Software Implementation of One VLSI Placement Optimization Problem

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Abstract

The statement of one problem of optimum accommodation of modules on a chip as a linear Euclidean combinatorial problem on poly- partial permutations and its special classes are presented. The software for the problem based on investigated polyhedral and combinatorial properties is considered.

Keywords

VLSI, module, chip, placement, linear combinatorial optimization, partially Boolean optimization, Euclidean combinatorial set, point configuration, combinatorial polytope, cutting-plane method

1. Introduction

Let us consider the issue of optimizing microcircuits, which are the main components of modern technology, which is relevant to the rapid growth of the capabilities of technical devices and the requirements for them. In particular, consider one problem of placing rectangular modules on a microcircuit. Such a problem in various settings and depending on the optimization criterion appeared in many sources [1], [2], [3]. Traditionally it is considered very difficult [1], [4], [5], [6], because it can always be considered as a two-dimensional packing problem, which is NP-hard [1], [2], [6].

2. Literature survey

In recent years, a number of methods have been proposed to solve this problem: based on search adaptation [7], [8], linear and quadratic programming [4], [9], [10], simulated annealing [11], based on evolutionary modeling methods [12], polyhedral approaches [5], [13]-[16], graph-theoretical techniques [17].

3. Issues

Since theses problems mostly NP-hard optimization problems [4], [9], [10], the development of algorithms are highly relevant, especially those that use specifics of all components of the models - targets function, search domain, feasible domain, properties of the corresponding combinatorial polytopes [5], [6], [18]-[19]. We propose a Euclidean Combinatorial Optimization [15], [20]. the approach that explores and utilizes algebraic-topological and geometric properties of the combinatorial set, on which optimization is carried out and subproblems are singled out that can be solved more efficiently due to the specifics.

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4. Problem statement

It is necessary to build mathematical models of one problem of the optimal arrangement of modules on a chip in the form of an optimization program over a finite point configuration belonging to a set of Boolean vectors (the Boolean set) and develop a polyhedral approach to solve this problem essentially used explored specifics of the problem and its special cases. Then we aim to build a software package for solving problems of this class implementing our specific approach, from a field of cutting plane methods [13], [21]-[25], to its solution based on the investigated properties of the corresponding combinatorial set embedded in Euclidean space and the corresponding polytope [14].

5. Main part

5.1. Problem 1

On a rectangular microcircuit with length L and height H, it is necessary to arrange n modules M_i with length $l_i > 0$ and height $h_i > 0$, respectively, in order to minimize the degree of chaos placements of modules, if it is possible to return modules by 90^0 ($i \in J_n, J_n = \{1,2,...,n\}$).

Remark 1 By the measure of the randomness of the placement: a) of a module, we will mean the number of other modules that are located diagonally from it; b) of all modules, it is implied the total number of modules situated diagonally from each other.

So, $x,y,T \in R^n$, $Z \in R^{4N}$, where $N = C_n^2$, need to be found such that:

$$x,y \geq 0, T \in B_n, Z \in A_{\overline{ms}}^{'\overline{K}}(\overline{G}), \tag{1}$$

where

$$z = \sum_{i,j=1 i<j}^{n} u_{ij} \to min \tag{2}$$

and the following constraints hold:

- placement of the chip:

$$x_i - h_i \cdot t_i \leq \overline{L}_i^{'}, y_i - l_i \cdot t_i \leq \overline{H}_i^{'}, i \in J_n; \tag{3}$$

- non-overlapping the modules:

$$\begin{gathered} x_i - x_j - h_i \cdot t_i + L \cdot z_{ij}^1 \leq L_i^{'}, \\ x_j - x_i - h_j \cdot t_j + L \cdot z_{ij}^2 \leq L_j^{'}, \\ y_i - y_j - l_i \cdot t_i + H \cdot z_{ij}^3 \leq H_i^{'}, \\ y_j - y_i - l_j \cdot t_j + H \cdot z_{ij}^4 \leq H_j^{'}, \\ i, j \in J_n, i < j, \end{gathered} \tag{4}$$

where

$$u_{ij} = \sum_{k=1}^{4} z_{ij}^k - 1, \tag{5}$$

- $x = (x_i)_{i \in J_n}, y = (y_i)_{i \in J_n}$ are vectors of coordinates of all left-bottom corners of modules M_i ($i \in J_n$) (the coordinate system origin is located at a left-bottom corner of the module);

