Shear Deformation of Compressed Elastic-Plastic Arrays with Collinear Systems of Cracks

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Abstract
The plastic strips propagation in an ideal elastic-plastic body with colinear system of shear cracks has been studied. The cracks are opposed to the stable stresses on the faces. Stress-and-strain state of the body, dependencies of the strips length on the loading for random distance between cracks and the level of their faces friction have been found. The value of critical loading has been found where the plastic strips are merging and some plastic fracture occurs.

Keywords 1
All-round compression, shear cracks, plastic zones, conformal representation.

1. Introduction

As a rule, the well-known investigations of plastic zones propagation for the bodies with cracks have dealt with their noninteracting and free of external stresses surfaces [1-3]. Under combined shear and compression stresses of three-dimensional arrays conditions the cracks opening is accompanied by the interaction of their faces. It is especially important in case of the cracks of 3d type as the distance between the surfaces during the crack formation process has been equal to 0 and the presence of compression has resulted in friction forces occurrence. The above-mentioned situation is typical in the problems of geomechanics, mechanics of earthquakes and rock formations due to the presence of big efforts of compression and widely spread shear mechanisms of deformation processes [4-6].

So, we will study the propagation of plastic deformations within the conception of plastic deformations location in the cracks plane [1]. The basic reason for this assumption is the interaction of cracks faces, as we have known that it causes the narrowing of the continuous plastic zone in perpendicular to the crack direction and contributes to the thin-strip location of plastic deformations [7]. Moreover, friction also reduces the size and slows down the propagation of plastic zones [7]. On the contrary, the interaction of cracks of collinear system has accelerated the propagation of plastic strips. Thus, it is necessary to take into account both of these competing factors of impact on the plastic strips to determine the conditions of domination of each of them.

2. Problem statement and formalization.

Let an unbounded ideal elastic-plastic body containing the system of tunnel collinear cracks $-l \leq |x + 2na| \leq l,\ y = 0 \quad (n \in Z)\ -\infty < z < \infty$ ($2l$ – length of cracks, $2a$ – distances between their

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centers), is loaded by the infinite shear efforts $\tau_{yz}^{\infty} = \tau_{\infty}$, $\tau_{xz}^{\infty} = 0$ (Figure). Under extra compression by normal stresses conditions on infinities $\sigma_{yy}^{\infty} = -p$ the crack is closed and it can't excite any normal homogeneous field of stresses in the environment: $\sigma_{yy}(x,y) = \sigma_{yy}^{\infty} = -p = \text{const}$. The interaction of cracks faces is accompanied by their friction resulted in some extra tangent stresses on the cracks faces which can oppose the shear and whose values are supposed to be stable and the same $\tau_{yz} = \tau_0 = f_0 p$ $(\tau_0 < k)$, where f_0 - static coefficient of the sliding friction. When $\tau_{\infty} > \tau_0$ the cracks faces are shifted, some extra tangent stresses are acting on their faces $\tau_{yz} = \tau_0 = f_0 p$ and some plastic strips are developing on the cracks continuation due to the concentration of stresses $-l - d \le |x + 2na| \le l + d$, $y = 0$ $\;(n \in Z)$, $-\infty < z < \infty$, whose length d should be found. In these plastic strips the yield criterion must be fulfilled: $\tau_{xz}^2 + \tau_{yz}^2 = k^2$, where k – shear limit of liquidity. Unlike linear case, due to the nonlinear character of the problem it cannot be reduced to the study of similar problems only in case of the given stress on infinities $\tau_{yz}^{\infty} = \tau_{\infty} - \tau_0$.

