Mathematical Modeling as a Tool for Interdisciplinary Training of Computer Sciences and Cybersecurity Students

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Abstract

The article characterizes the category of interdisciplinarity in education, the necessity of introducing interdisciplinary links of higher mathematics with other fundamental and professional disciplines to improve the theoretical and practical training of bachelors in "Computer Sciences" and "Cybersecurity". The content of fundamental mathematical training of students was determined based on the analysis of Ukrainian higher education standards for these specialities. On the basis of survey of graduates of educational programs of IT bachelors in Borys Grinchenko Kyiv University, Sumy State Pedagogical University named after A.S. Makarenko and Alfred Nobel University (Dnipro) and teachers of mathematical and special educational disciplines revealed the contradiction between the potential of the university mathematical training of students in standards and educational programs and low level of real mathematical competence of graduates. The main reasons for insufficient mathematical training of students in these specialities were revealed and the ways of their elimination were proposed. In particular, the expediency of using mathematical modeling as a tool to implement interdisciplinarity in the process of professional training of Computer Sciences and Cybersecurity students was substantiated.

Keywords

Interdisciplinarity, fundamental training, mathematical modelling, mathematical competence, future IT bachelors, computer science, cybersecurity.

1. Introduction

Interdisciplinarity is one of the key principles to which the educational component of modern computer science and cyber security training must correspond. The category of interdisciplinarity is broad and multidimensional, so it has many different definitions and characteristics. Interdisciplinarity in education is understood as an approach where perspectives on a common problem of two or more disciplines are integrated (combined) to obtain a full, deeper and more generalised understanding. Interdisciplinarity thus involves the mutual integration of epistemology, terminology, organisational concepts and methodological procedures for learning. The main features of interdisciplinarity in learning are: interaction, integration, focus and combination [1]. Undoubtedly, interdisciplinary approach is important for training students in any specialty at the university. But it is especially important in future computer science and cybersecurity training. The use of an interdisciplinary approach makes it possible to develop a unified scientific outlook for students of these specialities, based on a combination of knowledge of computer science, information technologies, physics, mathematics and other sciences. The interdisciplinary approach promotes the development of systematic ideas and concepts, enables the integrated use of knowledge and skills of various academic disciplines in collecting, systematising, storing and protecting information, implementing design and development of software products (mobile applications, hardware and software of computerized

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systems, including embedded ones), design and administration of computer networks, information and intelligent systems, use of computer graphics, virtual reality technology etc.

According to the current state standards [2, 3], one of the objects of study and future professional activity of students “Computer Sciences” and “Cybersecurity” are mathematical models of real phenomena, objects, systems and processes. At the same time, the analysis of the real practice of professional training of students shows that mathematical modeling methods are not often and effectively used in training, in particular, in the study of technical and physical disciplines.

2. Analysis of Previous Results

The problem of integration of the sciences, the interdisciplinary links between different disciplines is presented in the works [4-8]. Modern research has established the core of the concepts of “interdisciplinary integration”, and “interdisciplinary links”. Interdisciplinary links in the professional training of students have been found to be an example of the integration processes taking place in science and society. These links contribute to the effectiveness of students' fundamental and practical training, a feature of which is the development of generalised cognitive skills. Educational and future professional tasks are solved at a qualitatively new level exactly with the help of interdisciplinary connections. Special attention should also be paid to the forms of interdisciplinary links use: creating problematic situations on a basis of using the material from different disciplines, as well as conducting integrated lessons. The analysis of scientific papers also revealed problems in the implementation of interdisciplinary integration: insufficient methodological preparedness of some teachers, different visions of ways to form professional competences of students, inconsistency of academic disciplines in content and time, and so on.

The problem of using mathematical modelling in the educational process is the subject of a number of studies [9-13]. However, the use of mathematical modelling as a tool for implementing interdisciplinarity in education has not been sufficiently explored.

The aim of the article is to reveal the potential of mathematical modelling for interdisciplinary teaching of Computer Sciences and Cybersecurity students.

3. Research Methodology

The following methods were used during the research: theoretical - analysis, synthesis, systematization and generalization of scientific, methodological literature, and normative documents to clarify the specifics of mathematical training of future IT bachelors, definition of conceptual and categorical framework of research; generalization of progressive ideas and existing drawbacks in modern higher education to substantiate ways to improve mathematical training of future IT bachelors on the basis of interdisciplinary integration; empirical: observation, surveys, interviews to determine the real state of mathematical training of future IT bachelors and the formation of their mathematical competence. The research was carried out within the framework of the academic theme “Mathematical methods and digital technologies in education, science, engineering” (State Registration Number: 0121U111924) by the Department of Computer Sciences and Mathematics of Borys Grinchenko Kyiv University.

4. Results and Discussion

4.1. Analysis of the Mathematical Component of Students’ Vocational Training Standards

In recent years, specialities 122 “Computer Sciences” and 125 “Cybersecurity” have been among the most popular among applicants. The high competition among universities, which provide professional training for students of the mentioned specialities, encourages educational institutions to improve the quality of education. This leads to the search and implementation of new forms, teaching methods, updating the content of academic disciplines and so on.