- $\overline{L'} = \left(L_i'\right)_{i \in J_n} = \left(L - p_i\right)_{i \in J_n}$ and
- $\overline{H'} = \left(H_i'\right)_{i \in J_n} = \left(H - p_i\right)_{i \in J_n}$ are vector of additional parameters; $p_i = h_i + l_i$ is a semi-perimeter of M_i ($i \in J_n$);
- $T = \left(t_i\right)_{i \in J_n}$ is a vector of module's orientation parameters:

$$t_i = \begin{cases} 1, if\ M_i\ does\ not\ rotate \\ 0, if\ M_i\ rotates\ 90^0\ degrees \end{cases}, i \in J_n; \tag{6}$$

$Z = \left(Z_{ij}\right)_{i,j \in J_n, i<j}$, $Z_{ij} = \left(z_{ij}^k\right)_{k \in J_4}$ is a vector that determines the relative position of the modules $M_i, M_j \left(i, j \in J_n, i < j\right)$, namely:

- $z_{ij}^1 = 1$, if M_i is left from M_j ($x_i + l_i \le x_j$), otherwise $z_{ij}^1 = 0$;
- $z_{ij}^2 = 1$, if M_i is right from M_j ($x_j + l_j \le x_i$), otherwise $z_{ij}^2 = 0$;
- $z_{ij}^3 = 1$, if M_i is lower than M_j ($y_i + h_i \le y_j$), otherwise $z_{ij}^3 = 0$;
- $z_{ij}^4 = 1$, if M_i is higher than M_j ($y_j + h_j \le y_i$), otherwise $z_{ij}^4 = 0$;
- B_n is Boolean n-vector set [12], [19];
- $A_{52}^{'4}\left(G\right)$ is a subsets of a set $A_{52}^{4}\left(G\right)$ of 4-partial multipermutations [18] induced by a multiset $G = \left\{0,0,0,1,1\right\}$, where the sums of the first two and of the last two coordinates of the elements does not exceed one;
- $A_{\overline{ms}}^{'\overline{K}}(\overline{G}) = A_{52}^{'4}(G) \times \ldots \times A_{52}^{'4}(G)$ is a poly-4-partial multipermutation set that is a Cartesian product of N sets of type $A_{52}^{'4}\left(G\right)$ [10].

Here are two particular cases of the problem simplifying the main problem (1)-(4) (further referred to as Problem 1) to some extend.

5.2. Problem 2

Suppose that some of the modules have a square shape that is

$$\exists i \in J_n : l_i = h_i. \tag{7}$$

Without loss of generality, we can assume that square modules have the first indexes, i.e.,

$$\exists n' \in J_n^0 : l_i = h_i\ \forall i \in J_{n'} = I'; \tag{8}$$

$$l_i \ne h_i\ \forall j \in I = J_n \setminus J_{n'}^0\ (J_n^0 = J_n \cup \{0\}).$$

In this case, reorientation of $M_i\ \left(i \in J_{n'}\right)$ is inexpedient. Therefore the introduction of the variables $t_i\ \left(i \in J_{n'}\right)$ of the form (6) is not required.

