Stable Semantics for Epistemic Abstract Argumentation Framework

Gianvincenzo Alfano, Sergio Greco, Francesco Parisi and Irina Trubitsyna

Department of Informatics, Modeling, Electronics and System Engineering (DIMES), University of Calabria, Rende, Italy

Abstract

Dung's Abstract Argumentation Framework (AAF) has emerged as a central formalism in AI for modeling disputes among agents. A recent extension of the Dung's framework is the so-called Epistemic Abstract Argumentation Framework (EAAF), which enhances AAF by allowing the representation of some pieces of epistemic knowledge [1]. EAAF generalizes the concept of attack in AAF, introducing *strong* and *weak epistemic attacks*, whose intuitive meaning is that an attacked argument is epistemically accepted only if the attacking argument is possibly or certainly rejected, respectively. The semantics of EAAF has been defined and studied for several argumentation semantics but not for the stable one, which is arguably one of the most investigated semantics in argumentation. Motivated by this, in this paper, we propose an intuitive stable semantics for EAAF that naturally extends that for AAF and coincides with the preferred semantics in the case of odd-cycle free EAAFs (analogously to what happens in the case of AAF). We analyze the complexity of two argumentation problems: *existence*, i.e. checking whether there is at least one epistemic extension; and *acceptance*, i.e. checking whether an argument is epistemically accepted.

1. Introduction

In the last decades, Argumentation [2, 3, 4] has become an important research field in the area of autonomous agents and multi-agent systems [5]. Argumentation has applications in several contexts, including modeling dialogues, negotiation [6, 7], and persuasion [8]. It has been widely used to model agents' interactions [9, 10, 11, 12], especially in the context of debates [13, 14, 15].

Dung's Abstract Argumentation Framework (AAF) is a simple yet powerful formalism for modeling disputes between two or more agents [16]. An AAF consists of a set of *arguments* and a binary *attack* relation over the set of arguments that specifies the interactions between arguments: intuitively, if argument a attacks argument b, then b is acceptable only if a is not. Hence, arguments are abstract entities whose status is entirely determined by the attack relation. An AAF can be seen as a directed graph, whose nodes represent arguments and edges represent attacks. Several argumentation semantics—e.g. *grounded* ($\mathtt{gr}$), *complete* ($\mathtt{co}$), *preferred* ($\mathtt{pr}$), and *stable* ($\mathtt{st}$) [16]—have been defined for AAF, leading to the characterization of σ*-extensions*, that intuitively consist of the sets of arguments that can be collectively accepted under semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}\}$.

Arg&App 2023: International Workshop on Argumentation and Applications, September 2023, Rhodes, Greece

g.alfano@dimes.unical.it (G. Alfano); greco@dimes.unical.it (S. Greco); fparisi@dimes.unical.it (F. Parisi); i.trubitsyna@dimes.unical.it (I. Trubitsyna)

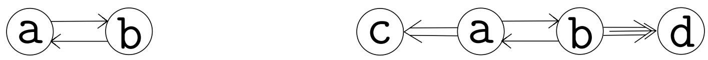

Figure 1: AAF Λ of Example 1 (left) and EAAF Δ of Example 2 (right).

Example 1. Consider an AAF Λ=$\langle\{\mathtt{a},\mathtt{b}\}, \{(\mathtt{a},\mathtt{b}), (\mathtt{b},\mathtt{a})\}\rangle$ whose corresponding graph is shown in Figure 1(left). Λ describes the following scenario. A party planner invites Alice (a) and Bob (b) to join a party. Due to their old rivalry (*i*) Alice replies that she will not join the party if Bob does, and (*ii*) Bob replies that he will not join the party if Alice does. This situation can be modeled by AAF Λ, where an argument x states that "*(the person whose initial is)* x *joins the party*". Under the stable semantics, there are two extensions $E_1 = \{\mathtt{a}\}$ and $E_2 = \{\mathtt{b}\}$ stating that only Alice or only Bob will attend the party, respectively. □