As practice shows, the professional activities of IT bachelors are closely connected with mathematical knowledge. Therefore, the question arises in determining the real state of the mathematical component of professional training of students - future IT bachelors and identifying ways to improve it.

First of all, it should be noted that specialities 122 “Computer Sciences” and 125 “Cybersecurity” belong to knowledge area 12 “Information Technologies.” Ukraine has developed higher education standards for both specialities [2, 3]. Considering that mathematical training of students is carried out mainly within the first (bachelor) level of higher education, let us analyse the current standards of the mentioned level.

Thus, the standard of speciality 122 “Computer Sciences” [2] specifies mathematical models of real phenomena, objects, systems and processes, subject areas, data representation and knowledge among the objects of study and activities. The mathematical component also appears among the learning objectives - training specialists capable of applying mathematical methods and algorithmic principles in modelling, design, development and maintenance of information technologies. Among the training methods, techniques and technologies, the standard highlights mathematical models, methods and algorithms for solving theoretical and applied problems arising in IT development.

In the standard of higher education for the speciality 125 “Cybersecurity” [3] when describing the theoretical content of the subject area, the knowledge of methods based on mathematical knowledge is indicated: means of detection, management and identification of risks; methods and means of technical and cryptographic protection of information.

The important place of the mathematical component of students' professional training is also traced in the description of professional competences.

For the Computer Science speciality, the following can be highlighted:

1. Ability to mathematically formulate and investigate continuous and discrete mathematical models, justifying the choice of methods and approaches to solve theoretical and applied problems in the sphere of the computer sciencse, analysis and interpretation
2. Ability to use modern methods of mathematical modelling of objects, processes and phenomena, to develop models and algorithms of numerical solution of mathematical modelling tasks, to consider errors of approximate numerical solution of professional tasks.
3. Ability to think logically, draw logical conclusions, use formal languages and models of algorithmic computation, design, develop and analyse algorithms, assess their efficiency and complexity, decidability and unsolvability of algorithmic problems to adequately model subject areas and create software and information systems.
4. Ability to carry out formalised description of operations research tasks in organisational-technical and socio-economic systems of different purposes, to determine their optimal solutions, to build optimal management models taking into account changes in economic situation, to optimise management processes in systems of different purposes and hierarchical levels.
5. Ability to think systematically, apply systems analysis methodology to investigate complex problems of different nature, methods of formalisation and solution of systemic problems with conflicting objectives, uncertainties and risks.
6. The ability to apply the theoretical and practical foundations of modelling methodology and technology to investigate the characteristics and behaviour of complex objects and systems, to conduct computational experiments with processing and analysis of the results.

For the Cybersecurity speciality, let us highlight such professional competency: the ability to apply methods and means of cryptographic and technical protection of the information on the objects of information activity.

Consider also the programme learning outcomes for the Computer Sciences speciality, which are based on the development of students' mathematical competences:

1. Use modern mathematical apparatus of continuous and discrete analysis, linear algebra, analytical geometry, in professional activities to solve theoretical and applied problems in the design and implementation of informatisation objects.
2. Use knowledge of patterns of random phenomena, their properties and operations on them, models of random processes and modern software environments to solve problems of statistical data processing and construction of predictive models.

3. Use methods of numerical differentiation and integration of functions, solution of ordinary differential and integral equations, features of numerical methods and possibilities of their adaptation to engineering problems, have skills of software implementation of numerical methods.

4. Understand the principles of modelling organisational and technical systems and operations; use operations research methods, solving single and multi-criteria optimisation problems of linear, integer, non-linear, stochastic programming.

When analysing the programme outcomes of the Cybersecurity speciality, we note that their mathematical component is not so explicitly emphasised, yet it is rather of a basic nature:

1. Organise your own professional activities, choose the best methods and techniques for solving complex specialised tasks and practical problems in professional activities and evaluate their effectiveness.

2. Solve the problems of protection of programmes and information processed in information and telecommunication systems by software and hardware, and assess the effectiveness of the quality of the decisions adopted.

3. To solve problems of continuity of business processes of the organisation on the basis of the theory of risks to ensure) the functioning of software and hardware systems for detecting intrusions of different levels and classes (statistical, signature, statistically signature).

As the analysis of normative documents shows, the standard for the Computer Sciences speciality shows the importance of mathematical training of students more clearly. In the standard for the Cybersecurity, the mathematical component is not so distinctly felt. However, analysis of the competencies and program outcomes listed in the standard suggests that they are predominantly based on mathematical competence.

Differences in standards in the context of mathematical knowledge, skills and competences presentation are also explained by the different composition of the developers of the current standards.

The importance of mathematical training of future IT bachelors is also confirmed by the academic programmes of universities. Let us analyse the academic programmes on the example of Borys Grinchenko Kyiv University.

Academic programme 122.00.01 “Informatics” [14] contains the following mathematical disciplines: “Linear algebra and analytic geometry” (6 credits), “Mathematical analysis” (5 credits), “Differential equations”, “Mathematical logic and theory of algorithms”, “Discrete mathematics”, “System analysis”, “Optimization and operations research methods”, “Probability theory and mathematical statistics” (4 credits each).

