Representational and Embodied Accounts of Abstract Concepts: A Framework for Prelinguistic Acquisition of Numerical Cognition

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Abstract. Abstract knowledge is regarded as a hallmark of human reasoning. It is considered to be an essential attribute of our cognitive capabilities that allows for further learning processes to take place. Yet, the origins of early acquisition of abstract reasoning remain unclear. Representational and embodied accounts have tried to develop frameworks for understanding the emergence of abstract reasoning. This paper investigates the assumptions of each framework and the conclusions drawn from empirical evidence by focusing on the concept of number. This supposedly abstract capability of organising quantities into embodied or mental representations is acquired progressively over multiple developmental stages. The evidence of each framework for describing abstract numerical concept acquisition is examined, especially at the early and prelinguistic stages of development, and a distinction is made between numerical cognition and the concept of number. Prelinguistic knowledge is discussed by asking whether infants are endowed with core numerical knowledge, and the extent to which prelinguistic numerical knowledge contributes to further cognitive developments is also analysed. This paper proposes that a renewed theoretical framework that extends beyond the representational-embodied dichotomy is needed, if a detailed understanding of numerical cognition is desired.

Keywords: Abstract Reasoning, Numerical Cognition, Cognitive Representation, Embodied Cognition, Core Knowledge, Categories

1 Introduction

Abstract reasoning is regarded as a fundamental aspect of human cognition. Abstract cognition has been defined as the ability to reason with concepts, ideas or entities that hold no direct relationship with sensory accessible information [3, 65]. Abstract concepts are therefore regarded as purely mental creations [3]. They refer to items that are not entirely limited by physical or spatial environments [65]. Examples of abstract concepts can imply notions of “truth” or “justice”. Here, we intend to investigate a different set of abstract concepts, namely core or common core knowledge concepts such as intuitive physics or intuitive psychology [61]. Common core knowledge [25, 36, 58] considers human cognition as organised around an elementary understanding of agents (intuitive psychology) and objects (intuitive physics) and their interaction in the world [61]. Core knowledge can imply intuitive but abstract notions of beliefs, desires, space, numbers, masses or forces [12, 16, 31, 33, 58, 61]. Spelke and Kinzler [58] further argue that core cognition is partly based on four to five representational systems used to characterise and reason with namely, objects, actions, numbers, space and social partners. Identified as core components of cognition [25, 58], numerical cognition and the development of the concept of number are therefore of interest when studying what aspects of cognition might be present at a prelinguistic level, and how these set a basis for reasoning that will subsequently shape the understanding and development of further learning and cognitive processes. Indeed, the ability to form conceptual representations is believed to provide the basis for a great amount of rich and complex cognitive tasks and learning abilities such as categorisation, generalisation, comparison, compositionality and prediction [42].

Yet, despite the extensive work on the subject, the reasons for the emergence of abstract reasoning and formation of abstract concepts at the very early stages of human development remain unclear. Theories of core cognition have therefore been put forward as a way to explain this early ability in infants to reason with physical and psychological components of their environment such as concepts of number, mass or agency [42, 58, 61]. Cognition, it appears, does not need ordinary language to perform early intuitive tasks. Proponents of the core cognition therefore argue for the existence of prelinguistic representations, as infants are not yet endowed with a full working language model but appear to be able to perform numerical tasks [25], reason about space [58], and attribute mental states to others [42].

Here we investigate the cognitive processes that underlie abstract reasoning, with a particular focus on numerical cognition, both from a representational and embodied perspective. Both frameworks develop theoretical assumptions in trying to account for the emergence of abstract reasoning. Each framework however seems to come with its limitations. On one hand, representational accounts face difficulties in explaining what exactly is represented when abstract reasoning is involved, and how that representation or internal model is taking place. How are numerical values mentally represented and what exactly do they represent? Is it by means of an internal symbolic system or a modal relationship with what is to be represented? This modality-specific framework leads to embodied accounts that encounter issues in relating abstract reasoning to modality-specific relationships. Abstract reasoning seems to bare no relationship with the environment or the body as such concepts appear to be purely mental and amodal constructions. Moreover, both frameworks are faced with challenging empirical results and limitations in understanding how such early and prelinguistic reasoning emerges. We will focus on the concept of number, and propose a framework based around understanding the implication and challenges faced by both theoretical approaches of human cognition. By discussing the representational and embodied accounts of numerical cognition and by distinguishing numerical cognition from the concept of number, we hope to clarify the claims of each approach. This will allow us to outline indications for further theoretical developments.

