Weighted Conditionals from Gradual Argumentation to Probabilistic Argumentation (Extended Abstract)

Mario Alviano[1], Laura Giordano[2] and Daniele Theseider Dupré[2]

[1] Università della Calabria, Italy
[2] Università del Piemonte Orientale, Italy

Abstract

Recently some new gradual argumentation semantics have been proposed based on different fuzzy multi-preferential semantics for weighted conditional knowledge bases with typicality. In this abstract we report about ongoing issues, including multi-valued variants of the gradual semantics, probabilistic semantics for weighted argumentation, and proof methods.

This extended abstract reports about some work [21, 22] investigating the relationships between the weighted conditional knowledge bases with typicality, under fuzzy semantics, and gradual argumentation semantics [13, 30, 17, 18, 2, 4, 3]. It then discusses extension of this work in the direction of allowing defeasible reasoning and probabilistic reasoning over argumentation graphs, and also considers many-valued semantics for weighted argumentation, for which ASP encodings of preferential entailment have been investigated [28].

In previous work [21, 22] some weighted argumentation semantics have been proposed, inspired by some multi-preferential semantics of commonsense reasoning, first introduced for description logics with typicality [26, 27]. Preferential description logics have been studied in the last fifteen years with the aim to model inheritance with exceptions in ontologies, based on the idea of extending the language of Description Logics (DLs) by allowing for non-strict forms of inclusions, called *typicality or defeasible inclusions*, of the form $\mathbf{T}(C) \sqsubseteq D$ (meaning “the typical C-elements are D-elements" or “normally C’s are D’s"), with different preferential semantics [23, 10] and closure constructions [12, 11, 24]. Defeasible inclusions correspond to Kraus, Lehmann and Magidor (KLM) conditionals $C \mathrel{|\!\sim} D$ [33, 34], and defeasible DLs inherit and extend some of the preferential semantics and closure constructions developed within preferential and conditional approaches to commonsense reasoning [37, 34, 20, 6].

A *concept-wise multi-preferential semantics* for weighted conditional DL knowledge bases (KBs) has been proposed [26, 27] to account for preferences with respect to different concepts, by allowing typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ with a rank [26] or weight [27], for some distinguished concepts C. The semantics exploits multiple preferences with respect to concepts, and different semantic closure constructions have been considered, in the spirit of Lehmann’s lexicographic closure [35] and Kern-Isberner’s c-representations [31]. The concept-wise multi-preferential semantics has also been proven to have some desired properties from the knowledge representation point of view (see [26] for the two-valued case). It has been shown [27] that the

---

AIˆ3: 6th Workshop on Advances In Argumentation In Artificial Intelligence, 2022, Udine, Italy

mario.alviano@unical.it (M. Alviano); laura.giordano@uniupo.it (L. Giordano); dtd@uniupo.it (D. Theseider Dupré)

multi-preferential semantics allows to describe the behavior of Multilayer Perceptrons (MLPs), after training, in terms of a preferential interpretation.

The relationships between preferential and conditional approaches to non-monotonic reasoning and argumentation semantics are strong. Let us just mention the work by Geffner and Pearl on Conditional Entailment [20]. To investigate the relationships between the fuzzy multi-preferential semantics for weighted conditionals and gradual argumentation, the notions of coherent, faithful and φ-coherent [21] (fuzzy) multi-preferential semantics have been proposed for weighted argumentation graphs, where positive and negative weights are associated to pairs of arguments.

Let us shortly report the φ-coherent semantics from [21]. Let a *weighted argumentation graph* be a triple $G = \langle \mathcal{A}, \mathcal{R}, \pi \rangle$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ and $\pi : \mathcal{R} \rightarrow \mathbb{R}$. A *labelling of the graph* G is a function $\sigma : \mathcal{A} \rightarrow [0,1]$ which assigns to each argument an *acceptability degree*, i.e., a value in the interval $[0,1]$.

For a weighted graph $G = \langle \mathcal{A}, \mathcal{R}, \pi \rangle$ and a labelling σ, we define a *weight* W^G_σ *on* $\mathcal{A}$, as a partial function $W^G_\sigma : \mathcal{A} \rightarrow \mathbb{R}$, assigning a positive or negative support to the arguments $A_i \in \mathcal{A}$ such that A_i has incoming edges, as follows: $W^G_\sigma(A_i) = \sum_{(A_j, A_i) \in \mathcal{R}} \pi(A_j, A_i)\ \sigma(A_j)$. Otherwise, $W^G_\sigma(A_i)$ is left undefined. The weight of an argument is exploited to define different argumentation semantics for a graph G. In particular, given a function $\varphi : \mathbb{R} \rightarrow [0,1]$, a labelling σ is a *φ-coherent labelling of* G if, for all arguments $A_i \in \mathcal{A}$ s.t. A_i has incoming edges, $\sigma(A) = \varphi(W^G_\sigma(A))$. The labelling of arguments without incoming edges in G is arbitrary, provided the constraints on the labellings of all other arguments can be satisfied.