Academic programme 125.00.01 “Security of Informational and Communication Systems” [15] contains the disciplines: “Mathematical analysis and numerical methods” (6 credits), “Decision-making in information and cybersecurity” (5 credits), “Linear algebra and analytical geometry”, “Discrete mathematics” (4 credits), “Probability theory and mathematical statistics” (3 credits).

Also both programs provide the study of disciplines “Physical processes in computing systems” (speciality 122) and “Physics” (speciality 125), which also need proper mathematical training.

The analysis of the mentioned standards and educational programmes allowed us to identify the blocks of mathematical knowledge that students study during their basic training:

- Theoretical and applied provisions of continuous and discrete analysis, including infinitesimal analysis, integral calculus, linear algebra, analytic geometry, differential equations, functional analysis, combinatorics, graph theory, Boolean algebra
- Regularities of random phenomena, their properties and operations on them, probabilistic methods of investigation of complex systems, basic concepts of mathematical statistics, methods of empirical data processing, methods of computational intelligence, machine learning, fuzzy data processing
- Basic concepts of algorithm theory, formal models of algorithms, primitively recursive, arcane recursive and partial recursive functions, computability issues, decidability and unsolvability of mass problems

Methodology of system analysis for system research of deterministic and stochastic models of objects and processes, design and operation of information systems, products, information technology services, other objects of professional activity.

4.2. Results of a Survey of Graduates of Academic Programmes

The next step of the study was to survey graduates of IT academic programmes at Borys Grinchenko Kyiv University, Makarenko Sumy State Pedagogical University and Alfred Nobel University (Dnipro). During September 2021, 16 university graduates, who already work in IT sphere in various positions, were interviewed: system administration technician, software development and testing technician, computer software development specialist, information and cybersecurity administrator, and information and communication systems security auditor.

1. The first question was related to clarification of whether mathematical knowledge is needed at all for successful implementation of a professional activity? The vast majority of respondents answered this question in the affirmative (87.5% of respondents). Thus, the respondents indicated that mathematics helps them to organize their own professional activities and that they often lack mathematical competence to choose the best methods and techniques for solving complex specialized tasks and practical IT problems and for assessing their effectiveness.
2. It was important for us to determine what kind of mathematical knowledge IT bachelors use in their professional activities and where exactly? As a result of summarizing the responses it was found that most often university graduates mentioned 2D / 3D graphics and physics in games, artificial neural networks, cryptography, load balancing, data analysis and prognostics, pattern recognition (images, audio, video), artificial intelligence, neural networks , machine learning, which are impossible to master without the basic sections of higher mathematics, studied in the junior years of study, such as mathematical analysis, algebra, geometry. Among other university disciplines, discrete mathematics, mathematical logic and probability theory and mathematical statistics were predominantly highlighted. Thus, respondent У notes: "In Data Mining and Machine Learning we need statistics, probability theory, linear algebra; in cryptography - linear algebra; in 3D modelling - geometry and mechanics; in bioinformatics - discrete mathematics, statistics; programmers who deal with data science, machine learning cannot do without mathematical analysis". Respondent Z said that he had to develop algorithms that use differential and integral calculus, to look for extremes. Some respondents said that "… mathematics lays the foundation for analysis and construction of algorithmic models", "... programming is actually automation of mathematical actions", "... programmer's main task is to explain step by step what a computer should do, so you should be able to write down mathematically these steps yourself", "... the ability to operate with abstract concepts", "... it is not the mathematics itself that is important, but the ability to use it", which also stresses the importance of mathematical training of students.
3. Next, we asked IT bachelors specialists to assess the level of their own mathematical knowledge to implement a successful professional activity, as well as to assess the quality of teaching mathematical disciplines in the university. The results of the survey are presented in Table 1.

Table 1
Assessment of mathematical knowledge for professional activity and quality of university mathematics teaching

Level	Own mathematical knowledge, %	Mathematical training in a university, %
High	12.5	37.5
Medium	25.0	62.5
Low	62.5	-

As the results of the survey show, former students rather highly assess the quality of mathematics teaching at the university, none of the respondents indicated such training as low. At the same time, respondents' self-assessment of the level of their own mathematical knowledge shows that only 12.5% of individuals consider this level to be high. The vast majority assess the level of their own mathematical training as low (62.5%), which clearly demonstrates the contradiction between the potential of university mathematical training of students in educational programs and the weak level of real mathematical competence of graduates.

Interestingly, many respondents also express that they have had various problems in the early stages of IT-related activities, with a significant number of respondents citing insufficient level of their own mathematical competence among the factors causing problems.