Thus, as prescribed by E_1 and E_2, in the previous example we have that the participation of Alice and Bob to the party is uncertain. To deal with uncertain information represented by the presence of multiple extensions, credulous and skeptical reasoning has been introduced. Specifically, an argument is credulously true (or accepted) if there exists an extension containing the argument, whereas an argument is skeptically true if it occurs in all extensions. However, uncertain information in AAF under multiple-status semantics proposed so far cannot be exploited to determine the status of arguments (which in turn influences the status of other arguments) by taking into account the information given by the whole set of extensions, as in the case of credulous and skeptical acceptance. To overcome such a situation, and thus provide a natural and compact way for expressing such kind of conditions, the use of *epistemic* arguments and attacks has been recently proposed in [1], leading to the definition of the so-called Epistemic Abstract Argumentation Framework (EAAF) which enhances AAF by allowing the representation of some pieces of epistemic knowledge. Informally, epistemic attacks allow considering all extensions and not only the current one. Thus, an epistemic attack from a to b is such that a defeats b if a occurs in at least one extension (*strong epistemic attack*) or in all extensions and at least one (*weak epistemic attack*), as illustrated in the following example.

Example 2. Consider the AAF Λ of Example 1 and assume that there are two more people: Carol (c) and David (d). Carol's answer is that she will not attend the party if it is sure (i.e. it is skeptically true) that Alice will, whereas David answers that he will not attend the party if the participation of Bob is possible (i.e. it is credulously true). Intuitively, the party planner should conclude that, as the participation of both Alice and Bob is uncertain, Carol will attend the party, whereas David will not.

This situation can be modeled by means of the Epistemic AAF (EAAF) shown in Figure 1(right) where a defeats c with a weak epistemic attack, whereas b defeats d with a strong epistemic attack (we use the two kinds of edges represented in the figure to denote weak and strong epistemic attacks). Under the stable semantics, there are two extensions: $E_1 = \{\mathtt{a},\mathtt{c}\}$ modeling the fact that Alice and Carol will attend the party, whereas Bob and David will not; and $E_2 = \{\mathtt{b},\mathtt{c}\}$ modeling the fact that Bob and Carol will attend the party, whereas Alice and David will not. Observe that the epistemic arguments c and d (i.e. the arguments defeated by an epistemic attack) are *deterministic* [17], that is, they have the same acceptance status in all extensions (true for c and false for d). □

Contributions. We introduce the stable semantics for Epistemic Abstract Argumentation Frameworks (EAAFs) and investigate the complexity of two fundamental problems (see below). The proposed EAAF semantics aims to let epistemic arguments be deterministic [17], that is, they have the same acceptance status in all extensions; the status of an argument depends on the credulous or skeptical acceptance of its attackers. Considering the dependence of the status of an argument on its attackers only is inspired by the well-known directionality property proposed for AAF [18, 19], which, if satisfied, then guarantees that the status of each argument depends only on that of its attackers. Specifically, our main contributions are as follows.

- We formally present EAAF stable semantics; it extends that of AAF and coincides with EAAF preferred semantics in case of odd-cycle free EAAFs (as it happens for the case of AAF).
- We investigate the complexity of the *acceptance* and *existence* problems under stable semantics Our complexity results are summarized in Table 2 (in Section 4).

2. Preliminaries

We first review the Dung's framework and then discuss and an extension of AAF with epistemic constraints.

2.1. Abstract Argumentation Framework

An *Abstract Argumentation Framework* (AAF) is a pair $\langle \mathrm{A}, \Omega \rangle$, where A is a (finite) set of *arguments* and $\Omega \subseteq \mathrm{A} \times \mathrm{A}$ is a set of *attacks* (also called *defeats*). Different argumentation semantics have been proposed for AAF, leading to the characterization of collectively acceptable sets of arguments called *extensions* [16].