So, after simplifying the constraints (1), (3)-(5) we came to a mathematical model of Problem 2: find

$$x, y \in R^n, T' \in R^{nn'}, Z \in R^{4N}$$

of the form of

$$x, y \geq 0, T' = (t_i)_{i \in I} \in B_{n'}, Z \in A_{ms}^{'\overline{K}}(\overline{G}), \tag{9}$$

that minimize (2) and satisfy conditions:

- of placing on the modules (placement constraints):

$$x_i \leq \overline{L}_i', \; y_i \leq \overline{H}_i', \; i \in I', \tag{10}$$

$$x_i - h_i \cdot t_i \leq \overline{L}_i', \; y_i - l_i \cdot t_i \leq \overline{H}_i', \; i \in I; \tag{11}$$

- nonoverlapping constraints:

$$\begin{aligned} & x_i - x_j + L \cdot z_{ij}^1 \leq L_i', \\ & x_j - x_i + L \cdot z_{ij}^2 \leq L_j', \\ & y_i - y_j + H \cdot z_{ij}^3 \leq H_i', \\ & y_j - y_i + H \cdot z_{ij}^4 \leq H_j', \\ & i, j \in I', i < j; \end{aligned} \tag{12}$$

$$\begin{aligned} & x_i - x_j + L \cdot z_{ij}^1 \leq L_i', \\ & x_j - x_i - h_j \cdot t_j + L \cdot z_{ij}^2 \leq L_j', \\ & y_i - y_j + H \cdot z_{ij}^3 \leq H_i', \\ & y_j - y_i - l_j \cdot t_j + H \cdot z_{ij}^4 \leq H_j', \\ & i \in I', j \in I, i < j; \end{aligned} \tag{13}$$

$$\begin{aligned} & x_i - x_j - h_i \cdot t_i + L \cdot z_{ij}^1 \leq L_i', \\ & x_j - x_i + L \cdot z_{ij}^2 \leq L_j', \\ & y_i - y_j - l_i \cdot t_i + H \cdot z_{ij}^3 \leq H_i', \\ & y_j - y_i + H \cdot z_{ij}^4 \leq H_j', \\ & i \in I, j \in I', i < j; \end{aligned} \tag{14}$$

$$\begin{aligned} & x_i - x_j - h_i \cdot t_i + L \cdot z_{ij}^1 \leq L_i', \\ & x_j - x_i - h_j \cdot t_j + L \cdot z_{ij}^2 \leq L_j', \\ & y_i - y_j - l_i \cdot t_i + H \cdot z_{ij}^3 \leq H_i', \\ & y_j - y_i - l_j \cdot t_j + H \cdot z_{ij}^4 \leq H_j', \\ & i, j \in I, i < j. \end{aligned} \tag{15}$$

As you can see, the dimension of Problem 2 as compared to Problem 1 is slightly less, but its advantage is that some of the constraints (3) turn into constraints on the (10) variables (boxing constraints), which simplifies its software implementation.

Remark 2 *If all modules are square, i.e.,* $I' = J_n \; I = \{\varnothing\}$*, we get a problem of finding*

$$0 \le x \le \overline{L}', 0 \le y \le \overline{H}', Z \in A_{\overline{ms}}^{'\overline{K}}(\overline{G}), \tag{16}$$

which delivers the minimum of (2) and satisfying the condition (12).

5.3. Problem 3

Let all modules are oriented along the boundaries of the microcircuit, then, as in Problem 2, it is unnecessary to introduce additional variables (6) and the problem (1)-(4) becomes: find x, y, Z minimizing (2) while satisfying the conditions (16), (17):

$$\begin{aligned} &x_i - x_j + L \cdot z_{ij}^1 \le L_i^{'}, \\ &x_j - x_i + L \cdot z_{ij}^2 \le L_j^{'}, \\ &y_i - y_j + H \cdot z_{ij}^3 \le H_i^{'}, \\ &y_j - y_i + H \cdot z_{ij}^4 \le H_j^{'}, \\ &i, j \in I_n,\ i < j. \end{aligned} \tag{17}$$

Remark 2 A certain simplification of the problem (1) can be derived if the modules form multisets. This means that among them, they are multiple of the same size. In this case, for each of a pair of such modules, one can introduce a certain ordering $M_i \prec M_j, i < j$, according to which M_i is always not to the right of M_j and not higher than it. Therefore, the set Z of additional variables and the number of the constraints (4) are reduced.