4. Of course, it was important for us to establish the basis of such a contradiction. As a result of interviews with IT bachelors the following was established:

- Insufficient motivation of students to study basic mathematical disciplines, which are taught mainly in I - II years of study. Students are still poorly oriented in future professional tasks and problems.
- Mathematics at university is often laid out "for the sake of mathematics itself", it is desirable to demonstrate more clearly the connection of mathematics with IT industry, physics, to fill university mathematical disciplines with tasks, which are practice-oriented, approximate to the solution of professional problems.
- The use of mathematical methods, creation of mathematical models is very rarely practised in special (professional) disciplines; more often a ready-made model (formula) is proposed, even without analysis of why this particular model is suitable in a given situation. Therefore there is no good opportunity to use mathematical knowledge, to develop skills in mathematical modelling. It is difficult to perceive mathematics in a distance learning environment, when technical resources for problem solving are limited. It is necessary that teachers introduce students to resources for self-education (e.g. at the level of digital resources and services), encourage the results of such independent, creative activity. Let us also note that in the previous work of the authors of the study it was established that effective formation of mathematical competence of students by means of e-learning is possible only in an experiential-oriented educational environment with active and interested participation of students and partnership interaction [16]
- It is desirable to popularise mathematics among non-mathematical students, to demonstrate the possibilities of this science for solving everyday and professional problems. It is important to introduce students to the positive experiences of IT bachelors who use mathematics in their professional activities.

Analysis of the results of the survey and discussion of problems of mathematical training of bachelors in "Computer Sciences" and "Cybersecurity" with teachers of mathematical and special educational disciplines allows us to highlight the main problems and weaknesses of such training. They are:

- Unjustifiably high level of abstractness in teaching, which leads to excessive formalisation of mathematical knowledge
- Lack of interdisciplinary links of higher mathematics with other fundamental and professional disciplines
- Insufficient attention to the use of mathematical methods and construction of mathematical models in the study of professional disciplines

4.3. Mathematical Training for Computer Sciences and Cybersecurity Students

The importance of the mathematical component in the training of technical professions needs no proof. After all, mathematics is the language of science, and because mathematical structures are models of phenomena and processes in various non-mathematical fields, it is an effective tool for learning the world.

However, before we can plan a system for teaching higher mathematics, we need to answer the question "why do we study it?" After all, the answer to the question "how?" is impossible if there is no answer to the question "why?". Depending on the category of students, the answers to the question "why?" would be different. If we talk about the students of technical specialities, in particular, "Computer sciences" and "Cybersecurity", they need mathematics as a tool to study real-world phenomena and processes in computer networks, computer systems, technical and software information and cyber security and effective use of digital technology to this end. It is therefore important for this category of future bachelors to master (in the shortest possible way) the mathematical knowledge and skills required for practical applications. It is superfluous to overload the content of mathematical disciplines with logical subtleties, to strive for maximum and complete rigor, because for these students mathematics will always be only an apparatus, a language to describe those or other phenomena, processes and facts, rather than the essence that interests them in itself. Sufficient (and necessary!) will be the ideological level of mastering mathematics, providing: a deep understanding of the content of concepts, the essence of theorems and methods, understanding, for what we need those or other mathematical structures and tools, how to use them in the construction and study of mathematical models of real processes. Strict, logically perfect proofs, for example, of the theorems of existence in mathematical analysis are completely unnecessary for them, unlike "pure" mathematicians. After all, the concept of, say, the area of a flat figure, or the speed of a moving point have a perfectly natural physical meaning, so their existence raises no doubt in any person with a healthy physical sense. It is quite possible, however, for fundamental mathematical disciplines to be ideologically sound using familiar pictorial representations, relying on empirical reasoning and common sense. Rejecting the 'canonical' rigour of the presentation of mathematical concepts and facts will free up time to cover a broader range of important mathematical structures that have applications. This is very important at a time when non-mathematics programmes are usually devastatingly short in terms of time for higher mathematics. Thus, concentrating solely on the ideological side of things, one can plan a single module to study integrals: one-dimensional, double, triple, curvilinear and surface integrals. Given the shortage of time and through "piety" towards "mathematical rigour" in multiples and curvilinear and surface integrals most often, unfortunately, do not get there.

The second key principle to which the educational component of engineering training must conform is its interdisciplinarity. An example of integration of views of mathematical and physical disciplines on the same physical phenomenon can be joint training modules which naturally combine learning of mathematical concepts of derivative, integrals (in particular, curvilinear and surface) with physical phenomena of divergence, rotor, circulation of vector field, its flow through the surface. Such integrated study, on the one hand, will improve students' conceptual awareness of the above abstract mathematical concepts, because it demonstrates their "vitality", and on the other hand, which is very important for future bachelors in technical branches - it will give them comprehensible mathematical tools for investigation of real physical processes.

Interdisciplinary connection of higher mathematics with special (professional) disciplines should be realized through filling mathematical courses with specific content of scientific and technical tasks of special disciplines and using mathematical methods in the courses of special disciplines.

