Mathematical Model and Method of Enterprise Financial Risk Assessment Based on Threshold Elements

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Abstract

To assess financial risk, it is necessary to restore a large set of many initial parameters, which aren't only determined by the criteria of authority, efficiency, and minimum capabilities, but also by using special information and their display functions. The authors propose a model of financial risk assessment of the enterprise, developed on the basis of mathematical apparatus of certain elements of decomposition functions to coordinate parameters and functions of determining the level of risk of a potential investor, which is a more accurate, unambiguous, and categorical during the assessment of financial risk by using rigidly defined threshold elements of input parameters loaded into response classes. The solution to the complex problem of accurate financial risk assessment becomes possible by obtaining several quantitative estimates of all separate classes of input data. The accuracy of financial risk assessment for the computer model developed in the article has been experimentally tested in small enterprises and is the highest in comparison to the normative methods of financial risk assessment. The special proposal of the proposed model consists of the restoration of many fundamental initial parameters aimed at assessing the financial risk, which is determined by the relevant capabilities of the enterprise and expert information. The model also uses the function of conversion of initial parameters for the assessment and a set of functional decompositions for compiling parameters, to influence the identification of financial risk which makes it universal for the use of enterprises operating in different sectors of the economy. Also, the developed computer model can be used both offline and when using a cloud environment.

Keywords

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Risk, information technology, risk assessment, financial risk, computer model, financial condition, risk function, threshold elements, 2-factor model of Gorvatov, multifactor regression analysis, experts, algorithm, assessment, enterprise.

1. Introduction

Problems of various risks of the financial struggle are actual in any country, especially in the period of its active digitalization and widespread introduction of information technology [1]. This is due, in particular, to the fact that there is a general digitalization of all spheres of human activity and society as a whole, which requires all the major integration of manufacturing enterprises with modern information systems and technologies. Information technology and computer models based on science-based methods are becoming increasingly important, which allow the assessment of the risk level not only quickly and accurately, but also autonomously, especially when it comes to the idea of assessing the financial risks of the enterprise. Financial risk not only affects the development of the company itself, but also the economic development of society, so the assessment of the financial risk of the company is partial.

The financial risks of the enterprise are determined by exogenous and endogenous factors. Exogenous factors are usually well controlled with the help of modern information technology. In contrast, endogenous factors are poorly taken into account in modern information systems to support decision-making. Therefore, the development of computer models to identify the financial risk of operating enterprises is an important direction in the development of modern information technologies.

The presence of computer models that can be used to monitor the financial condition of both the enterprise itself and its communicators in the future allows the creation of information systems to improve the efficiency of the economy as a whole. These information systems can also be localized in the cloud, which can significantly increase the number of their users.

An important circumstance that comes along this path is that computer models must be accessible enough to cover a wide variety of business areas.

Problems of economic development and activity of enterprises in the crisis under the influence of coronavirus in the world and Ukraine are exacerbated not only by uncertainty (demand, prices) and fierce competition but also by the lack of adequate financial risk assessment model taking into account current requirements. To survive in such conditions, business leaders need to make bold and unconventional decisions about innovative investments to increase the competitiveness of the company and improve its financial condition, which significantly increases not only the investment but also the financial risk of the company. And, if the first type of risk and its assessment were considered in previous works by individual authors [2, 3], the assessment of the financial risk of the enterprise, which is to blame for the direct impact and investment risk, was not considered at a sufficient level.

Traditionally, the decision-making problems involve creating a preference order to classify alternatives to choose the best one. For these decision-making problems where alternatives are compared to a single criterion, this can be easily done. However, in the most realistic cases, several different criteria should be evaluated. That is why in this paper a model for calculating the financial risk of the enterprise on the basis of the threshold elements of the tuple of selected financial criteria is developed.

2. Related work

We will analyze related work and consider the main models used to assess the financial risk of parameters in the works of both domestic and foreign scientists.

In paper [4] there is a proposed approach that integrates the decision-relevant information, which is subject to uncertainty, to multi-criteria decision-making. An approach must enable decision-makers to explore the uncertainty and risk involved in their decisions. It arose from the theory of risk-based decision-making and the generalization of particular risk-based solutions in different domains. The authors of this work consider the roar as a whole, without its financial specification.

As a rule, the financial risk of the enterprise is considered as the risk of bankruptcy of the enterprise. For example, the paper [5] presents the results of intelligent information system development for enterprise bankruptcy risk estimation on the basis of fuzzy logic and neural network technologies synthesis. The developed information system allows to make the current estimation of the risk of bankruptcy of the enterprise and gives the chance to trace how it impacts to separate indicators' changes. The paper [6] develops a genetic bankrupt ratio analysis tool using an agenetic bankrupt ratio analysis tool using a genetic algorithm to identify influencing ratios from different bankruptcy models and their influences in a quantitative form.

The paper [7] proposes a novel financial risk assessment model for companies based on heterogeneous multiple-criteria decision-making (MCDM) and historical data. Subjective and objective indexes are comprehensively taken into consideration in the financial risk assessment index system of the model, which combines fuzzy theory with quantitative data analysis. Moreover, the assessment information obtained from historical financial information of the company, credit rating agency, and decision-makers, including crisp numbers, triangular fuzzy numbers, and neutrosophic numbers. However, the authors do not take into account the threshold values of the selected criteria.

The article [8] discusses the bankruptcy prediction model using random forest based on the most influential ratios needed to predict bankruptcy. These coefficients are selected based on a genetic algorithm that filters out the most important of the various existing bankruptcy models.

The authors [9] developed the generalized computational models and methods for automated assessments of the risks due to uncertainties of influencing factors in the complicated deterministic stationary systems. The principal idea of these models and a method is based on computational solving the finite set of boundary value problems modeling the considered systems to represent the deterministic properties of researched possible risks in the general case.

