instances. The idea here is to consider universals (or concepts) as cognitive tools that remove details of its instances, through an abstraction process.

In other words, it is not that a sortal provides identity and a non-sortal doesn't, but that a non-sortal abstracts so much detail of its instances that, when used as "filters", they make it impossible to identify some of its instances.

As an example, consider again a configuration with two spheres, A and B, where A and B are physically apart, have the same diameter and have distinct colors. In this case, we might say that the spheres A and B instantiate the **Solid Object**, **Spherical Object**, and **Colored Object**. Each of the universals induce an abstraction on the configuration: as solid objects, only their distance from each other is relevant; as spherical objects, the distance and diameter are relevant; and as colored objects, only the distance and color are relevant. If we trim down the configuration according to the abstraction represented by each of these universals, we shall arrive in three distinct configurations. However only the abstraction given by the **Colored Object** universal keeps the spheres identifiable. The other two universals are too abstract and produce representations in which the spheres lack identity. Thus, a sortal, in the sense proposed here, is a universal whose instances are still identifiable even after its abstraction process.

3. Formal Characterization

A structural characterization of the notions of identity, individuality and sortality require a suitable notion of "structure" and of transformations that preserve this structure (homomorphisms). We call such as structure a particular structure, in the sense that its purpose is the representation of the particulars in the domain, its properties and relationships, and any other elements that are required to represent them. The particular structure we present here is a simplified version of the UFO model structure:

Definition 1. A *particular structure system* is a is a tuple $\mathfrak{P} = (\mathcal{PS},\ \mathcal{W}_{-},\ (\triangleleft_{-}),\ \Phi_{-,-})$ where

- $\mathcal{PS}$ is a non-empty set of particular structures;
- $\mathcal{W}_{\_} :: \mathcal{PS} \mapsto Set$ is a function that associates particular structures to possible world configurations, i.e. to sets of sets of particulars;
- $\mathcal{P}_{\_}$ is the function that associates a particular structure to its set of particulars, i.e. $\mathcal{P}_\Gamma = \bigcup \mathcal{W}_\Gamma$;
- $(\triangleleft_{\_}) \subseteq Set \times \mathcal{PS} \times Set$ is a ternary relation denoting an *inherence relation*, where $x_1 \triangleleft_\Gamma x_2$ denotes "in the particular structure Γ, the moment x_1 *inheres in* the endurant x_2".
- $\Phi_{\_,\_}$ denotes a family of functions, or morphisms, between particulars of particular structures in $\mathcal{PS}$, that determine the correspondence between endurant representations in different structures contained in $\mathcal{PS}$, i.e. for some particular structures Γ_1 and Γ_2 of $\mathcal{PS}$, Φ_{Γ_1,Γ_2} is a set of functions from the set of particulars of Γ_1 to those of Γ_2.

From these elements, we can define the following:

- the set of *moments* of a particular structure $\Gamma \in \mathcal{PS}$, written as $\mathcal{M}_\Gamma$, where

$$\mathcal{M}_\Gamma \equiv \{x \in \mathcal{P}_\Gamma \,|\, \exists y.\, x \triangleleft_\Gamma y\}\,; \tag{1}$$

- the set of *substantials* of a particular structure $\Gamma \in \mathcal{PS}$, written as $\mathcal{S}_\Gamma$, where

$$\mathcal{S}_\Gamma = \mathcal{P}_\Gamma - \mathcal{M}_\Gamma; \tag{2}$$

- the *existential dependency* relation, where $ed_\Gamma(x, y)$ denotes the statement that x and y are particulars of Γ and that x is existentially dependent upon y in Γ, i.e.

$$ed_\Gamma(x, y) \equiv \forall w \in \mathcal{W}_{\_}.x \in w \longrightarrow y \in w; \tag{3}$$