The style of teaching higher mathematics, which emerged in the twentieth century and prevails today, is of little use for technical disciplines. This style emerged as a result of the development of the logical foundations of mathematical science. However, it should be taken into account that strict definitions of fundamental mathematical concepts (real number, boundary, continuity, derivative, differential, integral, etc.) appeared as a result of rather long and very non-trivial logical analysis of previously created "non-strict", intuitively understandable theories. All of the above and many other mathematical concepts and facts are very challenging for the beginner. Therefore if acquaintance with them (in lectures or textbooks) begins with strict definitions in formal mathematical language, if such acquaintance precedes the realisation of the content and its real applications, this only artificially complicates the perception of the intuitive and quite obvious things. As the eminent mathematician, teacher-didact and populariser of mathematics Vladimir Arnold points out: "It is only possible to understand the commutativity of multiplication by counting and re-counting soldiers by ranks and files or by calculating the area of a rectangle in the two ways. Any attempt to do without this interference by physics and reality into mathematics is sectarianism and isolationism which destroy the image of mathematics as a useful human activity in the eyes of all sensible people" [17].

Moreover, teaching higher mathematics according to the scheme: "The definition-theorem-proof" contradicts the very purpose and objectives of both the teacher and the technical specialities students. Unmotivated theory cannot be assimilated to become a tool for action. A much more effective (and more natural!) way is active learning. The teacher organises the process of active learning so that students acquire knowledge as much as possible on their own. Active learning follows the scheme: "observation/experiment—model—model investigation—conclusion/result—verification by experiment."

The didactic model of such learning is based on the idea of emergent modelling. The idea is that the understanding of the essence of abstract mathematical concepts and theories comes through the mathematisation of meaningful life situations. The starting point is a real-life problem (task) which is purposefully sought and proposed by the teacher, and whose solution leads to a particular mathematical concept or method. That is, a mathematical model arises first as a model of a particular situation and only then becomes an entity in itself, a formal mathematical structure. This is especially important for higher mathematics training because most key mathematical concepts, structures and methods can first be introduced as intuitively understood, naive, "non-rigorous" models of real situations.

In the transition from the real world to abstraction, the strict, formal concept acquires a new quality. On the one hand, it bears the imprint of reality and, on the other hand, becomes an effective tool for its investigation, a "building block" for mathematical modelling. As Koeno Gravemeijer, one of the ideologists of the strategy of emergent modelling, has argued, formal mathematics should be created by students, and emergent modelling encourages and fosters this process of discovery (construction) of mathematical theory [18].

Consequently, we can consider the formation of "rigorous" mathematical concepts and their further application as a complete spiral cycle (Fig. 1):

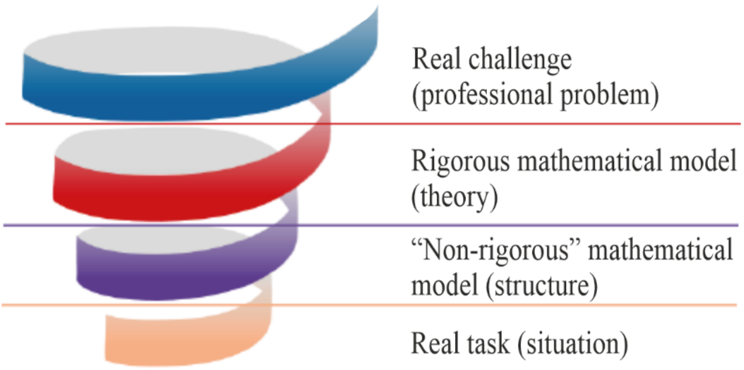

Figure 1: The cycle of the formation and development of "rigorous" mathematical concepts in students

Unfortunately, the limited time budget allotted to study higher mathematics in the initial years of undergraduate study often leads to the other extreme - the desire to limit oneself to a substantive treatment of certain mathematical structures arrived at in the process of solving a physical, say, problem (for example, the derivative is a mechanical speed). Such a "educed" cycle is detrimental to the realization of the essence, the true meaning of a particular mathematical concept, fact or method. After all, the same mathematical structure can serve as a model for many real processes, even very distant from each other in their specific real content. If instead of mathematics itself the student is taught only certain applications, then, faced with a problem in an unfamiliar situation, he will be helpless, despite the fact that for the solution of this problem uses the same model, which he used in other, familiar to him, situation. That is why it is important, on the basis of a mathematical model created for a concrete practical (real) problem, to pass necessarily to the corresponding abstract mathematical structure (the essence of the Pythagoras theorem does not depend on who uses it and in what field). And after that, again illustrate its application to solving applied problems by specific examples.

Thus, mathematical modelling becomes not only a tool for an interdisciplinary approach in teaching, but also a philosophy of learning, aimed primarily at improving conceptual understanding of mathematics and, as a consequence, developing the ability to solve professional problems in their (non-mathematical) field with the help of mathematics.

The use of mathematical modelling as an educational technology is effective for several reasons. It:

- Provides a cheap and safe experiment (computer-based)
- Teaches to evaluate the obtained results critically
- Teaches to penetrate into the essence of the studied concept or phenomenon, not limiting oneself to formal acquaintance
- Encourages continuous learning

The ability to evaluate the obtained result of mathematical modelling is critical in solving specialised professional tasks.

Let us argue the above with a concrete example of a mathematical model describing the flight of a basketball thrown by a player into a game basket.