The article [10] proves that the traditional practice of using a singular performance metric for classifier evaluation is not sufficient for imbalanced classification credit and bankruptcy risk. This paper proposes a multi-criteria decision-making (MCDM)-based approach to evaluate imbalanced classifiers in credit and bankruptcy risk prediction by considering multiple performance metrics simultaneously. Note that the estimation of crisis symptoms of the enterprise and diagnosing the possibility of a financial crisis is carried out long before the detection of its obvious signs [11].

The article [12] considered the assessment of the risk of bankruptcy of an enterprise according to indicators of financial and economic activity using Bayesian networks. The purpose of this study is to develop a model for predicting the financial problems of enterprises. A Bayesian network has been developed for analyzing and predicting the financial condition of industrial enterprises. Financial statements were used to analyze 3000 industrial enterprises in Ukraine. Five integral financial indicators were identified for building Bayesian networks (maneuvering coefficient, debt-to-equity ratio, the coefficient of autonomy, current liquidity ratio, financial stability ratio).

The results obtained in the study show the forecast of the quality and the practical application possibility of the developed Bayesian network in the decision support system for an intelligent assessment of forecasting the bankruptcy probability of an enterprise.

The key point in assessing the financial risk of any organization is to determine the level of the financial condition of the enterprise. The key point in assessing the financial risk of any organization is to determine the level of the financial condition of the enterprise. For this, such basic mathematical models as Horvathov's 2-factor model [13] and the Beaver model [14] are used. But both of them have several disadvantages. For example, Horvathova's model was developed for the US economy. In Ukraine there are high inflation rates, other cycles of macroeconomics and microeconomics, levels of capital intensity, energy intensity, and labor intensity of production, other taxation does not allow for a comprehensive estimation of the financial condition of enterprises and therefore significant deviations in estimates in the model. Beaver's coefficient [2, 13, 14] also has a number of disadvantages. First, the normative values of financial indicators do not take into account the industry specifics of enterprises. Secondly, the efficiency of capital use in enterprises (turnover, profitability) is not taken into account. Third, the calculation of the Beaver coefficient is carried out in statics, not taking into account the transient external and internal environment of enterprise valuation.

In paper [15] an improved financial credit risk assessment approach is presented. Based on the credit data from China Banking Regulatory Commission (CBRC), a multi-dimensional and multi-level credit risk indicator system is constructed. In particular, we present an improved sequential minimal optimization (SMO) learning algorithm, named four-variable SMO (FV-SMO), for the credit risk classification model. At each iteration, it jointly selects four variables into the working set and a theorem is proposed to guarantee the analytical solution of the sub-problem. The assessment is made on the China credit dataset and two benchmark credit datasets from the UCI database and CD-ROM database. Also, scientists from China are considering the assessment of financial risks based on a factor analysis model [17]. Based on factor analysis, the presented paper [17] establishes a financial risk assessment model at the company level, and determines the influence degree of the solvency, operation ability, profitability, development ability, and the ability to obtain cash flow, and collects a large amount of relevant information and data, calculates the index weight. Finally, based on the analysis of the actual situation of each real estate company in China, the SPSS software is used for empirical analysis and divides the risk levels of these 120 real estate companies. Unfortunately, these models cannot be used for Ukraine as they take into account the specifics of China.

Modeling the interaction of risk factors using Copula functions was discussed in the article [17, 18]. The procedure is proposed for analyzing the risk factor interaction in financial systems. The procedure is based upon the results of eigenvalues distribution analysis and distances between the eigenvalues for empirical and theoretical dependency matrices. Some results of the theory of random matrices are used to interpret the results achieved in the process of empirical studies for the correlation matrices of a different kind. The results of computational experiments show that for small eigenvalues the results of theoretical analysis for random matrices are similar to the empirical matrices. This result provides a possibility for determining correctly the number of principal factors to construct mathematical models necessary for practical applications.

Thus, an important part of the wider use of information technologies for the development of the country's economy is the creation of information systems for managing the financial condition of enterprises. Moreover, such systems should be flexible enough to, on the one hand, allow, from a single point of view, to identify the financial risks of a large number of enterprises operating in different sectors of the economy [19, 20]. However, on the other hand, these information tools should allow taking into account the specifics of the enterprise.

These conditions are conflicting. This contradiction is resolved by creating a bank of computer models. Users will take into account the conclusions of these models and will be able to select those that will give the most adequate results.

3. Formal problem statement

The aim of the work is to develop a computer model as an element of information technology for calculating financial risk, which is common enough for use by enterprises operating in various sectors of the economy. To assess financial risk, it is necessary to take into account a large set of many initial parameters, which are determined not only by the criteria of completeness, efficiency, and minimality but also by using expert information, as well as their display function. The authors propose a model for assessing the financial risk of an enterprise, developed on the basis of the mathematical apparatus of threshold elements, taking into account the decomposition function for folding the parameters and the function of determining the risk level of a potential investor, which is more accurate, unambiguous and categorical in assessing the level of risk through the use of rigidly defined threshold elements of the input parameters grouped into four classes. The solution to the complex problem of accurately assessing financial risk becomes possible when obtaining a quantitative assessment of all four classes of input data. The computer model proposed in the article is quite general for use by enterprises operating in various sectors of the economy and can be used both offline and in the cloud.

4. Building the structural and mathematical models of financial risk assessment

The process of financial risk (FR) calculating belongs to the category of complex problems [22] due to the need to take into account a powerful set of input parameters X and output parameters R, and their transformation functions F: $X \rightarrow R$. Promising is the way to decompose a complex function into a sequence simpler so that the functions of lower levels unambiguously identify certain parameters in the functions of higher levels [23].