The relationship of the φ-coherent semantics with the family of gradual semantics studied by Amgoud and Doder [2] has also been investigated, by slightly extending their *gradual argumentation framework* to deal with positive and negative weights to capture the strength of supports and of attacks. A correspondence between the gradual semantics based on a specific evaluation method M^φ and φ-coherent labelings has been proven [22]. Unlike the Fuzzy Argumentation Frameworks by Jenssen et al. [30], where an attack relation is a fuzzy binary relation over the set of arguments, here real-valued weights are associated to pairs of arguments.

Since a deep neural network can be mapped to a weighted conditional knowledge base [27], it can as well be seen as a weighted argumentation graph, with positive and negative weights, under the proposed semantics. This is in agreement with previous work on the relationship between argumentation frameworks and neural networks, first investigated by D'Avila Garcez, Gabbay and Lamb [14] and recently by Potyca [38] (we refer to [22] for comparison).

A *many-valued argumentation semantics* can as well be defined by replacing the set of truth degrees $\mathcal{S} = [0,1]$ in the fuzzy preferential semantics above with the finite set $\mathcal{C}_n = \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, \frac{n}{n}\}$, for some integer $n \geq 1$. This also requires introducing, for each activation function φ, a function φ_n which approximates $\varphi(x)$ to the nearest value in $\mathcal{C}_n$. An ASP approach for reasoning under finitely multi-valued φ-coherent semantics for weighted KBs has been proposed in [28], exploiting *asprin* [8] for defeasible reasoning, selecting preferred answer sets. As a proof of concept, the approach has been experimented for checking properties of some trained MLPs. This encoding can be used as the basis of other ASP solutions for the multi-valued φ-coherent argumentation semantics, which exploit state of the art ASP solving, including fuzzy ASP solving [1] and custom propagation based on the *clingo* API. An objective is to use

ASP solvers is the verification of conditional properties over weighted argumentation graphs, such as "does normally argument A_2 follow from argument A_1 with a degree greater than 0.7?", which can be formalized by defeasible implications of the form $\mathbf{T}(A_1) \rightarrow A_2 > 0.7$, both in the fuzzy and many-valued case, where $\mathbf{T}(A_1)$ refers to the typical situations in which A_1 holds.

Observe that a labelling of arguments in $[0, 1]$ (or in $\mathcal{C}_n$) can be extended to boolean combinations of arguments by using fuzzy combination functions, e.g., by letting $\sigma(A_1 \wedge A_2) = min\{\sigma(A_1), \sigma(A_2)\}$, using the minimum t-norm as in Gödel fuzzy logic. This suggests that a general approach can be developed to define a preferential interpretation of an argumentation graph, starting from a set of labellings Δ in a given gradual semantics, to allow for *defeasible reasoning over the argumentation graph.* The conditional properties of the graph can be formalized in a many-valued logic with typicality, i.e., a many-valued propositional logic in which arguments play the role of propositional variables, and a typicality operator is allowed, as in Propositional Typicality Logic [7], but with a multi-preferential semantics. This is in line with the correspondence between Abstract Dialectical Frameworks [9] and Nonmonotonic Conditional Logics studied by Heyninck, Kern-Isberner and Thimm [29], considering two-valued models, the stable, the preferred semantics and the grounded semantics of ADFs.

The fuzzy interpretation of arguments also allows a notion of probability to be associated to arguments in a weighted argumentation graph, based on Zadeh's *probability of fuzzy events* [39]. The approach has been considered for providing a probabilistic interpretation of SOMs after training, starting from their fuzzy interpretation [25], by exploiting a recent characterization of the continuous t-norms compatible with Zadeh's probability of fuzzy events (P_Z-compatible t-norms) by Montes et al. [36].

For a given gradual semantics with domain of argument valuation $[0, 1]$, it suggests an epistemic approach to probabilistic argumentation, in which arguments A_i play the role of fuzzy events, with membership function $\mu_{A_i} : \Delta \rightarrow [0, 1]$, where $\mu_{A_i}(\sigma) = \sigma(A_i)$ (and similarly for boolean combinations of arguments). Assuming a discrete probability distribution $p : \Delta \rightarrow [0, 1]$ over a set Δ of possible labellings σ of the graph in some gradual semantics (e.g., in the φ_n-coherent semantics), one can define the *probability of a boolean combination of arguments* α as: $P(\alpha) = \sum_{\sigma \in \Delta} \sigma(\alpha)\ p(\sigma)$. When the labellings are two-valued ($\sigma(\alpha)$ is 0 or 1), this definition relates to the probability of a boolean term α by Hunter and Thimm [19].