Given an AAF $\Lambda = \langle \mathrm{A}, \Omega \rangle$ and a set $S \subseteq \mathrm{A}$ of arguments, an argument $a \in \mathrm{A}$ is said to be *i*) *defeated* w.r.t. S iff $\exists b \in S$ such that $(b, a) \in \Omega$; *ii*) *acceptable* w.r.t. S iff $\forall b \in \mathrm{A}$ with $(b, a) \in \Omega$, $\exists c \in S$ such that $(c, b) \in \Omega$. The sets of defeated and acceptable arguments w.r.t. S are defined as follows (where Λ is understood):

- $\mathit{Def}(S) = \{a \in \mathrm{A} \mid \exists b \in S \,.\, (b, a) \in \Omega\}$;
- $\mathit{Acc}(S) = \{a \in \mathrm{A} \mid \forall b \in \mathrm{A} \,.\, (b, a) \in \Omega \text{ implies } b \in \mathit{Def}(S)\}$.

To simplify the notation, we will often use S^+ to denote $\mathit{Def}(S)$.

Given an AAF $\langle \mathrm{A}, \Omega \rangle$, a set $S \subseteq \mathrm{A}$ of arguments is said to be:

- *conflict-free* iff $S \cap S^+ = \emptyset$;
- *admissible* iff it is conflict-free and $S \subseteq Acc(S)$.

Given an AAF $\langle \mathrm{A}, \Omega \rangle$, a set $S \subseteq \mathrm{A}$ is an *extension* called:

- *complete* (co) iff it is conflict-free and $S = Acc(S)$;
- *preferred* (pr) iff it is a $\subseteq$-maximal complete extension;
- *stable* (st) iff it is a total complete extension, i.e. a complete extension such that $S \cup S^+ = \mathrm{A}$;
- *grounded* (gr) iff it is the $\subseteq$-smallest complete extension.

Figure 2: AAF Λ of Example 3 (left) and AAF Λ' of Example 4 (right).

The set of complete (resp. preferred, stable, grounded) extensions of an AAF Λ will be denoted by $\texttt{co}(\Lambda)$ (resp. $\texttt{pr}(\Lambda)$, $\texttt{st}(\Lambda)$, $\texttt{gr}(\Lambda)$). It is well-known that the set of complete extensions forms a complete semilattice w.r.t. $\subseteq$, where $\texttt{gr}(\Lambda)$ is the meet element, whereas the greatest elements are the preferred extensions. All the above-mentioned semantics except the stable admit at least one extension. The grounded semantics, that admits exactly one extension, is said to be a *unique-status* semantics, while the others are said to be *multiple-status* semantics. With a little abuse of notation, in the following we also use $\texttt{gr}(\Lambda)$ to denote the grounded extension. For any AAF Λ, $\texttt{st}(\Lambda) \subseteq \texttt{pr}(\Lambda) \subseteq \texttt{co}(\Lambda)$ and $\texttt{gr}(\Lambda) \in \texttt{co}(\Lambda)$.

Example 3. Let $\Lambda = \langle \mathrm{A}, \Omega \rangle$ be an AAF where $\mathrm{A} = \{\texttt{a}, \texttt{b}, \texttt{c}\}$ and $\Omega = \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a}), (\texttt{b}, \texttt{c})\}$, whose graph is show in Figure 2 (left). The set of complete extensions of Λ is $\texttt{co}(\Lambda) = \{E_0 = \emptyset, E_1 = \{\texttt{a}, \texttt{c}\}, E_2 = \{\texttt{b}\}\}$. E_0 is the grounded extension, while E_1 and E_2 are preferred and stable extensions. □

Given an AAF $\Lambda = \langle \mathrm{A}, \Omega \rangle$ and a semantics $\sigma \in \{\texttt{gr}, \texttt{co}, \texttt{pr}, \texttt{st}\}$, for $g \in \mathrm{A}$, the *credulous* (resp. *skeptical*) *acceptance* problem, denoted as CA_σ (resp. SA_σ) is deciding whether g is credulously (resp. skeptically) accepted, that is deciding whether g belongs to *any* (resp. *every*) σ-extension of Λ. Clearly, $CA_{\texttt{gr}}$ and $SA_{\texttt{gr}}$ coincide.