The process of assessing FR is the consistent implementation of a number of functions. The task of deciding on the evaluation of FR is to choose an adequate solution R from the set of solutions Z_j ($j=\overline{1,J}$). The choice is made with the help of FR estimates based on the set X of the estimation parameters x_i ($i = \overline{1,n}$, $n \in N$).

To assess the FR, it is necessary to determine the criteria for assigning the company to a particular class of financial condition. In addition, the specificity of the construction of such a system is the need to take into account the set of initial input parameters, which are the basis for calculating the evaluation parameters.

The peculiarity of the model is that it takes into account the set of initial input parameters $K = (k_c)$ ($c=\overline{1,C}$), which is determined by the relevant reporting of the enterprise and expert information; the set of evaluation parameters $X = (x_i)$ ($i=\overline{1,n}$) financial condition; function of conversion of initial parameters into estimating F_1: $K \rightarrow X$; the set of decomposition functions $D = (Y, \ldots S, P)$ of the collapse of the parameters by which the identification of the financial condition of the enterprise; the function of determining the level of risk F_2: $Z_j \rightarrow R_j$ of the potential investor, which corresponds to the Zj level of FR; the set of output parameters $R = (R_j)$ ($j = \overline{1,J}$):

$$R = \{K, F_1, X, D, Z, F_2\}, \quad (1)$$

To obtain the final result on the assessment of FR and the corresponding level of risk in decision-making, based on the initial input evaluation parameters *K*, it is necessary to implement the above functions in the following sequence:

$$K \xrightarrow{F_1} X \xrightarrow{D} Z_j \xrightarrow{F_2} R_j. \quad (2)$$

To determine the final assessment of the financial condition of the enterprise *Zj* and the corresponding level of risk Rj of the potential investor, it is proposed to consider a combination of complex functions - parameters $P_1 \dots P_q$ - financial condition, assessing groups of indicators of the highest level of the hierarchy:

$$Z_j = F(P_1, P_q) \quad (3)$$

In turn, the input data for calculating the complex parameters P_1 and P_q are a set of parameters that evaluate certain groups of indicators (($S_1 \dots S_p$), starting with financial stability (S_1) and ending with profitability (S_p), i.e.:

$$P_1 = F(S_1 \dots S_t), P_q = F(S_e \dots S_p), \quad (4)$$

where $t, e, p \in M$, and *M* is the set of functionals of generalizing parameters of the *P*-th level.

Taking into account the influence of a constantly changing set of factors of the external and internal environment means that the complex parameters of the penultimate level ($Y_1 \dots Y_m$) are functions of the corresponding evaluation parameters x_i of the financial condition, in particular:

$$Y_1 = f(x_1 \dots x_l) \ \cdots \ Y_m = f(x_k \dots x_n), \quad (5)$$

where $l, k, n \in N$.

The estimation parameters xi are determined on the basis of the set of initial input parameters *K* and the transformation function $F_1 : X = F_1(K), K = (k_c), c = \overline{1,C}; X = (x_i), i = \overline{1,n}$.

Based on the composite functions (1) - (5), it is necessary to form a set *X* of corresponding parameters for estimating FR. This set is formed using the set of initial input *K* parameters $k_1 \dots k_e$), where $e \in N$. The definition of this set K is carried out using the appropriate reporting of the enterprise, including balance sheet, statement of financial performance, etc., as well as expert estimates various issues.

The general structural model of the FR evaluation process (Fig. 1) consists from *A* levels, its decomposition occurs as follows.

At the first level, the set *K* of the initial input parameters is formed. The second level involves the formation of the set *X* of the evaluation parameters of the financial condition of the enterprise on the basis of the set *K* of the initial input parameters.

At the third and subsequent intermediate levels, in particular *S* and *P*, there is a formation of complex generalized indicators of evaluation of FR $Y_1 \dots Y_m$; $S_1 \dots S_p$; $P_1 \dots P_q$.At the highest level *A*, the solution Z_j, $j = \overline{1,J}$, is identified, which determines the FR from the set of possible states.