1.1 The Epistemological Problem of the Concept of Number

Concepts and numbers are related in that both seem to be considered as representational systems used by our cognitive apparatus in order to categorise patterns encountered in the environment [5, 14, 22]. Defining numbers with reference to categories therefore seems appropriate under a cognitive framework as categories and concepts are considered essential to reasoning processes. Numbers would thus constitute a tool used to reduce uncertainty by organising recuring patterns under specific entities [14, 22]. In line with the common core framework, numbers would be defined as tools that are part of the common core knowledge infants possess at the very early and prelinguistic stages of development, that allows them to make predictions and generalisations. For instance, five items encountered repeatedly in the environment might be grouped under the concept “five”, which can be generalised to all new sets consisting of five items. Numbers can thus be defined as categories used to determine the size or quantity of any given or observable set. Under a cognitive perspective, numbers therefore represent the ability to perform continuous discriminations of sets of items [10].

Other approaches relating to knowledge acquisition have been proposed to define concepts, yet they all seem to fall short in trying to describe numbers. For instance, under a rule-based approach, concepts are defined as properties about particular categories [49]. Concepts respond to formal or informal definitions that are stored as representations. When a new item or pattern is identified, it will be compared to the definition or properties that have been previously stored for that particular category. However, applying this model to the concept of number is problematic: What are the properties of numbers? Numbers do not seem to have physical properties. Numbers are tasteless, odourless, colourless and shapeless. Arabic numbers, admittedly, have a particular shape but the shape is one that takes place under a representational symbolic language used to denote numbers, i.e. numerals. Numbers do not have shapes in the environment as tables or apples do. They are therefore not directly accessible by the senses. Thus, a definition based on the properties of concepts is limited when reasoning about numerical concepts.

The exemplar model, as proposed by Nosofsky [51] as well as Medin and Schaffer [48] under the *Context Theory of Classification,* is another model that tries to explain people's categorisation processes. According to this model, examples of a particular category that is intended to be represented are stored in memory. Decisions are then made upon the relative similarity to the stored examples, which are then used for the categorisation of items. The exemplar model therefore pictures categorisation as a process of recognising salient patterns. Stored examples should be recognised in the environment, as examples of the prototype example that has been previously stored. The issue with this model regarding the concept of number is that infants do not encounter occurrences of numbers as they would encounter occurrences of tables for instance. They can surely perceive four apples, four trees or four chairs, but the number four in itself is arguably not perceivable, nor is it directly accessible in the infant's environment – as would concrete concepts such as trees or chairs upon which, after a few presentations of chairs or trees, the child is able to generalise the concept of "tree" or "chair" [42]. This process of generalisation over examples is one that appears to be less obvious when studying conceptual numerical acquisition. As a result, the exemplar model cannot be regarded as adequately suited for this description. Moreover, if the exemplar model accounts for a distinction made on similarity, how is the number one similar to the number two, other than it being a number? This issue brings back the initial problem, which asks what is a number? What is the prototype, the most typical example or representation of number? It appears as there is none, and that the only similarity-like inference infants are able to make, is to understand that numbers follow themselves in sequential order and are grouped under ordinality [11]. More than being able to distinguish for example 16 from 8, infants appear to be able to generalise that 16 is greater than 8 [11].