We let the conditional probability of A given B, where A and B are boolean combinations of arguments, to be $P(A \mid B) = P(A \wedge B)/P(B)$ (provided $P(B) > 0$). As observed by Dubois and Prade [15], this generalizes both conditional probability and the fuzzy inclusion index advocated by Kosko [32]. Then, $\sigma(A)$ can be interpreted as the conditional probability of argument A, given labelling σ (i.e., the degree of belief we put into A when we are in the state represented by labelling σ, a *subjective* probability).

Observe that, while it has been proven [39, 36] that this notion of probability satisfies Kolmogorov's axioms, some properties of classical probability may be lost (depending on the chosen combination functions), as a consequence of the fact that not all classical logic equivalences hold in a fuzzy logic. This approach has been experimented in the verification of properties of some feedforward neural networks under the φ-coherent semantics [28], in the finitely-valued case, using Gödel combination functions with standard involutive negation, and we aim to extend the experimentation of the proof methods to general argumentation graphs.

Acknowledgement: This research is partially supported by INDAM-GNCS Project 2022

“LESLIE”, by the Università del Piemonte Orientale, and by the LAIA lab (part of the SILA labs).

References

[1] M. Alviano and R. Peñaloza. Fuzzy answer set computation via satisfiability modulo theories. In *Theory Pract. Log. Program.*,15(4-5):588–603, 2015.

[2] L. Amgoud, J. Ben-Naim, D. Doder, and S. Vesic. Acceptability semantics for weighted argumentation frameworks. In *IJCAI 2017, Melbourne, Australia*, pages 56–62, 2017.

[3] L. Amgoud and D. Doder. Gradual semantics accounting for varied-strength attacks. In *Proceedings AAMAS ’19, Montreal, QC, Canada, May 13-17, 2019*, pages 1270–1278, 2019.

[4] P. Baroni, A. Rago, and F. Toni. How many properties do we need for gradual argumentation? In *Proc. AAAI 2018*, pages 1736–1743, 2018.

[5] F. Bartoli, M. Botta, R. Esposito, L. Giordano, D. Theseider Dupré. An ASP approach for reasoning about the conditional properties of neural networks: an experiment in the recognition of basic emotions, Datalog 2.0 2022, Sept. 5, Genova, CEUR vol. 3203, 54-67.

[6] S. Benferhat, C. Cayrol, D. Dubois, J. Lang, and H. Prade. Inconsistency management and prioritized syntax-based entailment. In *Proc. IJCAI’93, Chambéry,*, pages 640–647, 1993.

[7] R. Booth, G. Casini, T. Meyer, I. Varzinczak. On rational entailment for Propositional Typicality Logic. In *Artif. Intell. 277(2019).*

[8] G. Brewka, J. P. Delgrande, J. Romero, and T. Schaub. asprin: Customizing answer set preferences without a headache. In *Proc. AAAI 2015*, pages 1467–1474, 2015.

[9] G Brewka, H. Strass, S. Ellmauthaler, J. P. Wallner, and S. Woltran. Abstract dialectical frameworks revisited. In *Proc. IJCAI 2013, pages 803–809, 2013.*

[10] K. Britz, J. Heidema, and T. Meyer. Semantic preferential subsumption. In G. Brewka and J. Lang, editors, *KR 2008*, pages 476–484, Sidney, Australia, September 2008. AAAI Press.

[11] G. Casini, T. Meyer, I. J. Varzinczak, and K. Moodley. Nonmonotonic Reasoning in Description Logics: Rational Closure for the ABox. *DL 2013*, *CEUR* Vol. 1014, 600–615, 2013.

[12] G. Casini and U. Straccia. Rational Closure for Defeasible Description Logics. *JELIA 2010*, volume 6341 of *LNCS*, pages 77–90, Helsinki, Sept. 2010. Springer.

[13] C. Cayrol and M. Lagasquie-Schiex. Graduality in argumentation. *J. Artif. Intell. Res.*, 23:245–297, 2005.

[14] A. S. D’Avila Garcez, D. M. Gabbay, and L. C. Lamb. Value-based argumentation frameworks as neural-symbolic learning systems. *J. Log. Comput.*, 15(6):1041–1058, 2005.