- the *correspondence relation* between worlds of two particular structures: given $\Gamma_1, \Gamma_2 \in \mathcal{PS}$, $\varphi \in \Phi_{\Gamma_1,\Gamma_2}$, $w_1 \in \mathcal{W}_{\Gamma_1}$ and $w_2 \in \mathcal{W}_{\Gamma_2}$, we say that w_1 (from Γ_1) *corresponds to* w_2 (from Γ_2) through the morphism φ, written as $w_1 \Leftrightarrow_{\Gamma_1,\Gamma_2,\varphi} w_2$ and defined formally as:

$$w_1 \Leftrightarrow_{\Gamma_1,\Gamma_2,\varphi} w_2 \equiv \forall x \in \mathcal{P}_{\Gamma_1}.\, x \in w_1 \longleftrightarrow \varphi(x) \in w_2. \tag{4}$$

Additionally, the elements of a particular structure system must satisfy the following conditions:

- for any particular structure, its set of particulars must be finite;
- inherence implies existential dependency, i.e. for any particular structure $\Gamma \in \mathcal{PS}$ and any particulars $x, y \in \mathcal{P}_\Gamma$,

$$x \triangleleft_\Gamma y \longrightarrow ed_\Gamma(x, y); \tag{5}$$

- inherence is functional on the set of moments, i.e. for any particular structure $\Gamma \in \mathcal{PS}$ and any particulars $x, y, z \in \mathcal{P}_\Gamma$,

$$x \triangleleft y \wedge x \triangleleft z \longrightarrow y = z \tag{6}$$

- for any particular structures $\Gamma_1, \Gamma_2 \in \mathcal{PS}$ and any morphism $\varphi \in \Phi_{\Gamma_1,\Gamma_2}$, φ must satisfy the following constraints:

morphisms reflect inherence:

$$\forall x, y \in \mathcal{P}_{\Gamma_1}.\, \varphi(x) \lhd_{\Gamma_2} \varphi(y) \longleftrightarrow x \lhd_{\Gamma_1} y \tag{7}$$

morphisms do not change substantial into moments:

$$\forall x \in \mathcal{P}_{\Gamma_1}.\forall y \in \mathcal{P}_{\Gamma_2}.\, \varphi(x) \lhd_{\Gamma_2} y \longrightarrow \exists z \in \mathcal{P}_{\Gamma_1}.\, y = \varphi(z) \wedge x \lhd_{\Gamma_1} z \tag{8}$$

every world in the the source structure has at least one corresponding world in the target structure, and vice-versa:

$$\forall w_1 \in \mathcal{W}_{\Gamma_1}.\, \exists w_2 \in \mathcal{W}_{\Gamma_2}.\, w_1 \Leftrightarrow_{\varphi,\Gamma_1,\Gamma_2} w_2 \tag{9}$$

$$\forall w_2 \in \mathcal{W}_{\Gamma_2}.\, \exists w_1 \in \mathcal{W}_{\Gamma_1}.\, w_1 \Leftrightarrow_{\varphi,\Gamma_1,\Gamma_2} w_2 \tag{10}$$

  These constraints ensure that the meaning of the inherence relation is preserved among structures (constraints 7 and 8) and that existential dependency is preserved (consequence of constraints 9 and 10).
- for any $\Gamma \in \mathcal{PS}$, the identity function on $\mathcal{P}_\Gamma$ is on Φ, i.e., $id_{\mathcal{P}_\Gamma} \in \Phi_{\Gamma,\Gamma}$;
- for any $\Gamma_1, \Gamma_2, \Gamma_3 \in \mathcal{PS}$ and any morphisms $\varphi_{12} \in \Phi_{\Gamma_1,\Gamma_2}$ and $\varphi_{23} \in \Phi_{\Gamma_2,\Gamma_3}$, their composition is on Φ, i.e. $\varphi_{23} \circ \varphi_{12} \in \Phi_{\Gamma_1,\Gamma_2}$;

Morphisms can be classified according to their functional properties. A morphism $\varphi \in \Phi_{\Gamma_1,\Gamma_2}$ is:

- *injective* if it is an injection on $\mathcal{P}_{\Gamma_1}$;
- *surjective* if it is surjective onto $\mathcal{P}_{\Gamma_2}$;
- *bijective,* or an *isomorphism*, if its both injective and surjective, or, equivalently, if it is invertible;
  - the set of isomorphisms between Γ_1 and Γ_2 is denoted by $\Phi^{\cong}_{\Gamma_1,\Gamma_2}$;
  - if there is an isomorphism between Γ_1 and Γ_2, Γ_2 is said to be *isomorphic* to Γ_1 (and vice-versa);
- an *endomorphism*, if $\Gamma_1 = \Gamma_2$;
- a *permutation* if its both an endomorphism and an isomorphism.