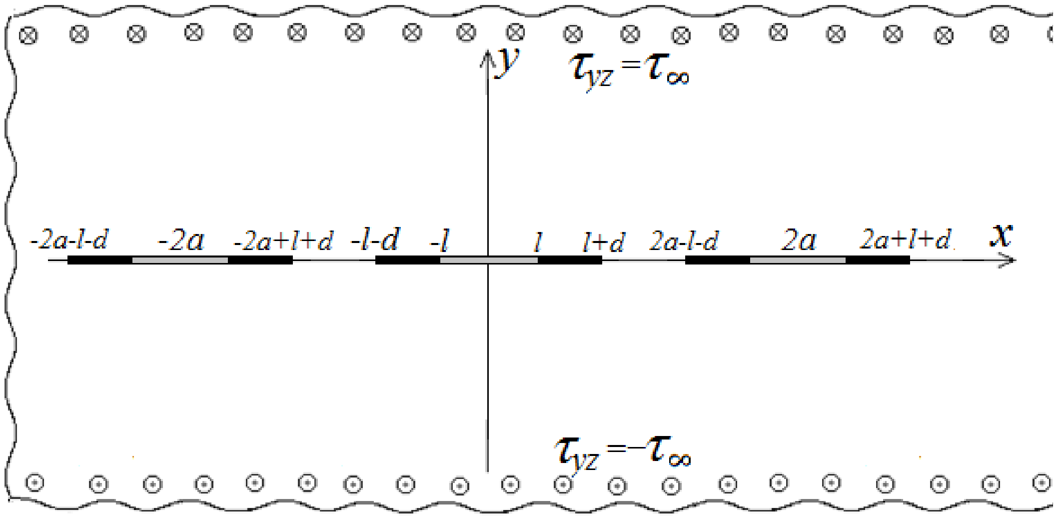

Figure 1: Cross section of the body.

Under the defined conditions together with the above-mentioned uniform field of compression stress some anti flat stress-and-strain state arises in the body which can be found by the shear $w(x,y)$. Two nonzero components of the stresses tensor are given by the formulae $\tau_{xz} = \mu\, \partial w/\partial x$ and $\tau_{yz} = \mu\, \partial w/\partial y$ (μ – shear modulus of the material). The shear $w(x,y)$ is symmetric referred to the lines $x = na$ $(n \in Z)$ and is antisymmetric about X-axis. That is why it can be determined only in a half-strip $0 \le x \le a$, $0 \le y < \infty$ (area D).

Due to the balance conditions and Hooke's law, the function $\tau(\zeta) = {}_{yz}(x,y) + i\tau_{xz}(x,y)$ is analytical one in the elastic part of the body. So, to determine stress-and-strain state of the body we will define a boundary problem for function $\tau(\zeta)$ in the area D, consisting in the necessity to fulfill four conditions.

1. Due to the symmetry we have obtained

$$\operatorname{Im}\tau(\zeta) = 0 \quad \big((\zeta = iy,\ y \ge 0) \cup (\zeta = x,\ l + d \le x \le a) \cup (\zeta = a + iy,\ y \ge 0)\big). \qquad (1)$$

2. On cracks faces the stress $\tau_{yz} = \tau_0 = \text{const}$, so

$$\operatorname{Re}\tau(\zeta) = \tau_0 \qquad (\zeta = x,\ 0 \le x \le l). \qquad (2)$$

3. In the area $l \le x \le l + d$ the yield criterion has been fulfilled, so

$$|\tau(\zeta)| = k \quad (\zeta = x,\ l \le x \le l + d). \qquad (3)$$

4. Stress-and-strain state on infinities is defined by the formula

$$\lim_{\zeta \to \infty} \tau(\zeta) = {}_{\infty}. \qquad (4)$$

As the function $\tau(\zeta)$ in the zone D is analytical and one-sheet, it conformally maps D to the part of circle $|\tau| \le k$, Re $\tau \ge \tau_0$, Im $\tau \ge 0$ (zone G Figure 2). In this case the following points match the areas boundaries D and G: $\zeta = \infty + ih \to \tau = \tau_\infty$, $\zeta = 0 \to \tau = \tau_0$, $\zeta = l \to \tau = \tau_0 - i\sqrt{k^2 - \tau_0^2}$, $\zeta = l + d \to \tau = k$. Section ($\zeta = iy,\ 0 \le y < \infty$) within the zone D is mapped to the interval ($\mathrm{Im}\tau = 0,\ \tau_0 \le \mathrm{Re}\ \tau \le \tau_\infty$); interval ($\zeta = x,\ 0 \le x \le l$) – to $\mathrm{Re}\ \tau = \tau_0,\ -\sqrt{k^2 - \tau_0^2} \le \mathrm{Im}\ \tau \le 0$, section ($\zeta = x,\ l + d \le x \le a$)$\cup$($\zeta = a + iy,\ y \ge 0$) – to the interval ($\mathrm{Im}\ \tau = 0,\ \tau_\infty \le \mathrm{Re}\ \tau \le k$). The interval ($\zeta = x,\ l \le x \le l + d$), corresponding to the plastic strip, is mapped to the circle arch ($|\tau| = k,\ -\arccos(\tau_0/k) \le \arg\tau \le 0$).