The model should make it possible to:

a) determine the position of the ball at any given time (x and y coordinates);

b) determine the accuracy of the ball hitting the basket after the shot at different initial parameters.

Initial data:

- mass (m) and radius (R) of the ball;
- the initial coordinates, the initial velocity and the angle at which the ball is thrown;
- coordinates of the centre and radius of the basket ring.

Having made a number of assumptions (we consider the ball to be a material point of mass m, the ball motion occurs in the field of gravity with constant free-fall acceleration g, the ball moves in a plane perpendicular to the surface of the Earth and passing through the throwing point and the centre of the basket; we neglect air resistance and disturbances caused by the ball's own rotation around the centre of mass) and, relying on Newton's classical laws of mechanics, we obtain a mathematical model in the form of a Cauchy problem of a system of ordinary differential equations:

$$\begin{cases} m\dfrac{dv_x}{dt} = 0, \\ v_x = \dfrac{dx}{dt}, \\ m\dfrac{dv_y}{dt} = -mg, \\ v_y = \dfrac{dy}{dt} \end{cases}$$

with initial conditions: $x(0) = x_0, y(0) = y_0, v_x(0) = v_0 \cos\alpha_0, v_y(0) = v_0 \sin\alpha_0$, where v_x and v_y are the projections of the velocity vector on the coordinate axes.

The solution to the problem is as follows:

$$x(t) = x_0 + v_0 \cos\alpha_0 \cdot t,$$

$$y(t) = y_0 + v_0 \sin\alpha_0 \cdot t - \frac{gt^2}{2},$$

$$v_x(t) = v_0 \cos\alpha_0,$$

$$v_y(t) = v_0 \sin\alpha_0 - gt.$$

Or, by removing the variable from the first two equations *t*:

$$y = y_0 + \mathrm{tg}\,\alpha_0 \cdot (x - x_0) - \frac{g(x - x_0)^2}{2v_0^2 cos^2\alpha_0}$$

This is the parabola equation. That is, the thrown ball will move along a trajectory close to a parabolic one, which is confirmed by experimental data. That is, we can assert the qualitative adequacy of the modelling result. However, experimental verification shows a quantitative discrepancy between the modelling result and the real situation, as the assumption of no air resistance is too rigid and implausible. Therefore, there is a need to correct the model, taking into account the force of air resistance.

The above example prompts the important caveat that one should never rely unconditionally on the result obtained by the mathematical modelling. After all, any model is always simpler than the original. And when building a mathematical model, a certain idealisation, simplification occurs: some parameters are not considered, some facts, the validity of which is only approximate or hypothetical, are considered to be absolutely accurate, the results of measurements are also considered to be accurate, but they are always approximate. Although this “arbitrary” treatment of the characteristics of the original in very many cases leads to correct results (in science this phenomenon is called the “Wigner principle” [19]), still the model built must be tested for adequacy and the results obtained for consistency with the physical meaning and dimensionality. It is important that the students, according to the complexity of the problem to be solved and to the extent of their knowledge, do so. Different systems of computer mathematics (MatLab, Mathematica, Maple, Derive, Wolfram SystemModeler, VisSim) are useful for this purpose.

4.4. Examples of Interdisciplinary Learning

Here are examples of different types of professionally oriented tasks (the simplest tasks on application of mathematical theory, task on creation of mathematical model and interdisciplinary project) that we use in the educational process of "Computer Sciences" and "Cybersecurity" specialities [20,21].

Example 1. The task is designed to form skills of applying matrix calculus to solve applied cryptography problems.

One way to encode and decode messages is to use a matrix key. For example, there is an alphabet containing 28 symbols. Each symbol is assigned a code – $0, 2, \ldots, 26, 27, 28$, respectively. The numeric equivalent of the message is then converted into a matrix by writing the numbers in columns. At the same time we form a matrix, in case there are not enough digits to form a matrix we fill the numeric message at the end with zeros. By multiplying the nondegenerate (hence the inverse) key matrix A by the message matrix, the messages are encoded. Using matrix inverse to A one decodes messages.

Problem. Suppose we have the following alphabet:

A	B	…	Z	.	!	_
0	1		25	26	27	28

Encrypt the phrase *"I LOVE YOU"* with a 2x2 key matrix.

1. Select the key matrix. The matrix must be nondegenerate, i.e. the determinant is not equal to zero, and $\det A \neq 0$ (mod 29).

At the encoding stage, if the matrix multiplication yields numbers greater than 29, then all matrix calculations are performed by mod $N = 29$.

Let an encoding matrix be:

$$A = \begin{pmatrix} 2 & 11 \\ 1 & 12 \end{pmatrix}$$

To find the inverse matrix by mod 29, look for the union matrix to A and multiply it by the inverse determinant of matrix A by mod 29: $\det A = 13$ (mod 29), then $(1 / \det A) = 9$ (mod 29). We obtain matrix inverse to A by mod 29 for further decoding:

$$A^{-1} = \begin{pmatrix} 21 & 17 \\ 20 & 18 \end{pmatrix} \text{(mod29)}$$

2. Proceed with the encryption.

Numerical equivalent of message: *I LOVE YOU!* $= (8\ 11\ 14\ 21\ 4\ 24\ 14\ 20\ 27)$.