Note that since the set of particulars of any particular structure is finite, every *injective morphism* between isomorphic particular structures is also an *isomorphism.* In particular, *injective endomorphisms* are *permutations.*

Finally, particular structure systems must satisfy the condition that for any particular structure Γ, there must be an infinite number of isomorphic particular structures in $\mathcal{PS}$.

Structural Properties

Using the notion of a particular structure and of its morphisms, we define two structural properties of particulars in a structure: **non-collapsibility**, which captures the distinguishability of a particular, and **non-permutability**, which characterizes the asymmetry of a particular with respect to the other objects in the configuration. They are defined as follows:

Definition 2. A particular x in a particular structure Γ is said to be *non-collapsible* in Γ if and only if, for every endomorphism φ on Γ, the following holds:

$$\forall y \in \mathcal{P}. \varphi(x) = \varphi(y) \longleftrightarrow x = y. \tag{11}$$

A non-collapsible particular is said to be a particular that *possesses individuality* in the structure.

Definition 3. A particular x in a particular structure Γ is said to be *non-permutable* in Γ if and only if, for every endomorphism φ on Γ, $\varphi(x) = x$ and, for every $y \in \mathcal{P}$, $\varphi(y) = x$ implies that $y = x$.

We can also use the set of morphisms to define a notion of identifiability that captures the idea that the identification of a particular does not depend on the particular labels assigned to them:

Definition 4. Let $\mathcal{PS}$ denote the set of particular structures, $\mathbf{P} = \bigcup\{\mathcal{P}_\Gamma \mid \Gamma \in \mathcal{PS}\}$ denote the set of all particulars, Γ denote some particular structure and x denote a particular of Γ. We call a predicate $P :: \mathcal{PS} \times \mathbf{P} \mapsto bool$ an *isomorphically invariant identifying predicate* of x in Γ if and only if, for any isomorphism φ from Γ to some other particular structure Γ^*, P "guesses" correctly what is the image of x under φ, i.e.

$$\begin{aligned} \mathrm{IdPred}(P, \Gamma, x) \equiv \forall \Gamma' \in \mathcal{PS}. \forall \varphi \in \Phi^{\cong}_{\Gamma,\Gamma'}. \, P(\Gamma', x) \longleftrightarrow \\ (\forall y \in \mathcal{P}_{\Gamma'}. \forall z \in \mathcal{P}_\Gamma. \, y = \varphi(x) \longleftrightarrow z = x) \,. \end{aligned} \tag{12}$$

In other words, assuming that Γ is an admissible particular structure ($\Gamma \in \mathcal{PS}$) and that x is a particular of Γ, P correctly guesses the particular that corresponds to x in any other particular structure in $\mathcal{PS}$, irrespective of which correpondence morphism is being considered, i.e. P correctly *captures the identity of* x.

This characterization of a predicate that describes the identity of a particular differs from the approach that relies on Leibniz principle of Identity of Indiscernibles because it does not rely on a quantification over predicates. Instead it defines such a predicate as being one that is able to pick the exact element that represents that particular in the each possible representation of the structure of reality. This approach avoids the possibility of considering trivial and non-informative predicates as identity predicates, since if we simply use logical identity to define an identity predicate in a trivial way, the resulting predicate would not be invariant with respect to the morphisms that stand for possible *changes in the representation form.* We can then define the notion of an identifiable particular by means of the existence of a suitable identifying predicate, i.e. of a predicate that can identify the particular irrespective of the specific labeling used in the representation:

Definition 5. An *identifiable particular* of a particular structure Γ is a particular of Γ for which there is at least one identifying predicate, i.e.

$$\mathcal{P}^{id}_\Gamma \equiv \{x \in \mathcal{P}_\Gamma \mid \exists P. \, \mathrm{IdPred}(P, \Gamma, x)\} \,.$$

The notions of permutability and identifiability are linked by the following theorem:

Theorem 1. *For any particular structure* Γ *and any particular* x *of* Γ*, the following statements are equivalent:*

1. x is an identifiable (non-identifiable) particular of Γ;
2. x is non-permutable (permutable) in Γ.