We are therefore faced with two epistemological concerns when trying to understand the concept of number. Numbers cannot be defined with reference to properties, nor can they be defined by means of comparison. They may be acquired via environmental perceptions (e.g. perceiving four trees) but they are devoid of any sensory modality in themselves. Moreover, there seems to be no particular perception of numbers as they appear to be stored and processed across sensory modalities [12]. As a natural conclusion from these issues in trying to define the acquisition of numerical knowledge, numbers have been defined as abstract entities that can only be conceived as purely mental representations. As a result, acquisition of the concept of number will also take place using mental representations[1]. Proponents of the embodied cognition approach have tried to argue otherwise. They propose to define concepts either as notions that have been abstracted away from experience [3] or as generalisations from bodily and environmental experiences that hold direct relationships with action, as well as emotional and perceptual processes [21]. As has been argued previously, numbers appear to be devoid of any perceptual or bodily modality, therefore it seems as these definitions, too, cannot account for the emergence of numerical concepts in humans.

[1] Although there exists other philosophical positions on the ontological existence of numbers and their relation to epistemology (see Horsten [38]), the arguments discussed in this paper are shared by most researchers in the field of cognitive science.

In the following section, we will review the empirical evidence for both representational and embodied approaches on numerical cognition and the concept of number. An important clarification that should be made beforehand is that numerical cognition must be distinguished from the concept of number. Indeed, perceiving numerosities does not imply that infants have a notion of the concept of number, even if they do appear to have a notion of numerosities, i.e. of bigger or smaller stacks of items. Similarly, perceiving numerical changes in the environment does not imply that infants do have numerical representations of the concept of number. Therefore, non-symbolic items such as numerosities and magnitudes must be distinguished from the symbolic representation of the concept of number [32]. The ability of subitizing, we suggest, consistent with Gallistel and Gelman [32], would then be the matching of the symbolic abstract representation to numerosities or orders of magnitudes.

2 Representational and Embodied Approaches: An Empirical Review

2.1 The Language of Abstract Concepts: Prelinguistic Thoughts and Common Core Knowledge

Traditional representational approaches on cognition consider the brain as being the locus of cognition which is described as an information processing system. This allows individuals to form representations of the environment as a way of building internal models of the world, a process that is believed to be essential for understanding the complexity of the environment. One way of representing knowledge is by developing concepts. Concepts are used by various cognitive functions [14]. They are essential to agents and appear to be the building blocks of various cognitive operations. This is consistent with Barsalou et al. [5], stating that concept acquisition and categorisation are regarded as fundamental units of knowledge organisation, with direct influence on cognitive processes. Thus, the organisation of knowledge constitutes a core component of human cognition, that allows for further cognitive reasoning and learning. Concepts, it appears, are to be regarded as the fundamental building blocks of cognition, upon which further knowledge will be developed. For instance, concepts allow us to make categorisations, which in turn help in reducing the information being processed as generalisations can be drawn from similar experiences that will be categorised under similar concepts [14]. Moreover, inferences are drawn from representations that are used as predictions of future events from past knowledge. Individuals are therefore able to generalise and group sets of items under the same category, and infer that all new items can be grouped under a similar abstract entity [14]. In the example of core numerical cognition, this is illustrated by the ability to group five chairs, five fingers and five dots under the same representation of the abstract concept of number five, even if the word “five” or the actual understanding of the concept of “five” is not yet mastered at the early stages of development.

Basic number concepts have been shown to be acquired and understood before language acquisition [27, 32, 39]. Language is useful for further and more complex mathematical reasoning and learning, however, it appears that initial thoughts and predictions regarding numbers and numerosities are made without the use of any language model and independently of any particular enculturation [27]. Thought processes are, at least initially, processed by a language of their own, independent of ordinary language. Evidence for this claim outlines the existence of a number sense used in fundamental and prelinguistic reasoning [27, 32, 39]. Infants therefore demonstrate a great amount of prelinguistic cognitive abilities that have been grouped under the notion of core reasoning [41, 58, 61]. These allow individuals to reason, learn, and understand the world, by building representational models of the environment without any proper language model. Numbers, according to this framework, are not accessible via the body nor can they be shaped by it. They appear to be purely abstract entities people use to reason about, model, generalise and simplify their environment, as seems to be the purpose of concepts outlined previously. This is done by the capacity humans have in building internal models of the world, as a way of reducing the unknown by organising the indeterminate nature of patterns encountered in their environment [22]. Considering this early cognitive mapping of the brain, models have tried to represent the organisation of what appears to be a cross-connected system of various representations of the environment that are responsible for the ability to perform complex reasoning and actions as well as generalisation to new information [20, 41]. This ability to generalise across modalities has been investigated with the concept of number, and have led Lake et al. [41] to believe that learning processes are better thought of as internal model buildings rather than pattern-recognition processes.