Recently, a satisfaction problem for AAF called *determinism* (DS_σ) has been introduced [17]. Given a σ-extension E, an argument $g \in \mathrm{A}$ is said to be: *accepted* if $g \in E$; *rejected* if $g \in E$; *undecided* otherwise ($g \notin E \cup E^+$). For a semantic σ, an argument is said to be deterministic if all σ-extensions assign the same status (either accepted, rejected, or undecided) to it.

Finally, the *existence* (resp. *non-empty existence*) problem denoted as Ex_σ (resp. $Ex_\sigma^{\neg\emptyset}$) is deciding whether there exists at least one (resp. at least one non-empty) σ-extension for AAF Λ.

For AAFs, the complexity of the existence and acceptance problems has been investigated (see [20] for an overview). The complexity of the determinism problem is investigated in [17]. The complexity results of these problems are summarized in the left-hand side part of Table 2.

Example 4. Consider the AAF Λ of Example 3. Under preferred and stable semantics, both arguments $\texttt{a}$ and $\texttt{b}$ are credulously accepted. None of them is skeptically accepted, nor deterministic.

Considering the AAF Λ' obtained from Λ by adding the self-attack $(\texttt{c}, \texttt{c})$ (see Figure 2 (right)), there are three complete extensions $E'_0 = \emptyset$, $E'_1 = \{\texttt{a}\}$ and $E'_2 = \{\texttt{b}\}$. Both E'_1 and E'_2 are preferred extensions, but only E'_2 is stable. □

2.2. AAF with Epistemic Constraints

An Epistemic Argumentation Framework (EAF) has been proposed in [21]. An EAF is a triple $\langle A, \Omega, C \rangle$, where $\langle A, \Omega \rangle$ is an AAF and C is an epistemic constraint, that is, a propositional formula extended with the modal operators $\mathbf{K}$ and $\mathbf{M}$. Here, the constraint is the belief of an agent which must be satisfied. Intuitively, $\mathbf{K}\phi$ (resp. $\mathbf{M}\phi$) states that the considered agent believes that ϕ is always (resp. possibly) true. EAF semantics is given by sets of feasible extensions of the underlying AAF, called ω-extension sets (ω-labeling sets in [21, 1]), consisting of maximal sets of arguments that satisfies the constraint. There could be different ω-extension sets (ω-sets) for the same epistemic formula, as shown in the following example.

Example 5. Consider the AAF $\Lambda = \langle A = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \Omega = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{c}, \mathtt{d}), (\mathtt{d}, \mathtt{c}), (\mathtt{b}, \mathtt{c})\rangle$ having 5 complete extensions $E_0 = \emptyset$, $E_1 = \{\mathtt{a}\}$, $E_2 = \{\mathtt{a}, \mathtt{c}\}$, $E_3 = \{\mathtt{a}, \mathtt{d}\}$ and $E_4 = \{\mathtt{b}, \mathtt{d}\}$. E_0 is the grounded extension, while E_2, E_3 and E_4 are preferred and stable extensions. Under the preferred semantics, considering the epistemic constraint $C_1 = \mathbf{K}\mathtt{c}$, there exists a unique ω-set $\{E_2\}$ for EAF $\langle A, \Omega, C_1\rangle$, whereas considering $C_2 = \mathbf{K}\mathtt{c} \vee \mathbf{K}\mathtt{d}$ there are the two alternative ω-sets $\{E_2\}$ and $\{E_3, E_4\}$ for EAF $\langle A, \Omega, C_2\rangle$. □

We point out that despite the name Epistemic Argumentation Framework is used, the role of epistemic formulae is only that of introducing constraints over the set of feasible extensions, that is it is similar to that of constraints or preferences in AAF [22, 23, 24, 25, 26].

3. Epistemic Abstract Argumentation Framework

We augment AAF with epistemic attacks, leading to the concept of Epistemic Abstract Argumentation Framework (EAAF).

3.1. Syntax

We start by recalling the syntax of EAAF [1].