Proof. Identifiability implies non-permutability because the identity predicate can only identify a particular x's mapping from a particular structure Γ into an isomorphic structure Γ^* through some unknown morphism if all morphisms from Γ to Γ^* agree with respect to the image of x. On the other direction, non-permutability implies identifiability because since the mapping of x is the same for all morphisms, it is sufficient to identify the mapping using any morphism, e.g. the isomorphism itself. □

This theorem clarifies the intuition, presented in the discussion regarding the identification game, that the identifiability of a particular can be determined structurally (non-permutability), without considering the existence of a suitable predicate. It suffices to demonstrate the existence of a non-identical permutation of a particular to prove its non-identifiability.

3.1. Sortality

To characterize sortality, we need first to characterize the notions of instantiation, which here includes an aspect of invariance modulo isomorphisms, and of *trimming*, i.e. of the abstraction process of a universal over the elements of the representation of a configuration:

Definition 6. Let $P :: \mathcal{PS} \times \mathbf{W} \times \mathbf{P} \mapsto$ *bool* be a ternary predicate over the set of all particular structures, the set of all possible worlds and the set of all particulars. We call P an *instantiation predicate* if and only if the particulars it applies to are invariant under isomorphisms of the underlying particular structure and correspondence between possible worlds, i.e. for any particular structures Γ_1 and Γ_2 with an isomorphism φ between Γ_1 and Γ_2, for any possible worlds w and for any particular x, the following holds:

$$P(\Gamma_1, w, x) \longrightarrow w \in \mathcal{W}_{\Gamma_1} \wedge x \in w \tag{13}$$

$$P(\Gamma_1, w, x) \longleftrightarrow \forall w' \in \mathcal{W}_{\Gamma_2}.\, w \Leftrightarrow w' \longrightarrow P\left(\Gamma_2, w', \varphi(x)\right) \tag{14}$$

Similarly to the concept of an isomorphically invariant identification predicate, this notion of instantiation predicate also captures the idea that the conditions that rule the applicability of a universal should not depend on the specific labeling (or representation) of the particulars. With this notion of instantiation, we define a representation of a configuration and of the instantiation relation between the particulars in the configuration and a set of universals:

Definition 7. An *instantiation system* is a tuple

$$\mathcal{I} = (\mathfrak{P}, \mathcal{U}_s, \mathcal{U}_m, \{\mathrm{iof}_u\}),$$

where $\mathfrak{P} = (\mathcal{PS}, \mathcal{W}_{\_}, (\lhd_{\_}), \Phi_{\_,\_})$ is a particular structure system, $\mathcal{U}_s$ is called the set of *substantial universals* of $\mathcal{C}$, $\mathcal{U}_m$ is called the set of *moment universals* of $\mathcal{C}$, $\{\mathrm{iof}_u\}$ is a family of instantiation predicates, indexed by substantial and moment universals, i.e. $u \in \mathcal{U}_s \cup \mathcal{U}_m$. An instantiation system must satisfy the following conditions: for any universal $u \in \mathcal{U}_s \cup \mathcal{U}_m$, any particular structure Γ, any possible world $w \in \mathcal{W}_\Gamma$ and any particular $x \in \mathcal{P}_\Gamma$:

substantial universals are only instantiated by substantials:

$$u \in \mathcal{U}_s \wedge \mathrm{iof}_u(\Gamma, w, x) \longrightarrow x \in \mathcal{S}_\Gamma \tag{15}$$

moment universals are only instantiated by moments:

$$u \in \mathcal{U}_\mathcal{M} \wedge \mathrm{iof}_u(\Gamma, w, x) \longrightarrow x \in \mathcal{M}_\Gamma \tag{16}$$