Infant numerical performance was shown to be costless when tested across auditory and visual modalities [39]. Izard et al. [39] were able to group a fixed number of phonemes from an auditory sample with the identical amount of visual items. For instance, when presented with a sequence of four phonemes (“ba-ba-ba-ba”), infants stared longer at four triangles, when they were presented with two patterns of either four or twelve triangles [39]. Barth et al. [8] have also demonstrated that infants could distinguish with accuracy and without any differences in performance between dots presented as auditory stimuli and dots presented as visual stimuli. Their study also demonstrated a cross-modal addition and comparison ability, where the result of a visual addition of dots was effectively compared with an auditory stimuli of the numerical result of the subsequent addition. Infants were also able to use auditory and visual stimuli in conjunction to do sums of auditory and visual patterns. This is consistent with Feigenson [26] showing that infants were able to distinguish numbers across modalities with no difference in performance. Moreover, the intuitive component of the performance was further demonstrated with adults that could instinctively compare approximate numerosities independent of the task being across or within format or modality, and so, without resorting to counting or using learned symbolic representations [8]. This constitutes further evidence that, even when a language model has been acquired and enculturation has eventually shaped adult cognition in ways that might differ from early developmental stages, core numerical abilities are language and culture independent, and are performed at an abstract and representational level. These representations then set the core structure for further addition and comparison tasks, as well as further and more complex numerical reasoning [8]. These results therefore strongly suggest a cross-modal and non-symbolic abstract representation of numerical concepts in the early prelinguistic stages of development that is still effective in human adults in which numerical stimuli are perceived on a modality-independent and format-independent basis. Consistent with Barth et al. [8], Ferrigno et al. [27] also suggested that the emergence of numerical perception is spontaneous, independent of any particular cultural predispositions, and independent of alternatives for judging quantities. These studies further suggest that even when language is acquired, there might exist a *number sense* that is independent of language. Therefore, it has been argued that humans are endowed with two systems responsible for this numerical cognition [25], the Approximate Number System (ANS), and a number system for exact representations. The ANS is a representational system that allows individuals to perform numerical tasks such as distinguishing orders of magnitude and numerosities. Sets of dots are presented to subjects and it appears as they are able to discriminate between smaller and larger sets without actually counting the dots [25]. On the other hand the precise number system is responsible for subitizing, where individuals are able, until a certain threshold set around four, to determine the exact quantity of dots presented to them without actually counting the dots. The main characteristics of the core numerical system are therefore that (1) the accuracy of the task is determined by the ratio, with a higher ratio correlating with better performances and (2) as numbers presented increase, predicting the exact number of dots decreases [13], which outlines a linear relationship between the amount of dots presented and the subitizing capacity.

There is therefore an extensive body of evidence for the mental representation of numerical concepts in humans. The overall consensus of this framework is to regard the concept of number as a purely mental and abstract representation that humans use in order to make sense of the world they encounter. However, as has been noted previously, the ability to perform operations on numerical representations does not necessarily suggest the existence of a number concept [33]. Moreover, the patterns presented to the subjects are very limited in comparison to the rich diversity that can be encountered and experienced in the world. This has led recent investigations to consider numerical cognition under a different theoretical perspective.