Definition 1 (Epistemic AAF)**.** *An Epistemic AAF is a quadruple $\Delta = \langle A, \Omega, \Psi, \Phi\rangle$ where A is a set of arguments, $\Omega \subseteq A \times A$ is a set of (standard) attacks, $\Psi \subseteq A \times A$ is a set of weak (epistemic) attacks, and $\Phi \subseteq A \times A$ is a set of strong (epistemic) attacks such that $\Omega \cap \Psi = \Omega \cap \Phi = \Psi \cap \Phi = \emptyset$.*

In the following, we represent attacks $(a, b) \in \Omega$ by $a \rightarrow b$, $(a, b) \in \Psi$ by $a \Rightarrow b$, $(a, b) \in \Phi$ by $a \Rrightarrow b$. An EAAF $\langle A, \Omega, \Psi, \Phi\rangle$ can be seen as a directed graph, where A denotes the set of nodes and Ω, Ψ, and Φ denotes three different kinds of edges. Arguments defeated through epistemic attacks are called *epistemic arguments*.

We say that there is a path from an argument $a \in A$ to argument $b \in A$ if either (*i*) there exists an attack (a, b) in Δ or (*ii*) there exists an argument $c \in A$ and two paths, from a to c and from c to b. We say that an argument $b \in A$ *depends* on an argument $a \in A$ if b is *reachable* from a in Δ, that is, if there exists a path from a to b in Δ. Moreover, an argument a depends on attack $\gamma \in (\Omega \cup \Psi \cup \Phi)$ if there exists a path in Δ that contains γ and reaches a.

We now introduce well-formed and plain EAAFs.

Definition 2. *An EAAF Δ is said to be:*

- *well-formed if there are no cycles in Δ with epistemic edges.*
- *in plain form if every epistemic argument is attacked by a single (epistemic) attack.*

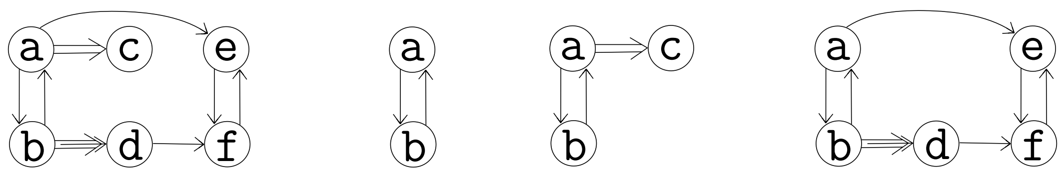

Figure 3: (From left to right) EAAFs Δ, Δ', Δ'' and Δ''' of Example 9.

In the following we assume that our EAAFs are well-formed. The reason for such a restriction is to guarantee that there exists at most one *world view* (c.f. Theorem 1). In the following *we also assume that our EAAFs are in plain form.* As it will be clear after introducing EAAF semantics, for well-formed EAAFs in plain form, epistemic arguments are deterministic (c.f. Proposition 2).

Example 6. The EAAF $\Delta = \langle A = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \Omega = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a})\}, \Psi = \{(\mathtt{a}, \mathtt{c})\}, \Phi = \{(\mathtt{b}, \mathtt{d}\}\rangle$ of Example 2, whose graph is shown in Figure 1 (right), is well-formed and in plain form. $\square$

The semantics of EAAF is given by relying on the concept of sub-framework (sub-EAAF), which is defined as follows.

Definition 3. *Given two EAAFs Δ and Δ', we say that Δ' is a sub-EAAF of Δ (denoted as $\Delta' \sqsubseteq \Delta$) if Δ' is obtained from Δ by deleting a subset S of the set of epistemic arguments of Δ and all the arguments depending on an argument in S w.r.t. Δ. Moreover, we write $\Delta' \sqsubset \Delta$ if $\Delta' \sqsubseteq \Delta$ and $\Delta' \neq \Delta$.*

Clearly, in Definition 3 by deleting arguments we also delete attacks having as a source or target element a deleted argument.