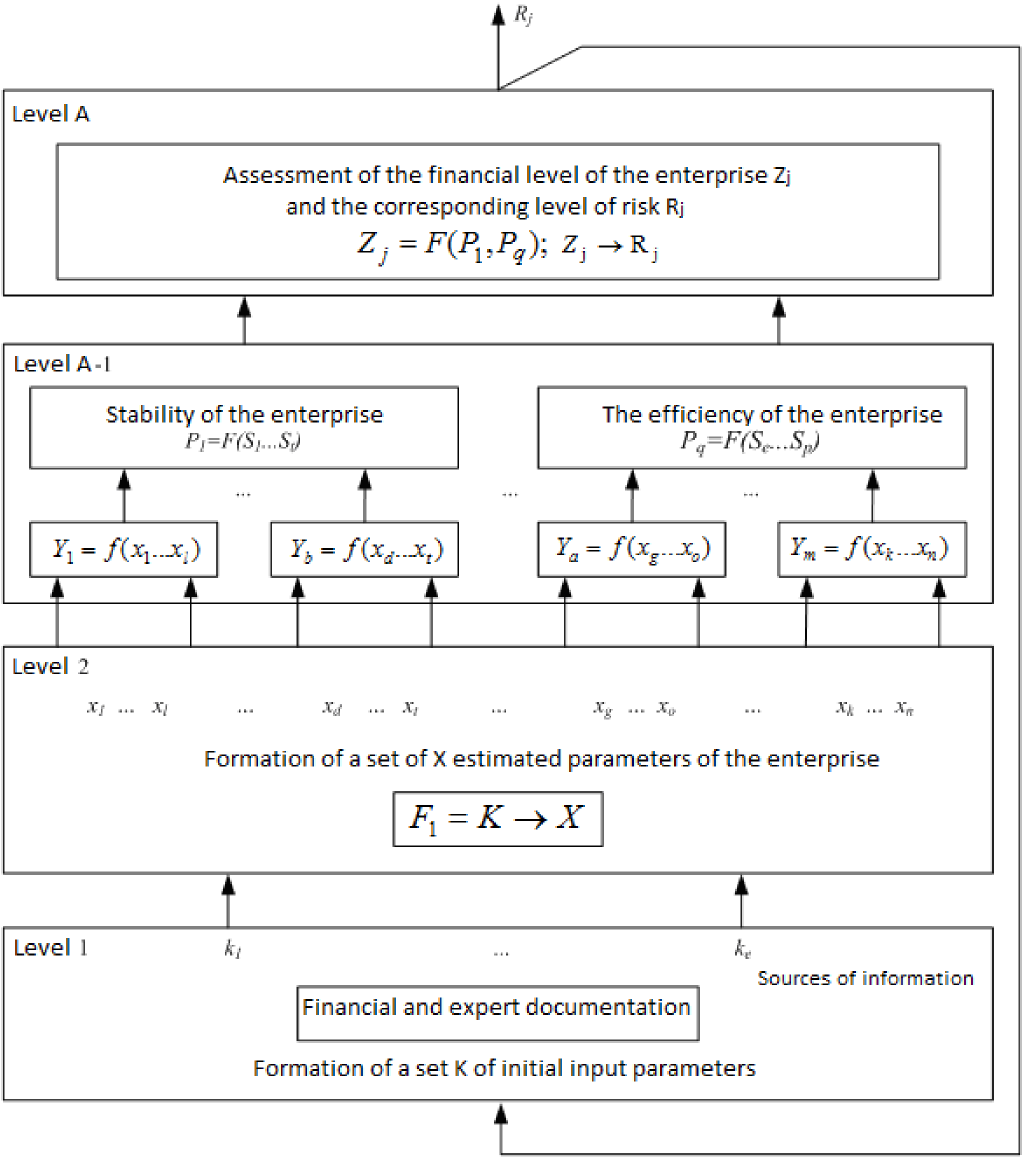

Figure 1: General structural model of the FR evaluation process

This model structure allows you to both add and remove indicators, taking into account the industry, the state of economic development and the ever-changing impact of both external and internal environments of the entity in a crisis. In addition, the hierarchy of the model proposed by the authors simplifies the evaluation process using a modern mathematical apparatus.

To determine the functions (1) - (5) it is necessary to form sets of input and output parameters. They must cover a wide range of influencing parameters, as well as meet the conditions of completeness, effectiveness and minimality.

The set of evaluation parameters X provides the formation of such complex parameters as financial stability (Y_1), liquidity and solvency (Y_2), business activity (Y_3) and profitability (Y_4).

Financial stability, which is a function $Y_1 = f(x_1 ... x_5)$,, is determined by the parameters: x_1 - coefficient of independence; x_2 - financial stability ratio; x_3 - coefficient of financial stability; x_4 – coefficient of maneuverability of own funds; x_5 - the ratio of own working capital.

Liquidity and solvency are a function $Y_2 = f(x_6 ... x_{10})$. It is identified by the following parameters: x_6 - monetary solvency ratio; x_7 - coefficient of estimated solvency; x_8 - critical liquidity ratio; x_9 - the ratio of receivables and payables; x_{10} - asset mobility ratio.

Business activity, which is a function $Y_3 = f(x_{11} ... x_{16})$, is determined by a set of parameters: x_{11} - asset turnover ratio; x_{12} - receivables turnover ratio; x_{13} - turnover ratio of accounts payable; x_{14} - turnover ratio of inventories; x_{15} - turnover ratio of fixed assets, x_{16} - turnover ratio of equity.

Profitability is a function $Y_4 = f(x_{17} ... x_{20})$. It is determined on the basis of the following parameters: x_{17} - cost-effectiveness; x_{18} - return on sales, x_{19} - return on all assets, x_{20} - return on equity.

These evaluation parameters are calculated on the basis of the relevant reports of the enterprise in accordance with the requirements of national legislation.

Based on these input parameters, a set of X quantitative parameters of the firm is formed (table 1).

Table 1

The set of evaluation parameters of FR

The name of the indicator		Formula for calculation
Financial stability		
Coefficient of independence	x_1	k_1/k_2
Coefficient of financial stability	x_2	k_1/k_3
Coefficient of financial firmness	x_3	$(k_1+ k_7)/k_2$
Coefficient of maneuverability of own means	x_4	$(k_1- k_{14})/k_1$
Ratio of own working capital	x_5	$(k_1- k_{14})/k_3$
Liquidity and solvency		
Monetary solvency ratio	x_6	k_4/k_5
Estimated solvency ratio	x_7	k_6/k_5
Critical liquidity ratio	x_8	k_8/k_5
Ratio of receivables and payables	x_9	$k_{10} /(k_5+ k_7)$
Asset mobility ratio	x_{10}	k_8/k_{14}
Business activity		
Asset turnover ratio	x_{11}	k_9/k_2
Receivables turnover ratio	x_{12}	k_9/k_{10}
Accounts payable turnover ratio	x_{13}	k_9/k_{11}
Inventory turnover ratio	x_{14}	k_{12}/k_{13}
Fixed assets turnover ratio	x_{15}	k_9/k_{14}
Equity turnover ratio	x_{16}	k_9/k_1
Profitability		
Cost-effectiveness	x_{17}	k_{15}/k_{12}
Profitability of sales	x_{18}	k_{16}/k_9
Return on all assets	x_{19}	k_{16}/k_2
Return on equity	x_{20}	k_{16}/k_1

Thus, the set of estimating parameters x_i $i = \overline{1,20}$ is determined, namely $x_1...x_{20}$), the values of which are calculated on the basis of the input initial parameters $k_1... k_{16}$).