3. Study of plastic strips propagation

The solution of the boundary problem (1)-(4) is reduced to the construction of the described conformal mapping [8]. We will introduce some additional complex plane t, where the areas D and G match the upper half-plane $H = \{\mathrm{Im}t \ge 0\}$ (see Figure 2) and we will find the function $\tau(\zeta)$ in parametric form

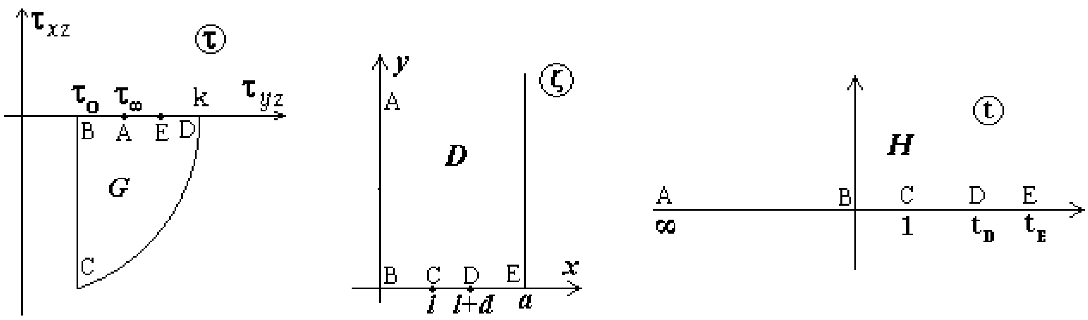

Figure 2: Example figure

$$\tau = \tau(t), \quad \zeta = \zeta(t) \quad (t \in H) \tag{5}$$

Function $\tau(t)$ is given as a composition of elementary mappings:

$$\tau(t) = k\frac{t_6(t)\exp(i\psi_0) + \exp(-i\psi_0)}{t_6(t) + 1}, \tag{6}$$

where

$$t_6(t) = t_5(t)^{\psi_0/\pi},\ t_5(t) = \frac{t_3(t) + M}{t_3(t) - M},\ t_3(t) = \sqrt{\frac{1 - (s+1)t}{(s+1)t}},$$

$$M = -\mathrm{tg}\left(\frac{\pi}{2\psi_0}\left(2\mathrm{arctg}\frac{\sqrt{k^2 - \tau_0^2}}{\tau_\infty - \tau_0} - \pi\right)\right),\ \psi_0 = \arccos\frac{\tau_0}{k},\ s = -\frac{M^2}{M^2 + 1}.$$

As z^q $(0 < q < 1)$ we consider the analytical in the upper half plane function receiving actual added values at the same values of z.

In the finishing point of the strip $\tau = k$. So, from the formula (6) we have obtained the complex number of the certain point of the additional plane: $t_E = 1/(s + 1)$.

The function determined by composition of elementary mappings $\zeta(t)$ looks like:

$$\zeta(t) = \frac{2a}{\pi}\arcsin\left(\sqrt{t}\sin\frac{\pi l}{2a}\right). \tag{7}$$

As $\zeta(t_E) = l + d$, the length of plastic strips can be obtained from the last formula

$$d = \frac{2a}{\pi}\arcsin\left(\frac{1}{\sqrt{s+1}}\sin\frac{\pi l}{2a}\right) - l. \tag{8}$$

The length of plastic strips as functions of stresses have been calculated for different stresses and distance between cracks and for different levels of the faces friction and are shown Figure 3.

The strips length can't exceed the half distance between the tops of neighboring cracks. Stress $\tau_\infty = \tau_\infty^*$ when $d = a - l$ can cause some ductile fracture of the body. From the formula (8) it is clear