2.2 The Interaction of Organisms with the World: The Embodied Perspective

Embodied cognitive science is a framework that proposes a different account of human cognition. Proponents of the embodiment framework reject the representational and information processing framework, by arguing that the brain is not the centre of cognition –or at least not the only one. This has been introduced by Gibson [35], arguing that our direct access to the world is mediated through our senses. Cognition is not a matter of information being distorted in sensory interpretation from the environment that must be processed and reconstructed by means of representations, but rather constitutes a directly accessible bodily and environmentally mediated process. Embodied accounts of cognition emphasise the organism's relationship with both its body and the world it inhabits, with the assumption that bodily and environmental states actively shape brain states [1, 23]. The claim is that it appears more important to deal with the problem of what the head is inside of, than what is inside of the head [47]. A more radical assumption pictures cognition as enactive [63]. Internal mental representations are replaced by direct phenomenal access to the world by means of bodily perceptions, motor acts and environmental sensations, that are all pictured as cognitive tools to perform tasks, learn and understand the world [64]. This body-environment relationship actively participates in shaping cognitive ability and replaces or adds to the brain as the controller of cognition. Action, therefore, is the primary component of cognition. Reasoning and learning are directed to and mediated by action, where the world is perceived with regard to what can be achieved through bodily and environmental modalities.

However, abstract concepts seem to challenge this framework. As have been demonstrated in the previous sections, abstract concepts do not seem to bare any relationship with bodily or environmental states. Specifically, the concept of number appears to be a purely mental representation. Lakoff and Johnson [44] have challenged this assumption by arguing that abstract states of the brain find their basis in bodily states, and that both states are interrelated and interdependent. Abstract numerical knowledge is therefore situated in concrete concepts that are accessed and acquired via sensory and physically mediated states [34, 40, 44]. Abstract concepts are grounded in, accessed via and mediated by concrete bodily and environmental enactments. Mathematical reasoning is to be understood as emerging from individual human experiences [45]. However, while Lakoff and Nunez [45] account might provide insights into how further complex mathematical concepts can be acquired, it cannot describe the acquisition of prelinguistic mathematical knowledge which is assumed to be mentally conceptualised rather than originating from physical grounds. How these prelinguistic factors might play a role in the acquisition of numerical knowledge and the concept of number is something that is not regarded in Lakoff and Nunez's work [52, 53]. Barsalou [4, 6] has also argued for the interdependence of abstract concepts on bodily states. According to Barsalou [6], concepts are given by perceptual symbols. Numerical concepts would therefore be grounded in bodily and environmental perceptions. Barsalou [4, 6] and Barsalou and Wiemer-Hastings [7] proposed an approach based on the analysis of abstract concepts in terms of the relationship of the concept to its content. Contrary to abstract representations that are amodal non-symbolic systems of numerical cognition in that the relationship to their contents is arbitrary, Barsalou proposes a modality-specific description of numerical abilities. Conceptual representations, whether they are abstract or not, are considered as embodied in sensory and motor activities. They are further processed at the sensorimotor level instead of being processed in an abstract and amodal system of concept representation that involves no direct relationships to modality-specific states. Contrary to what has been shown previously, where empirical evidence demonstrated a modality-independent relationship to numerical cognition, proponents of the embodiment framework suggest that concepts are dependent of a variety of modalities including perceptual, emotional or motor states.

Empirical evidence for sensorimotor processing can be found in what has been theorised as the *spatial-numerical association of response codes* effect (SNARC). A body of evidence that has been well replicated [66] has suggested that sensorimotor habits influence numerical cognition. For instance, Dehaene et al. [17] have argued that there might exist a “mental number line” that is responsible for parity and number magnitudes representations. The outcome is still a mentally processed representational account of magnitudes, but one that finds deep influences into bodily and sensorimotor states. According to Dehaene et al. [17], smaller numbers are processed quicker with the left side of the body, when larger numbers are processed quicker with the right side. The numbers in themselves do not seem to affect the SNARC representation but rather, it seems as the magnitude of presented numbers relative to other magnitudes is what matters. For instance, in Dehaene et al. [17] study, the numbers 4 and 5 were processed with right-hand movements when the list was made of numbers ranging from 0 to 5, but the same numbers were processed with left-hand movements when the list was made of numbers ranging from 4 to 9. Moreover, this spatial representation of numerical concepts seems to be language and culture dependent, where it has been shown that in left-to-right writing systems, the effect is represented from left to right, whereas in right-to-left writing systems, the effect is represented from right to left [17, 56]. However, Rugani and de Hevia [54] have shown that the mental number line was present in pre-verbal infants, therefore suggesting that language is not a necessary component of fundamental understanding of numerical magnitudes. The origins of the prelinguistic “mental number line” are yet to be found. It seems as the sensorimotor influence on the SNARC effect is less prominent in prelinguistic subjects. Upon familiarisation and after testing, the subjects did not associate body states with mental representations in determining the spatial representation of numerical magnitudes, but rather directly represented this “mental number line” by building consistent expectations between changes in magnitude across number, time and space [54].