[15] D. Dubois and H. Prade Fuzzy sets and probability: misunderstandings, bridges and gaps. Proc. 2nd IEEE Int. Conf. on Fuzzy Systems, pages 1059-1068, vol.2, 1993.

[16] P. M. Dung. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artif. Intell.*, 77:321–357, 1995.

[17] P. Dunne, A. Hunter, P. McBurney, S. Parsons, M. Wooldridge. Weighted argument systems: Basic definitions, algorithms, and complexity results. *Artif. Intell.*, 175(2):457–486, 2011.

[18] S. Egilmez, J. G. Martins, and J. Leite. Extending social abstract argumentation with votes on attacks. In *TAFA 2013, Beijing, China, Aug. 3-5, LNCS 8306*, pages 16–31. Springer, 2013.

[19] A. Hunter, S. Polberg and M. Thimm. Epistemic graphs for representing and reasoning with positive and negative influences of arguments. *Artif. Intell.*, 281: 103236 (2020).

[20] Hector Geffner and Judea Pearl. Conditional entailment: Bridging two approaches to default reasoning. *Artif. Intell.*, 53(2-3):209–244, 1992.

[21] L. Giordano. From weighted conditionals of multilayer perceptrons to a gradual argumentation semantics. In *5th Workshop on Advances in Argumentation in Artificial Intelligence, Milan, Italy, November 29, 2021*, vol. 3086 of *CEUR Workshop Proc.*. CEUR-WS.org, 2021.

[22] L. Giordano. From weighted conditionals with typicality to a gradual argumentation semantics and back. In *NMR 2022, Haifa, August 7-9*, CEUR vol. 3203, 54–67, 2022.

[23] L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Preferential Description Logics. In *LPAR 2007*, volume 4790 of *LNAI*, pages 257–272, Armenia, October 2007. Springer.

[24] L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Semantic characterization of rational closure: From propositional logic to description logics. *Artif. Intell.*, 226:1–33, 2015.

[25] L. Giordano, V. Gliozzi, and D. Theseider Dupré. A conditional, a fuzzy and a probabilistic interpretation of self-organising maps. *J.Log. Comput., 32(2): 178-205 (2022).*

[26] L. Giordano and D. Theseider Dupré. An ASP approach for reasoning in a concept-aware multi-preferential lightweight DL. *Theory Pract. Log. Program.*, 20(5):751–766, 2020.

[27] L. Giordano and D. Theseider Dupré. Weighted defeasible knowledge bases and a multi-preference semantics for a deep neural network model. In *Proc. JELIA 2021, May 17-20*, volume 12678 of *LNCS*, pages 225–242. Springer, 2021.

[28] L. Giordano and D. Theseider Dupré. An ASP approach for reasoning on neural networks under a finitely many-valued semantics for weighted conditional knowledge bases. *Theory Pract. Log. Program.*, 22(4):589-605 (2022).

[29] J. Heyninck, G. Kern-Isberner, and M. Thimm. On the correspondence between abstract dialectical frameworks and nonmonotonic conditional logics. In *Proc. of the FLAIRS 2020, May 17-20, 2020*, pages 575–580. AAAI Press, 2020.

[30] J. Janssen, M. De Cock, and D. Vermeir. Fuzzy argumentation frameworks. In *IPMU 2008*, pages 513–520, 2008.

[31] G. Kern-Isberner. *Conditionals in Nonmonotonic Reasoning and Belief Revision - Considering Conditionals as Agents*, volume 2087 of *LNCS*. Springer, 2001.

[32] B. Kosko. Neural networks and fuzzy systems: a dynamical systems approach to machine intelligence Prentice Hall, 1992.

[33] S. Kraus, D. Lehmann, and M. Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artif. Intell.*, 44(1-2):167–207, 1990.

[34] D. Lehmann and M. Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55(1):1–60, 1992.

[35] D. J. Lehmann. Another perspective on default reasoning. *Ann. Math. Artif. Intell.*, 15(1):61–82, 1995.

[36] I. Montes, J. Hernández, D. Martinetti and S. Montes, Characterization of continuous t-norms compatible with Zadeh's probability of fuzzy events, *Fuzzy Sets Syst.* 228(2013), 29–43.

[37] J. Pearl. System Z: A natural ordering of defaults with tractable applications to nonmonotonic reasoning. In *TARK'90, Pacific Grove, CA, USA*, pages 121–135, 1990.

[38] N. Potyka. Interpreting neural networks as quantitative argumentation frameworks. In AAAI 2021, February 2-9, 2021, pages 6463–6470. AAAI Press, 2021.

[39] L. Zadeh. Probability measures of fuzzy events. J.Math. Anal. Appl. 23:421–427 (1968).