Furthermore, we require that a moment instantiation has to be *rigid*, i.e. all moment universals considered here are rigid universals, or universals that are necessarily instantiated by their instances in all possible worlds in which these instances exist:

$$\forall u \in \mathcal{U}_m. \forall m. \forall w_1, w_2 \in \mathcal{W}_\Gamma. \, \mathrm{iof}_u(\Gamma, w_1, m) \wedge m \in w_2 \longrightarrow \mathrm{iof}_u(\Gamma, w_2, m). \tag{17}$$

The relationship between the substantial universals, that represent abstractions of the objects in the configuration, and the moment universals, which represent the properties of the objects, is given by the *characterization* relation:

Definition 8. Given a particular structure Γ, a substantial universal $u_s \in \mathcal{U}_s$ and a moment universal $u_m \in \mathcal{U}_m$, we say that u_m *characterizes* u_s in Γ if, and only if, for all possible worlds $w \in \mathcal{W}_\Gamma$ and for all substantials $s \in \mathcal{S}_\Gamma$, if $\mathrm{iof}_{u_s}(\Gamma, w, s)$ then there exists a moment $m \in \mathcal{M}_\Gamma$ such that $m \lhd_\Gamma x$, $m \in w$ and $\mathrm{iof}_{u_m}(\Gamma, w, m)$. We call $\mathrm{Char}_\Gamma(u_s)$ the set of characterizing universals of u_s according to Γ, i.e.

$$\mathrm{Char}_\Gamma(u_s) \quad = \quad \{u_m \,|\, \forall w, x. \, \mathrm{iof}_{u_s}(\Gamma, w, x) \longrightarrow \exists y. \, y \lhd_\Gamma x \wedge \mathrm{iof}_{u_m}(\Gamma, y, w)\} \tag{18}$$

We can then use the characterization relation to identify the moments that specify an object *beyond what is abstracted way* by an universal, i.e. the moments that provide further details on a particular, with respect to a universal:

Definition 9. Given a particular structure Γ and a substantial universal $u_s \in \mathcal{U}_s$, we call the set of *detailing moments of* u_s *in* Γ, written as $\Delta_{u_s,\Gamma}$, the set of all moments that inhere in instances of u_s but do not instantiate a moment universal that characterizes u_s, i.e.

$$\Delta_{u_s,\Gamma} = \left\{ \begin{array}{l} m \,|\, \exists x, w. \, m \lhd_\Gamma x \wedge \mathrm{iof}_{u_s}(\Gamma, w, x) \wedge \\ \qquad \left(\forall u_m \in \mathrm{Char}(u_s) . \forall w'. \, \neg \mathrm{iof_{u_m}}\left(\Gamma, \mathrm{w}', \mathrm{m}\right)\right) \end{array} \right\}. \tag{19}$$

We can then define the trimming operation, which captures the abstraction process of a universal, as the removal of the detailing moments of a universal:

Definition 10. Given a particular structure Γ, a substantial universal $u_s \in \mathcal{U}_s$ and a particular structure Γ^*, we say that Γ^* is Γ *trimmed by* u_s, written as $\Gamma^* = \Gamma \downarrow u_s$, if and only if:

- the set of worlds of Γ^* consists on the worlds of Γ with u_s's detailing moments removed, i.e.

$$\mathcal{W}_{\Gamma^*} = \{w - \Delta_{u_s,\Gamma} \mid w \in \mathcal{W}_\Gamma\}\,; \tag{20}$$

- inherence is restricted to moments that are not detailing moments of u_s. i.e.

$$x \lhd_{\Gamma^*} y \longleftrightarrow x \lhd_\Gamma y \land x \notin \Delta_{u_s,\Gamma}. \tag{21}$$

Before we present the definition of sortality, we just add some a few restrictions on the particular structure:

Definition 11. We call a particular structure *complete with respect* to an instantiation system if and only if the following conditions are met:

- every particular in Γ is non-collapsible;
- every universal has at least one instance in Γ, i.e.