According to another embodied account, numerical cognition is not strictly attached to abstract processing but is best defined as occupying a space between abstract and modal reasoning, with no clear cut distinction between the two. With reference to the “mental number line”, where right and left-hand body movements affect numerical reasoning, Fisher and Shaki [29] propose that numbers are embodied concepts. They also expose a body of work [19, 28, 30] that has demonstrated evidence for the extensive use of fingers as markers for associating various perceptual and bodily mediated inferences with numerical understanding. One of the reported studies [19] outlines that fingers appear useful for representation, processing and communication of numerical knowledge. Di Luca and Pesenti [19] show that children across cultures use fingers as a way of representing numerical tasks in their visual and spatial environment. Fuson [30] has further reported that preschooler’s use of fingers for mathematical tasks remains one of the main strategies observed. Finger use therefore greatly influences the acquisition of numerical cognition and the concept of number, as even when symbolic representations are acquired, adults still tend to use finger representations [19]. However, prelinguistic use of fingers has never been reported in infants trying to make sense of numerical patterns presented to them. In the experiments reviewed in section 2.1, infants seemed to only refer to mental representations when trying to access numerical knowledge. It would therefore seem as finger counting and representation, as much as language, are powerful tools that enhance and help in shaping further and more complex mathematical reasoning, as well as help in acquiring the concept of number that, it has been argued, is not necessarily present even if evidence for numerical cognition is present [32].

Abstract numerical concepts therefore still pose a problem for embodied frameworks in that they appear to be devoid of a strong physically accessible reference, particularly in the early prelinguistic stages of development. Numbers cannot be reached or touched, and are as a result only accessed via thinking. It is unclear to which degree information from motor systems are involved in the acquisition of the meaning and understanding of numbers. Motor activity might only be involved in a limited degree and in an *a posteriori* relationship, where numerical cognition has, to some extent, already been acquired. It would appear as motor systems therefore influence numerical cognition as language does. Both provide tools for further learning mechanisms to take place, while the core components have already been learned.

Embodied cognition also suggests that over time, sensitivity to numerosities is described as predominantly mediated by visuospatial cues, where the constant interaction with these cues is essential to the development of more accurate representations [67]. The study of change over time in cognitive systems has been accounted for by applying the methods of dynamical systems to the study of various cognitive abilities such as language [15], perception [59] or action and motor activity [55, 59]. The dynamical organisation of numerical cognition can therefore be framed as a non-linear and ever evolving system [9]. The body, the environment and the nervous system are seen as one unique dynamical system and not as three distinctive systems however interdependent they may be [9]. Under this characterisation, numerical cognition is therefore described as an interaction between states that change over time. Complexity in numerical reasoning acquisition is framed as an emergent property that finds its roots in perpetual variations over time with outputs of the past connections feeding the following ones [67]. In distinguishing orders of magnitude and numerosities, Hope et al. [37] have shown that quantities are compared on the basis of changes in environmental stimulus. The perceived changes are accounted for by what is described as Weber's law, a hypothesis that applied to numerical cognition, allows to explain the variability of numerical perception with regard to changes in the environment by means of two distinct features [37]. While the *distance effect* explains why error increases in distinguishing magnitudes as the distance between these magnitudes shrinks, the *size effect* describes the increase of accuracy as the difference between magnitudes within a set increases. These are properties that have been identified in section 2.1 as core characteristics of the ANS. According to Zorzi and Testolin [67], it is believed that they are inherently dynamical properties as sensory experience constantly improves the accuracy of the ANS. Another account of variability over time has been put forward by Laski and Siegler [46]. Upon analysing the causal connections in magnitude discriminations and categorisations, they found that representations of magnitudes could benefit from children's subjectivity in categorising numbers. A causal relationship was identified between children's development of accurate linear representations of numbers, an ability that is believed to have an influence on further mathematical performance. The extraction of regularities across patterns is grouped under higher-order processing mediated by semantic knowledge acquisition [60]. Regularities are generalised to novel items, such that progressive alignments occur where small rates are associated by means of analogy to higher rates [57, 62]. Linear representations are therefore improved as children are exposed to variabilities in orders of magnitude, and learn to generalise over these previously acquired linearity scales [57, 62].