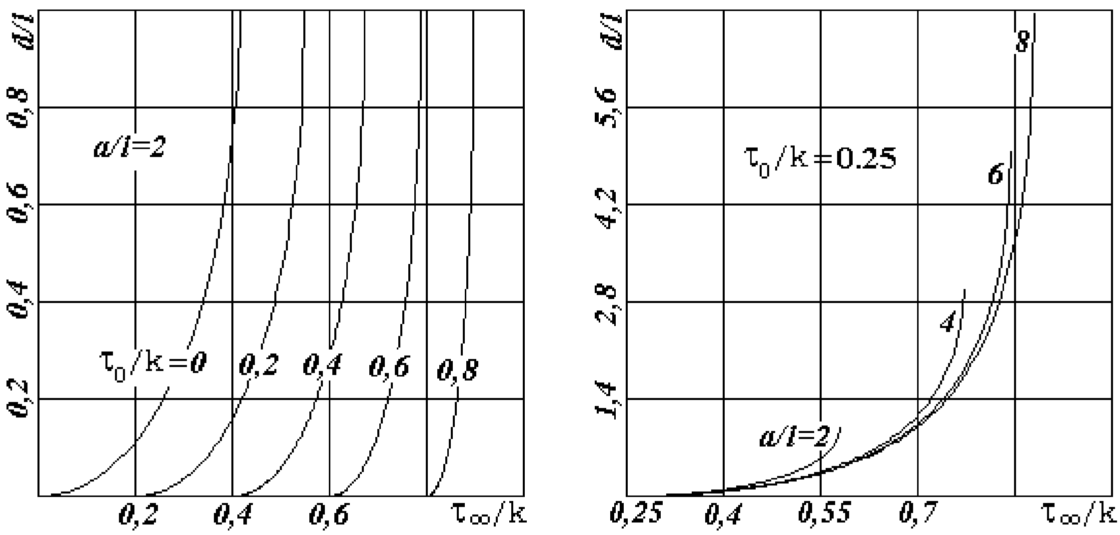

Figure 3.

that the value of critical loading $\tau_\infty = \tau_\infty^*$ has satisfied the equation $\sin(\pi l/(2a)) = \sqrt{s+1}$. Hence,

$$\tau_0^* = \tau_0 + \sqrt{k^2 + \tau_0^2}\,\mathrm{tg}\left(\frac{\psi_0}{2}\left(1 - \frac{l}{a}\right)\right). \qquad (9)$$

Under fixed levels of the friction of faces conditions ($\tau_0 = \mathrm{const}$) the last par has given the dependence of critical loading on the distance between cracks (Figure 4). The interaction of faces has very strong impact on the value of critical loading for the cracks locating very close to each other. The bigger distance between cracks the more weaken is the interaction impact.

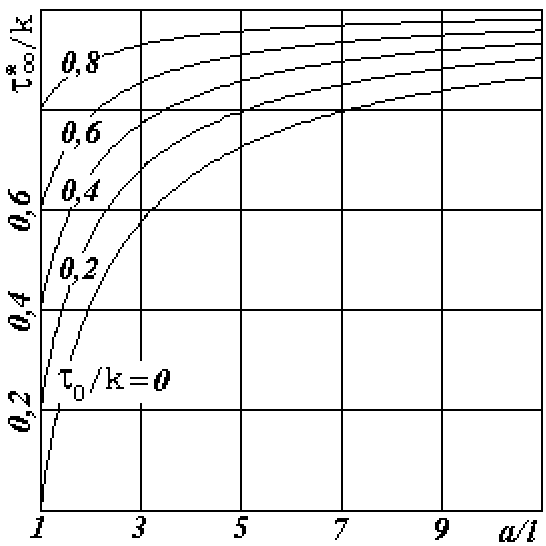

Figure 4.

Without the interaction of faces ($\tau_0 = 0$) the formulae (8), (9) have given the known dependencies [9]:

$$d = \frac{2a}{\pi}\arcsin\left(\frac{k^2+\tau_\infty^2}{k^2-\tau_\infty^2}\sin\frac{\pi l}{2a}\right) - l, \qquad \tau_\infty^* = k\,\mathrm{tg}\left(\frac{\pi}{4}\left(1 - \frac{l}{a}\right)\right).$$

In case of large distance between cracks the function stresses has been expressed by the formulae (5) where $\tau(t)$ is the same as for the general case and $\zeta(t) = l\sqrt{t}$. The length of plastic strips has been described by the following expression $d = l((s+1)^{-1/2} - 1)$.

The conducted study has proved a considerable impact of cracks interaction on plastic strips development near their tops. Therefore, the important problem requiring a special attention is the study of cracks interaction located in neighbourhood in an elastic-plastic body resulted in possible plastic strips coalescence and plastic fracture.

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