$$\forall u \in \mathcal{U}_s \cup \mathcal{U}_m.\, \exists x, w.\, \mathrm{iof}_u(\Gamma, w, x); \tag{22}$$

- the characterizing set of every substantial universal is unique, i.e.

$$\forall u_1 \in \mathcal{U}_s.\forall u_2 \in \mathcal{U}_S.\mathrm{Char}_\Gamma\left(u_1\right) = \mathrm{Char}_\Gamma\left(u_2\right) \longrightarrow u_1 = u_2; \tag{23}$$

- characterizing sets determine the instantiation relation on substantials, i.e.

$$\forall u_s \in \mathcal{U}_s.\forall w \in \mathcal{W}_\Gamma.\forall x \in \mathcal{P}_\Gamma.\mathrm{iof}_{u_s}\left(\Gamma, w, x\right) \longleftrightarrow \\ \left(\forall u_m \in \mathrm{Char}_\Gamma\left(u_s\right).\, \exists y.\, y \lhd_\Gamma x \land \mathrm{iof}_{u_m}\left(\Gamma, w, y\right)\right); \tag{24}$$

  - every moment universal characterizes at least one substantial universal, i.e.

$$\forall u_m \in \mathcal{U}_m.\exists u_s \in \mathcal{U}_s.\, u_m \in \mathrm{Char}_\Gamma\left(u_s\right); \tag{25}$$

  - it has all trimmings, i.e. for any substantial universal $u_s \in \mathcal{U}_s$, $(\Gamma \downarrow u_s) \in \mathcal{PS}$ and the identity function on particulars of $(\Gamma \downarrow u_s)$ is a morphism from $(\Gamma \downarrow u_s)$ to Γ.

Now we can present the formal definition of sortality of a universal, relative to an instantiation relation and a complete partial structure:

Definition 12. Given a complete particular structure Γ and a substantial universal $u_s \in \mathcal{U}_s$, we call u_s a *sortal* (*with respect to* Γ) if and only if all instances of u_s in Γ are identifiable in the particular structure obtained by trimming Γ by u_s, i.e.

$$\mathrm{Sortal}\left(u_s\right) \longleftrightarrow \left(\forall x, w.\, \mathrm{iof_{u_s}}\left(\Gamma, \mathrm{w}, \mathrm{x}\right) \longrightarrow \mathrm{x} \in \mathcal{P}^{\mathrm{id}}_{(\Gamma\downarrow\mathrm{u_s})}\right). \tag{26}$$

The properties of the proposed characterization of sortality are described by the following lemmas and by the 2:

Lemma 1. *All endomorphisms in a complete particular structure are permutations in that structure.*

Proof. Since the particular structure is non-collapsible, every endomorphism must be injective and, since the set of particulars is finite, the endomorphism must also be bijective and, as such, a permutation. □

Lemma 2. *For any complete particular structure Γ, any substantial universal u_s, and any permutation φ on Γ, the restriction of φ to $\mathcal{P}_{(\Gamma \downarrow u_s)}$ is a permutation on $\Gamma \downarrow u_s$.*

Proof. The trimming operation removes all moments that do not instantiate certain moment universals (that do not characterize u_s) *and* that inhere in some substantial that instantiate u_s. Since instantiation is invariant under isomorphisms and permutations, the image of $\Delta_{u_s,\Gamma}$ under is going to be $\Delta_{u_s,\Gamma}$ itself. Consequently, since φ is a permutation, the image of $\mathcal{P}_\Gamma - \Delta_{u_s,\Gamma}$ is $\mathcal{P}_\Gamma - \Delta_{u_s,\Gamma}$ itself. Thus, the restriction of φ on the set of particulars of $\Gamma \downarrow u_s$, i.e. on $\mathcal{P}_\Gamma - \Delta_{u_s,\Gamma}$, is a permutation on $(\Gamma \downarrow u_s)$. □