Define the set of initial parameters $Z = (Z_1,\ldots,Z_j)$, which determine the corresponding level of financial risk of the enterprise Z_j ($j = \overline{1,3}$), as follows: Z_1 - low level of FR; Z_2 - average level of FR; Z_3 - high level of FR.

Given the above justification of the sets of parameters, the authors present the following refined structural model of the FR evaluation process (Fig. 2).

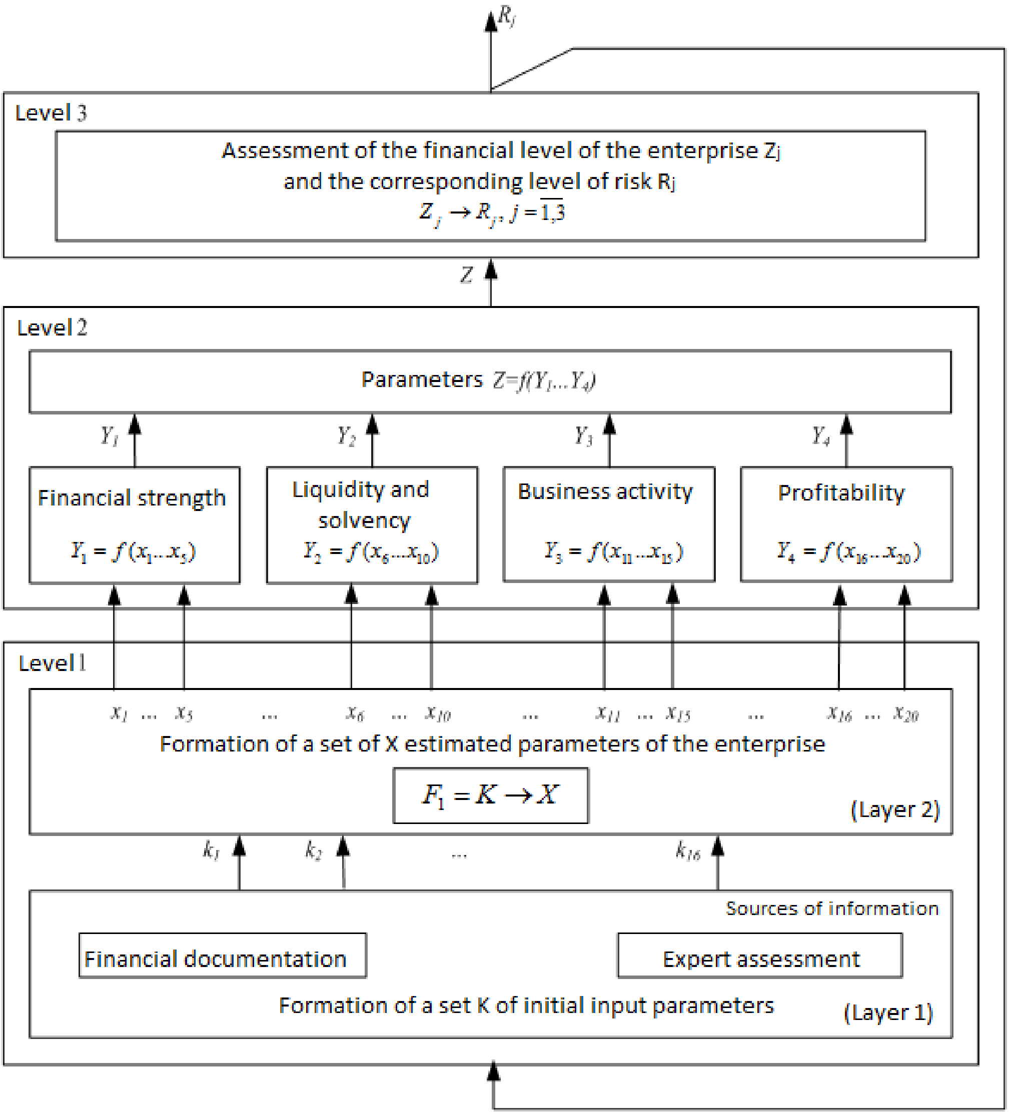

Figure 2: Refined structural model of the process of evaluation of FR enterprise R_j

5. Method for assessing financial risk based on threshold elements

The specificity of the assessment of the level of risk for the financial condition of the enterprise necessitates the need to take into account the evaluative financial parameters with varying degrees of influence on the resulting decision. For this purpose, the mathematical apparatus of the threshold elements will be optimal due to the presence of various influential parameters in the absence of the possibility of their complete search in decision-making. This allows you to increase the speed of information processing when building a risk assessment system for the financial condition of the enterprise.

At the *first* stage, we determine the interval $[x_{i\min}; x_{i\max}]_j$ of change of estimation parameters x_i, $i=\overline{1,20}$ for each level of financial condition Z_j ($j=\overline{1,J}$, J = 3).To assess the FR we will use the following possible financial conditions: low level - j = 1, average level - j = 2, high level - j = 3.

Each financial condition is characterized by evaluative parameters x_i, $i=\overline{1,20}$.

With the help of expert assessments, the authors determined the ranges of change $[x_{i\min} \cdots x_{i\max}]$ of these parameters in accordance with the possible FR Z_j are given in table 2.