Then the matrix of the original message

$$P = \begin{pmatrix} 8 & 11 & 14 & 21 & 4 \\ 24 & 14 & 20 & 27 & 0 \end{pmatrix}$$

The matrix of the encrypted message:

$$S = A \times P = \begin{pmatrix} 280 & 176 & 248 & 339 & 8 \\ 296 & 179 & 254 & 345 & 4 \end{pmatrix} = \begin{pmatrix} 19 & 2 & 16 & 20 & 8 \\ 6 & 5 & 22 & 26 & 4 \end{pmatrix} \text{(mod 29)}$$

The encrypted text takes the following form: ***TCQUIGFW.E***

To decrypt, the rightful recipient translates the received text into a matrix and applies into it the matrix inverse to *A.*

$$P = A^{-1} \times S = \begin{pmatrix} 501 & 127 & 710 & 862 & 236 \\ 488 & 130 & 716 & 868 & 232 \end{pmatrix} = \begin{pmatrix} 8 & 11 & 14 & 21 & 4 \\ 24 & 14 & 20 & 27 & 0 \end{pmatrix} \text{(mod 29)}$$

This task is good to solve in pairs: each of the pair picks a matrix key, encrypts his/her message and passes it on to his/her partner for deciphering.

Example 2. The task is designed to form skills of applying systems of linear algebraic equations to study the flow of some quantity through a network.

Systems of linear algebraic equations arise when studying flow of some quantity through a network (automobile transport in urban transportation networks, electric current flow in energy networks, some goods from supplier to consumer in trade networks).

A network is a set of nodes and a set of branches connecting all or some nodes. On each branch, the direction of flow is indicated, and its magnitude is designated as unknown.

In the network flow problems, it is assumed that:

1) the total input flow in the network is equal to the total input flow out of the network;
2) the total input flow in the node is equal to the total source flow out of the node.

Problem [22]. Given an electrical circuit (Figure 2) containing four resistors and two voltage sources:

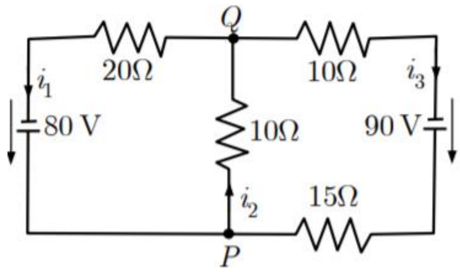

Figure 2: Electrical circuit

The directions of the currents are indicated on each section of the circuit by arrows. Determine their strength.

To solve the problem, use two Kirchhoff laws:

1) at any point in a circle, the sum of input currents is equal to the sum of output currents (current law).
2) in any loop, the sum of all voltage drops equals the EMF (electromotive force) (law of voltages).

$$Node\ P: i_1 - i_2 + i_3 = 0.$$
$$Node\ Q: -i_1 + i_2 - i_3 = 0.$$
$$Right\ loop: 10i_2 + 25i_3 = 90.$$
$$Left\ loop: 20i_1 + 10i_2 = 80.$$

As we can see the equations for the P and Q nodes are proportional, so leaving one of these equations, we get the system:

$$\begin{cases} i_1 - i_2 + i_3 = 0, \\ 10i_2 + 25i_3 = 90, \\ 20i_1 + 10i_2 = 80. \end{cases}$$

Solving this system, we have: $i_1 = 2\ A,\ \ i_2 = 4\ A,\ \ i_3 = 2\ A$.

Example 3. A task for creating a mathematical model (can be offered to the first year students at the beginning of the "Integral" theme learning).

Road traffic regulations stipulate that sign No. 1.33 "Children" shall be installed at road sections where children from a childcare facility (pre-school, school, health camp, etc.) directly adjacent to the road may appear. The warning sign shall be placed outside built-up areas at the distance of $150 - 300\ m$, in built-up areas – at the distance of $50 - 100\ m$ before the beginning of the dangerous section. And it is repeated at least $50\ m$ before the beginning of the danger area.

Is this distance between the second sign and the dangerous section justified, assuming that a pedestrian crosses the road at T seconds?

Solution. Let $v(t)$ be the speed of the car, which is known to change according to the law $v(t) = v_0 + a \cdot t$, where v_0 is the initial speed and a is the acceleration. The path that the car will travel in steady motion during the time T:

$$S = \int_0^T (v_0 + at)dt = v_0 + \frac{at^2}{2}\Big|_0^T = v_0 T + \frac{aT^2}{2}.$$

There we have a mathematical model. Let us investigate it with specific values of parameters.