It is seen that the problem (1) is a linear constrained partially combinatorial optimization problem. It is offered to apply the combinatorial cut-off method [20] to its solution. This method is based on the derived properties of a polytope being the convex hull of the set $A_{\overline{ms}}^{\overline{K}}(\overline{G})$, such as its H-representation and the vertex adjacency criterion.

6. Software implementation

To implement the above approach to solving the (1) problem, a web service was developed based on ASP.NET WebForms and the programming language C#.

The block for generating the system of constraints and auxiliary information is based on the implemented class Matrix with its methods, operators, indexers and properties.

Further, capital letters $A,..,D$ denote matrices, and lowercase letters $a,...,d$ denote vectors,

$$A(m,n) = (a_{ij})_{i \in J_m; j \in J_n},\ A(n) = (a_{ij})_{i,j \in J_n},$$

$a(n) = (a_i)_{i \in J_n}$, E, e are unit matrix and a vector of units.

To solve the problem, a system of constraints (11)-(17) is generated.

The following functions are implemented in the $Matrix$ class:

- static methods that implement matrix concatenation:

$$\left(\begin{array}{l} A(m,n1+n2) = Matrix.ExtendMatrix(\\ A1(m,n1), A2(m,n2)), \\ B(m1+m2,n) = Matrix.ExtendMatrixInDown(\\ B1(m1,n), B2(m2,n)) \end{array} \right.;$$

- the identity matrix $E(n)$ is generated using the static method of the class $UnitMatrix(n)$
- operator $(A(m,n),a(n))$ multiplies matrix columns by elements of a given vector $(B(m,n)=A(m,n)\ a(n))$;
- methods $MultiplyColsForX(n,N,a(n))$, $MultiplyColsForY(n,N,a(n))$ are used to generate the matrices B1-B4. Input parameters are: n,N is matrix dimension and a (n) is data array, arranged in columns;
- generation of the specific matrix is carried out using the constructor of the $Matrixclass(iRows,iCols,iColValues)$, where $iRows,iCols$ define the dimension of the matrix, $iColValues$ is an array of elements.

Also, matrix multiplication and matrix-scalar multiplication are implemented.

The constraint system is generated using combinations of the above methods and operators of the Matrix class.

Matrices D1, D2 are generated using the operator $(D1(n)=(-E(n),h(n)),D2(n)=(-E(n),l(n))$.

We form matrix A by combining matrices of lower dimension using the static functions *ExtendMatrix* and *ExtendMatrixInDown*. We generate matrices of lower dimensions utilizing the constructor of zero arrays of type $Matrixclass(n,N)$, with the last unit component and the operator $^{*}(E',(-1))$. We form the matrix B1 using the $MultiplyColsForX$ (n,N , h) ($h=h(n)$) function.

Similarly, a matrix B2 is created using the $MultiplyColsForY$ (n,N , h) function. We form matrices B3 and B4 utilizing the functions *MultiplyColsForY* and *MultiplyColsForX* with the parameters (n,N , l) ($l=l(n)$).

We form the right side vector of the system using the $GetB(H,L,h,l,n)$ function, which uses some auxiliary methods, which outputs are combined with the help of *ExtendMatrixInDown* into a column matrix. The additional methods perform the following operations:

- generation ($e=e(n),h,l$);
- generation of vectors $c1,c2,c3,c4$.

The resulting system of constraints along with auxiliary information is displayed in a spreadsheet for further analysis. In orfer to find the optimal solution to the problem, the GPL.AlgLib library is integrated into the service. It is adapted to solve problems of this type using the proposed combinatorial cutting plane methods [14].

7. Conclusion

The work presents a mathematical model of a problem of systematized placement of rectangular modules on a microcircuit. For it solution, a service was developed based utilizing ASP.NET technology and the C # programming language. The approach underlying the offered algorithm the problem solution is the preliminary study of geometric and extremal properties of point configurations being a search domain and its convex hulls [20] with further application in the combinatorial cutting-plane method [21].

8. References

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