Table 2

Ranges of change of parameters $x_1 ... x_{10}$, $x_{17} ... x_{20}$ for 3 levels of financial risk

Parameter	Financial risk		
	low level $[x_{i\min} \cdots x_{i\max}]$	Average level $[x_{i\min} \cdots x_{i\max}]$	High level $[x_{i\min} \cdots x_{i\max}]$
x_1	[0,4;+∞)	[0,2–0,4)	(-∞;0,2)
x_2	[0,7;+∞)	[0,25–0,7)	(-∞;0,25)
x_3	[0,4;+∞)	[0,2–0,4)	(-∞;0,2)
x_4	[0,4;+∞)	[0,2–0,4)	(-∞;0,2)
x_5	[0,1;+∞)	[0,05–0,1)	(-∞;0,05)
x_6	[0,15;+∞)	[0,05–0,15)	(-∞;0.05)
x_7	[1,5;+∞)	[0,5–1,0)	(-∞;0,5)
x_8	[0,75;+∞)	[0,25–0,75)	(-∞;0,25)
x_9	[0,6;+∞)	[0,2–0,6)	(-∞;0,2)
x_{10}	[0,4;+∞)	[0,2–0,5)	(-∞;0,2)
x_{17}	[0,13,;+∞)	[0,04–0,13)	(-∞;0,04)
x_{18}	[0,075;+∞)	[0,025–0,075)	(-∞;0,025)
x_{19}	[0,1;+∞)	[0,02–0,1)	(-∞;0,02)
x_{20}	[0,25;+∞)	[0,8–0,25)	(-∞;0,8)

For data processing, a mathematical apparatus based on threshold elements is used, which works on Boolean algebra. To do this, the evaluation parameters must be presented in two-digit form, so we convert the vector X to the vector G = (g_i), where:

$$g_i = \begin{cases} 0, \text{ if } x_i \notin [x_{imin}; x_{imax}]_j ; \\ 1, \text{ if } x_i \in [x_{imin}; x_{imax}]_j . \end{cases} \quad (6)$$

In the second stage, to rank the evaluation parameters of x_i using the knowledge of experts whose competence was defined above.

I – g_1, g_2, g_3, g_5, g_8 (high level),
II – $g_4, g_7, g_{10}, g_{18}, g_{19}, g_{20}$ (average level),
III – $g_6, g_9, g_{11}, g_{13}, g_{16}, g_{17}$ (low level),
IV – g_{12}, g_{14}, g_{15} (very low level),

Therefore, each of the parameters of I, II or III levels is equal to the set of parameters of the lower level. For example, g6 is compensated by the parameters: $\{g_{12}, g_{14}, g_{15}\}$; $g_4 - \{g_6, g_9, g_{11}, g_{13}, g_{16}, g_{12}, g_{14}, g_{15}\}$; and $g_1 - \{g_4, g_7, g_{10}, g_{18}, g_{19}, g_6, g_9, g_{11}, g_{13}, g_{16}, g_{12}, g_{14}, g_{15}\}$.

In the third stage, determine the relationship between the weights w_i, $i = \overline{1,20}$ by the following rule: let $g_{l,r}$ have the weight $w_{l,r}$, then for variables of the same rank $g_{1,r}, g_{2,r}, ..., g_{p,r}$, the weights will be the same:

$$w_{1,r} = w_{2,r} = ... = w_{p,r} \tag{7}$$

and the weights of the variables of the r-th rank are greater than the weights of the variables ($r+i$)-th, $i = \overline{1, R-r}$,

$$w_{l,r} > w_{j,r+1} \quad l = \overline{1, p_r}, \quad j = \overline{1, p_{r+1}}, \forall l, j \tag{8}$$

Therefore, according to ranking (5) we define the ratio between the weights as follows:

$$w_{12} = w_{14} = w_{15} < w_6 = w_9 = w_{11} = w_{13} = w_{16} = w_{17} < w_4 = w_7 = w_{10} = w_{18} = \quad = w_{19} = w_{20} < w_1 = w_2 = w_3 = w_5 = w_8.$$

At the fourth stage we will make system of inequalities for weights of variables by rules:

1) Each rank forms an inequality. Let the set of evaluation parameters in the rank have the form $g_{k1} g_{k2} ... g_{kr}$. Each logical variable has its own weight $g_{ki} \rightarrow w_{ki}$. Then the rank can be rewritten as an inequality:

$$w_{k1} + w_{k2} + ... + w_{kr} \geq Q \tag{9}$$

where Q is the sum of the weights of the financial parameters at which the assessment of the financial condition of the enterprise Z_j becomes fair.

2) Each rank forms an inequality. In this case, for the implicants $g_{l1} g_{l2} ... g_{ls}$ the corresponding inequality will look like:

$$w_{m1} + w_{m2} + ... + w_{mt} < Q \tag{10}$$

where $m_1, m_2, ..., m_t$ are indices of variables that are not included in this rank.

So, we get the following abbreviated system of inequalities:

$$\begin{cases} w_1 + w_2 + w_3 + w_5 + w_8 \geq Q \\ w_1 + w_2 + w_3 + w_5 + w_4 + w_7 + w_{10} + w_{18} + w_{19} + w_{20} \geq Q \\ w_1 + w_2 + w_3 + w_5 + w_4 + w_7 + w_{10} + w_{18} + w_{19} + w_6 + w_9 + w_{11} + w_{13} + w_{16} + w_{17} \geq Q \\ w_1 + w_2 + w_3 + w_5 + w_4 + w_7 + w_{10} + w_{18} + w_{19} + w_6 + w_9 + w_{11} + w_{13} + w_{16} + w_{12} + w_{14} + w_{15} \geq Q \\ w_{12} + w_{14} + w_6 + w_9 + w_{11} + w_{13} + w_{16} + w_4 + w_7 + w_{10} + w_{18} + w_{19} + w_1 + w_2 + w_3 + w_5 < Q \end{cases} \tag{11}$$

In the fifth stage, it is necessary to rewrite the abbreviated system of inequalities obtained in the previous stage, taking into account the following inequality:

$$w_{l,r} = w_{S,R} + \sum_{i=1}^{R-r}\delta_i,\ S = \overline{1, p_R} \quad (12)$$

where δ_i are positive integers greater than zero.