3 Beyond the Embodied-Representational Dichotomy

We must recognise that when it comes to numerical cognition, embodied and representational accounts are not as separated as they might appear. Proponents of representational accounts have also demonstrated evidence for bodily and culturally influenced behaviours in distinguishing magnitudes [17, 56]. However, when it comes to determining the prelinguistic numerical abilities found in infants, it seems as a purely embodied approach is insufficient. The purely mental and abstract number sense might be further developed into a clear, accessible and symbolically-mediated number concept by means of finger representations [2, 24], but the initial prelinguistic numerical reasonings appear to be abstract and disembodied. Even if embodied actions might reinforce numerical cognition under diverse conditions, the early stages of numerical cognition seem to remain entirely abstract and amodal.

Perhaps the debate between representational and embodied scholars is bound by a vocabulary misunderstanding. There may be no such thing as representations, but the theoretical assumption of the cognitivist framework is also one that regards cognition as an information processing system. This claim is rejected by most embodied frameworks, who also seem to argue against the brain being the locus of cognition [50]. However, it does not seem as proponents of the representational theory would deny the fact that humans are embedded in an interconnected feedback loop between the environment, the limbs and the brain, as has been demonstrated with the SNARC effect. Bodily states can influence and affect representational patterns. Bodily preferences and influences on number reasoning can modify information processing states such that individuals with different right-left body biases will result in different reasoning strategies for dealing with numbers.

However, what representational and embodied accounts seem to have in common is that, contrary to most mathematicians who would hold a platonic view of mathematics, where numbers exist on their own and are to be found somewhere in the world, they hold that mathematics is a product of human cognitive abilities and are therefore to be regarded, studied and understood under those conditions. What distinguishes both fields is how to study the cognitive apparatus and what frameworks we can develop to explain the emergence of numerical cognition. Proponents of the embodied approach stand against the representational and computer metaphor by rejecting information processing. They argue that the brain cannot be compared to a computer, even on a metaphorical standpoint, and that the brain is not the locus of cognition or at least not the only one. Yet, they do not seem to offer a genuine replacement to computational cognitive sciences [50]. The debate between both frameworks is however fruitful in that it allows for the clarification of what is meant by representations and concepts, and more fundamentally, of what cognitive systems are at the higher-level.

Theoretical frameworks are to some extent similar to models in that they try to account for the totality of human behaviours but in the process may only account for a portion of reality. Frameworks must be combined and overcome to be able to account for the diversity of cognitive systems and subsequent functions. For instance, the “triple-code theory” proposed by Dehaene [18] assumes that some numerical representations are driven by symbolic contextual information, such as visual and auditory stimuli triggered by verbal counting or Arabic numerals mapping. At the same time, the model presupposes the existence of a modality-independent and thus abstract “number sense” used to map semantic numerical knowledge such as orders of magnitudes and numerosities. Some features of numerical cognition are still in discussion, and the relationship between sensorimotor activity and abstract processing is far more complex than can be accounted for by each individual finding. As much as prelinguistic concepts might be present at the very early stages of development, it is the further grounding by a great amount of learning mechanisms that reinforces these concepts and provide foundations for further cognitive development. Embodied and representational accounts therefore appear to be inseparable, and both seem valuable albeit at different stages of investigation and under varied empirical assumptions.

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