Lemma 3. *Non-identifiable substantials are kept under trimming of a complete partial structure, i.e. for any complete particular structure Γ, any substantial $x \in \mathcal{P}_\Gamma$ and any substantial universal u_s:*

$$x \in \mathcal{P}^{id}_\Gamma \longrightarrow x \notin \mathcal{P}^{id}_{(\Gamma \downarrow u_s)}. \tag{27}$$

Proof. By 1, since non-identifiability is equivalent to permutability, x is also non-permutable in Γ. Thus, there is must be an endomorphism φ on Γ such that either (1) $\varphi(x) \neq x$ or (2) there is a particular y in Γ such that $y \neq x$ and $\varphi(y) = x$. By 1, φ is a permutation on Γ and, . trimmed by U, i.e. $\varphi \in Perm(\Gamma \downarrow U)$. Of course, since φ is a $(\Gamma \downarrow U)$-permutation, it is also a $(\Gamma \downarrow U)$-endomorphism.

To show that x is permutable in $(\Gamma \downarrow U)$ as well, consider the cases (a) and (b): in case (a), since the trimming operation only removes moments, x is a also a substantial in $(\Gamma \downarrow U)$, and since φ is a $(\Gamma \downarrow U)$-endomorphism and $\varphi(x) \neq x$, then is also permutable in $(\Gamma \downarrow U)$; in case (b), we have that y is a substantial in Γ, since particular structure morphisms preserve substantials and, thus, that y is also in $(\Gamma \downarrow U)$ and, since $\varphi(y) = x$, $y \neq x$ and φ is a $(\Gamma \downarrow U)$-endomorphism, we have that x is permutable in $(\Gamma \downarrow U)$. □

Lemma 4. *If a particular is identifiable in a complete particular structure Γ trimmed by U, then it is also identifiable in Γ, i.e.*

$$\mathcal{P}^{id}_{(\Gamma \downarrow u)} \subseteq \mathcal{P}^{id}_\Gamma.$$

Proof. This is is just the contrapositive of 3. □

Theorem 2. *Particulars that instantiate a sortal in a complete particular structure are identifiable in that structure.*

Proof. Directly from 4 and Definition 12. □

4. Some Practical Implications

The falsifiability of the assertions constructed using the concepts and relations provided by an upper ontology is an important factor in the application of the later in the practice of conceptual modeling. Unless the upper ontology provides a clear and precise method for determining the truthfulness of an assertion such as "X is a part of Y", or "U is a rigid universal" or, in the case discussed in this paper, "U is a sortal", the use of this concepts shall introduce an undesirable element of subjectivity in the ontology or conceptual model. In the specific case of UFO, the lack of a definition for the notion of sortality, in contrast with other concepts provided by UFO, is a gap that has implications in the validation of OntoUML models.

However, our proposed characterization leads naturally to a clear *falsifiability criterion* for the notion of sortality (and of identity and individuality): to prove that a substantial universal is not a sortal, one first constructs the trimmed version of the intended model, and then presents a morphism that permutates one of the instances of the universal in the trimmed model.

5. Conclusion

In this paper, we presented a novel characterization of the notions of identity, individuality and sortality in the context of the Unified Foundational Ontology (UFO). We presented an informal discussion regarding these concepts and pointed out the issues regarding a characterization that relies on the existence (or definability) of suitable predicates. As an alternative, we propose a characterization that is determined solely by the properties of the configuration of objects itself, without requiring the consideration of the existence of a predicate, nor it carries a hidden variable representing the choice of a language and of an expression for the predicate. We also presented a formal characterization of these concepts and demonstrated: (1) the equivalence between the structural and the tradition approaches for characterizing identity; (2) the properties of the formal characterization of sortality. We also described how a falsifiability criterion for sortality can be derived from the proposed definition, filling an important gap present in the original UFO theory and enabling the objective validation of OntoUML models.

The structural approach towards identity described in this paper extends the one proposed in [11] with the characterization of the notion of individuality. This approach is similar to the one used to characterize grades of discernibility in fragments of first-order logic presented in [12]. The notions of particular with individuality and of a particular of identity presented in this paper correspond, respectively, to the notions of a two-distinguishable particular and a one-distinguishable particular presented in that work.

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