However, for certainty we consider $S = 1$, ie the weight of the first variable of the R-th rank is taken as the base.

Rewrite the resulting system of inequalities taking into account (12):

$$\begin{cases} w_{12} = w_{14} = w_{15} = w_{12}, \\ w_6 = w_9 = w_{11} = w_{13} = w_{16} = w_{17} = w_{12} + \delta_1, \\ w_4 = w_7 = w_{10} = w_{18} = w_{19} = w_{20} = w_{12} + \delta_1 + \delta_2, \\ w_1 = w_2 = w_3 = w_5 = w_8 = w_{12} + \delta_1 + \delta_2 + \delta_3, \end{cases} \quad (13)$$

where δ_1, δ_2, and δ_3 are positive integers greater than zero.

Thus, the abbreviated system of inequalities will take the form:

$$\begin{cases} 5w_{12} + 5\delta_1 + 5\delta_2 + 5\delta_3 \ge Q, \\ 10w_{12} + 10\delta_1 + 10\delta_2 + 4\delta_3 \ge Q, \\ 15w_{12} + 15\delta_1 + 9\delta_2 + 4\delta_3 \ge Q, \\ 17w_{12} + 14\delta_1 + 9\delta_2 + 4\delta_3 \ge Q, \\ 16w_{12} + 14\delta_1 + 9\delta_2 + 4\delta_3 < Q. \end{cases} \quad (14)$$

In the sixth stage, we determine the values of the threshold Q and w_i for the system of inequalities, which was obtained in the previous stage and gives a minimum of linear form.

After solving the system of inequalities (14), we obtain that it is compatible at the following minimum values: $w_{12} = 1$; $\delta_1 = 2$; $\delta_2 = 15$; $\delta_3 = 90$. Substituting these values, this system of inequalities takes the form:

$$\begin{cases} 540 \ge Q, \\ 540 \ge Q, \\ 540 \ge Q, \\ 539 < Q. \end{cases} \quad (15)$$

Thus, the minimum threshold is $Q_{min} = 540$.

Determine the weight of 20 parameters for estimating the FP, taking into account expression (13).

Table 3

Values of weights of financial parameters and minimum threshold

w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	
108	108	108	18	108	3	8	108	3	18	Q_{min}
w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}	w_{17}	w_{18}	w_{19}	w_{20}	
3	1	3	1	1	3	3	18	18	18	540

Thus, the minimum threshold element for assessing the level of FR is: [108,108,108,18,108,3,8,108,3,18,3,1,3,1,1,3,3,18,18,18; 540].

Decision-making in the built system will be carried out according to the following algorithm.

Algorithm for estimating the financial condition of the enterprise, built using threshold elements.

Step 1. Form logical vectors $G_j = [g_1,...,g_n]$, ($j = \overline{1,3}$), representing the values of the estimation parameters in the Boolean form (1 or 0). The logical variable g_i describes the value of the parameter x_i in the range $[x_{i\min}; x_{i\max}]_j$. To estimate it, we use rule (6).

Step 2. Calculate the value of the threshold function H_j by the formula:

$$H_j(g_1,...,g_n) = \sum_{i=1}^{n} w_i g_i, \quad (16)$$

If $H_j(g_1,...,g_n) \geq Q$, where Q is the threshold, then the financial condition of the enterprise, described by the vector G_j, is characterized by the Z_j-th level of FR. Otherwise, it belongs to another level of FR.

Step 3. The risk to the financial condition of the enterprise Z_j is determined using the following expression:

$$b_j = \frac{H_j}{Q}, \ j = \overline{1,J} \ (J=3). \quad (17)$$

The value of FR is Z_j, which is described by the largest of the values of b_j.

$$b_j = max\{b_1, b_2, b_3, b_4, b_5\}. \quad (18)$$

When checking the validity of the expression $H_j \geq Q_{min}$ for all levels of FR, sometimes there may be a situation where it is not possible to unambiguously identify the risk of financial condition. To eliminate this shortcoming, it is proposed to introduce a level of accuracy of decision-making, which for most economic problems is q = 0,4. Further increase in accuracy is unjustified for the studied class of problems.

6. Implementation of the model

It illustrates the application of the developed models and methods of financial risk assessment on the example of entities in various industries, namely construction, industry, agriculture, trade, transport and communications, forestry, procurement, education.

Consider as an example the indicators of Enterprise 1. For this enterprise, the input data are as follows: k_1 = 63,8; k_2 = 328,3; k_3 = 264,5; k_4 = 7,7; k_5 = 264,5; k_6 = 92,15; k_7 = 0; k_8 = 24,65; k_9 = 280,2; k_{10} = 23,65; k_{11} = 199,9; k_{12} = 204,7; k_{13} = 67,5; k_{14} = 234,55; k_{15} = 75,5; k_{16} = 0.

Using table 1, we determine the values of the evaluation parameters on the basis of these input data. The results are entered in table 4.