Let the width of a part of the road on the traffic side be $3.75\ m$ (the width of one lane in Ukraine is $3.50 - 3.75\ m$, in Europe $3.00 - 3.25\ m$). An average human speed of $4\ km/h = 1.11\ m/s$. Then $T = 3.4\ s$. Acceleration a depends on: type of road surface (dirt road, asphalt, cement concrete pavement, etc) and its condition (dry, wet, icy), brand of car, quality of tyres (winter, summer, new, with worn tread). Let $a = -6\ m/s^2$ (average acceleration under emergency braking on a horizontal road of $4 \ldots 8\ m/s^2$). Let us assume that the driver is law-abiding and responsible, so at the first sign he reduces the speed, say, to $60\ km/h$. If there is a person at the crossroads, the driver needs to start braking at the second sign (with the vehicle speed smoothly decreasing) to ensure that he can stop while the pedestrian is on the road. Thus, we need to find out how much distance the car will travel, braking from the second sign, in time T at an initial speed of $60\ km/h$ ($16.67\ m/s$), and whether this distance will not be more than $50\ m$.

$$S = \int_0^{3.4} (16.67 - 6t)dt = \left(16.67 \cdot t - \frac{6 \cdot t^2}{2}\right)\Big|_0^{3.4} = 16.67 \times 3.4 - 3 \times (3.4)^2 \approx 22\ (m).$$

If the driver does not apply the brakes and the initial speed is $120\ km/h$ ($33.3\ m/s$), then the braking distance is $79\ m$. What is the permissible maximum speed of the car before braking so that the braking distance under the same conditions does not exceed $50\ m$?

$$v_0 T + \frac{aT^2}{2} \le 50;$$
$$3.4 v_0 - 3 \times (3.4)^2 \le 50;$$
$$v_0 \approx 25 (m/s) = 90 (km/h).$$

The answer: $90\ km/h = 25\ m/s$.

Example 4. Hackathon

An example of implementing an interdisciplinary learning approach based on mathematical modelling is the study practice of the students specialising in Computer Sciences, joined by the students of the Mathematics speciality. This practice has been held for three consecutive years at the Borys Grinchenko Kyiv University (Figure 3) in the form of Hackathon [23].This practice format brings the students as close as possible to their future real professional activities. It is a project work. Teams develop, within the framework of the suggested topics, an idea for a project and implement it.

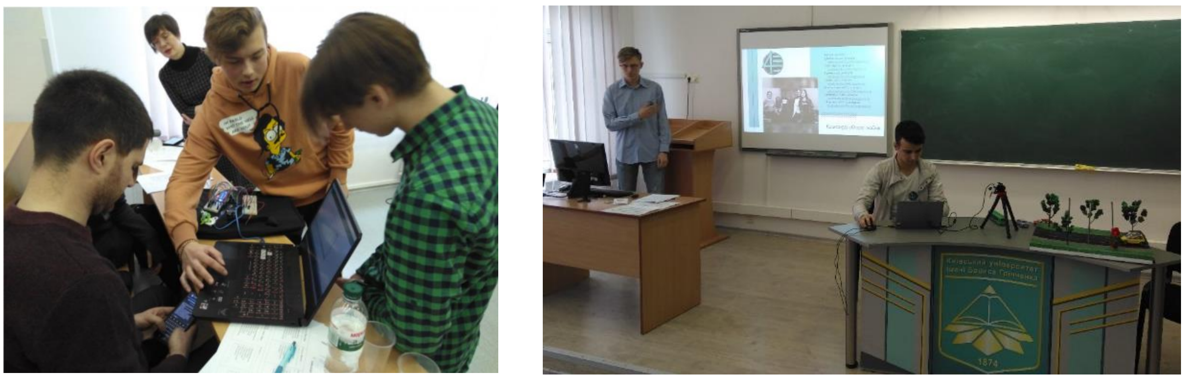

Figure 3: Development and presentation of the devices created at the Hackathon

They see (and implement!) interdisciplinary links in practice, gain experience in teamwork, self-organisation, improve self-education skills (the event itself is preceded by thorough preparatory work) and presentation of work results, increase motivation for learning.

5. Conclusions

1. The mathematical component of Ukrainian higher education standards for specialities "Computer Sciences" and "Cybersecurity" at the level of professional competencies and program learning outcomes was analyzed. This made possible to determine the content of fundamental mathematical training of students in the above specialities of the first (bachelor) level and its reflection in the corresponding academic programs of Borys Grinchenko Kyiv University.
2. As a result of the survey of graduates of academic programs of IT bachelors it was found that the level of mathematical training is important for professional activities. In particular, the vast majority of respondents lack mathematical competence to choose the best methods and techniques for solving complex specialized tasks and practical problems in IT sphere, to evaluate their effectiveness, etc.
3. The reasons for insufficient mathematical training, according to the teachers of mathematics and special academic subjects, are the following: unreasonably high level of abstractness in teaching higher mathematics, which leads to excessive formalization of mathematical knowledge; insufficient use of interdisciplinary links of higher mathematics with other fundamental and special academic disciplines; insufficient attention to using mathematical methods and construction of mathematical models in the study of special disciplines.
4. The expediency of using mathematical modeling as a pedagogical technology for interdisciplinary training of students in "Computer Sciences" and "Cybersecurity" bachelors was substantiated. Such approach improves conceptual understanding of mathematics and, as a consequence, formation of ability to solve professional tasks in IT sphere using mathematics. Examples of interdisciplinary problems are given.

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