Table 4

Estimated values of evaluation parameters

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0,19	0,24	0,19	-2,68	-0,65	0,03	0,35	0,09	0,09	0,11
x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
0,85	11,85	1,4	3,03	1,19	4,39	0,37	0	0	0

We apply the method of risk assessment of the financial condition of the enterprise using the mathematical apparatus of threshold elements for entities of different industries. Based on the previously calculated values of the evaluation parameters (tab. 4) we will make certain vectors G_j for 3 levels of financial risk, which are given in table 5.

Table 5

Vectors G_j for low, average and high levels of FR

Financial risk	Indicator									
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
low	0	0	0	0	0	0	0	0	0	0
average	0	0	0	0	0	0	0	0	0	0
high	1	1	1	1	1	1	1	1	1	1
Financial risk	Indicator									
	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}	g_{17}	g_{18}	g_{19}	g_{20}
low	0	1	0	0	0	1	1	0	0	0
average	0	0	0	0	1	0	0	0	0	0
high	0	0	0	0	0	0	0	1	1	1

Find the values of the threshold function H_j for each level of FR using formula (11) and reduce them to table. 6.

Table 6

The value of H_j, $j = \overline{1,J}$ $(J = 3)$ for the Enterprise 1

Financial risk	Low	Average	High
Value	7	1	644

In order to determine the affiliation of the enterprise to a certain financial condition, we calculate b_j by formula (12):

$$b_1 = \frac{7}{540} = 0{,}01, \; b_3 = \frac{1}{540} = 0{,}002, \; b_3 = \frac{644}{540} = 1{,}19.$$

Based on the required accuracy of decision-making, we choose the approximation factor q = 0,4.

Thus, since $b_3 = \max\{b_j\}$ and $b_3 > q$, where $q = 0.4$, the Enterprise 1 is characterized by a high level of financial risk R_3.

To verify the adequacy of the proposed methods of assessing FR, we compare the results of existing regulatory approaches - the model of multivariate regression analysis, integrated indicator and bankruptcy forecasting model and the proposed model.

Seven enterprises from different fields of activity (industry, transport, communications, etc.) were considered. The results are summarized in table 7.

Table 7
Comparative characteristics of normative and proposed approaches to the assessment of FR

N for Enterp-rise	Normative methods			The proposed method	The real level FR
	Multifactorial regression analysis	Integral indicator	Bankruptcy forecasting	Threshold elements	
1	average levels of FR	low levels of FR	average levels of FR	low levels of FR	low levels of FR
2	average levels of FR	average levels of FR	average levels of FR	average levels of FR	average levels of FR
3	low levels of FR	average levels of FR	high levels of FR	high levels of FR	high levels of FR
4	average levels of FR	low levels of FR	average levels of FR	low levels of FR	low levels of FR
5	high levels of FR	high levels of FR	high levels of FR	high levels of FR	high levels of FR
6	average levels of FR	average levels of FR	high levels of FR	average levels of FR	average levels of FR
7	high levels of FR	high levels of FR	high levels of FR	high levels of FR	high levels of FR
Number of erroneous estimates	3	1	3	0	-

Table 7 shows the adequacy of the proposed models. At the same time, in contrast to the normative ones, the developed approaches allow for a deeper analysis, accelerate the decision-making process, reduce its risk and increase the efficiency of assessment for such poorly structured decision-making tasks. This allows you to automatically display the set of input evaluation parameters to the set of output results of the assessment of financial risk of the enterprise by decomposing and formalizing the decision-making process based on the appropriate mathematical apparatus.

7. Conclusion

The financial risks of the enterprise are caused by both exogenous and endogenous factors. Exogenous factors are usually well controlled by modern information technology. On the contrary, endogenous factors are poorly taken into account in modern information systems to support decision-making. That is why the development of computer models to identify the financial risk of existing enterprises is an important direction in the development of modern information technology. An important factor in this is that computer models need to be general enough to cover a wide variety of business areas.

A mathematical model for assessing the risk of the financial condition of the enterprise within the principle of decomposition division of a complex function into a sequence of simpler ones is constructed. The feature of this model is that it takes into account the set K of the initial input parameters *kc* ($c = \overline{1,C}$), which is determined by the relevant reporting of the enterprise and expert information; the set *X* of the evaluation parameters *xi* ($i = \overline{1,n}$) of the financial condition; function of conversion of initial parameters into estimating $F1$: $K \rightarrow X$; the set of decomposition functions $D = (Y, \ldots S, P)$ of the collapse of the parameters by which the identification of the financial condition of the enterprise; the function of determining the level of risk of a potential investor F_2: $Zj \rightarrow Rj$, which corresponds to *Zj* the level of the financial condition of the enterprise; the set of initial parameters of financial risk *Rj*. This simplifies the process of assessing the financial risk of the company, which will face a potential investor in a fast-paced external and internal environment.

The sets of estimating input and output parameters for the definition of risk of a financial condition of the enterprise are made and substantiated. They take into account a wide range of external and internal influencing factors. The choice of the set of evaluation parameters is justified by the criteria of completeness, effectiveness, and minimality. The set of initial parameters allows to identify the level of risk of the business subject and is determined by the criteria of completeness and effectiveness, which allows to fully satisfy the consumers of the system.

A method of assessing the financial risk of the enterprise on the basis of the mathematical apparatus of threshold elements, the use of which greatly simplifies and increases the accuracy of assessing the financial condition of economic entities in various industries by taking into account the importance of valuation parameters. The practical implementation of the proposed mathematical models and methods of financial risk assessment on the example of real enterprises is considered.

The advantages of these models and methods are that, unlike normative valuation methods, they allow to take into account a wide range of valuation parameters, industry, and its specifics and make decisions without considering all combinations of parameters, which simplifies the process of assessing financial risk *Rj* and computer simulation. Also, the developed computer model can be used both offline and when using a cloud environment.

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