Example 7. Consider the EAAF $\Delta = \langle\{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{e}), (\mathtt{d}, \mathtt{f}), (\mathtt{e}, \mathtt{f}), (\mathtt{f}, \mathtt{e})\}, \{(\mathtt{a}, \mathtt{c})\}, \{(\mathtt{b}, \mathtt{d})\}\rangle$ shown in Figure 3 (left). We have four sub-EAAFs $\Delta^* \sqsubseteq \Delta$, as shown in the figure: the first one (from left to right) coincides with Δ, the others are obtained by deleting all arguments depending on: (i) both arguments $\mathtt{c}$ and $\mathtt{d}$, (ii) only $\mathtt{d}$, and (iii) only $\mathtt{c}$, respectively. $\square$

3.2. Semantics

We first introduce the stable semantics of EAAF and then present some results concerning properties of the proposed framework.

For any EAAF $\Delta = \langle A, \Omega, \Psi, \Phi\rangle$, a set W of sets of arguments in A is called *world view* of Δ. Informally, a world view can be seen as a set of extensions that are to be used to compute the status of epistemic arguments. Given EAAF $\Delta' = \langle A', \Omega', \Psi', \Phi'\rangle \sqsubseteq \Delta$, we denote by $W_{\downarrow\Delta'} = \{S \cap A' \mid S \in W\}$ the projection of W over A'.

With the aim of providing EAAF semantics by extending AF semantics, we first extend the definitions of defeated and acceptable arguments for EAAF by taking into account the additional concept of world view, that is a candidate set of extensions, which is used to decide if an argument is epistemically defeated/acceptable. Given an EAAF Δ, a world view W of Δ, and a set $S \in W$, the sets of arguments defeated (resp. accepted) w.r.t. S and W are defined as follows:

- $Def(W, S) = \{b \in A \mid (\exists a \in S \,.\, a \rightarrow b) \vee$
  $(\exists T \in W \,.\, \exists a \in T \,.\, a \nRightarrow b) \vee$
  $(\forall T \in W \,.\, \exists a \in T \,.\, a \Rightarrow b)\}$.
- $Acc(W, S) = \{b \in A \mid \forall a \in A \,.$
  $((a \rightarrow b)$ implies $a \in Def(W, S)) \wedge$
  $((a \nRightarrow b)$ implies $\forall T \in W \,.\, a \in Def(W, T)) \wedge$
  $((a \Rightarrow b)$ implies $\exists T \in W \,.\, a \in Def(W, T))$.

Example 8. Considering the EAAF Δ of Example 7 and the world view $W = \{S_1 = \{\mathtt{c}\}$, $S_2 = \{\mathtt{a}, \mathtt{c}\}$, $S_3 = \{\mathtt{b}, \mathtt{c}\}\}$, we have that:

- $Def(W, S_1) = \{\mathtt{d}\}$ and $Acc(W, S_1) = \{\mathtt{c}\}$;
- $Def(W, S_2) = \{\mathtt{b}, \mathtt{d}\}$ and $Acc(W, S_2) = \{\mathtt{a}, \mathtt{c}\}$; and
- $Def(W, S_3) = \{\mathtt{a}, \mathtt{d}\}$ and $Acc(W, S_3) = \{\mathtt{b}, \mathtt{c}\}$. □

Given an EAAF $\Delta = \langle A, \Omega, \Psi, \Phi \rangle$ and a world view W of Δ, a set $S \in W$ is:

- *W-conflict-free* if $S \cap Def(W, S) = \emptyset$;
- *W-admissible* if it is W-conflict-free and $S \subseteq Acc(W, S)$;
- *W-complete* (W-co) if it is W-conflict-free and $S = Acc(W, S)$.

Moreover, a W-complete set S is said to be :

- *W-preferred* (W-pr) if S is $\subseteq$-maximal;
- *W-stable* (W-st) if $S \cup Def(W, S) = A$;
- *W-grounded* (W-gr) if S is $\subseteq$-minimal.

We are now ready to define EAAF semantics. The meaning of EAAF under the grounded, complete and preferred semantics has been introduced in [1]. For the sake of completeness and to easy readability we include those semantics in the next definition, where the meaning of EAAF under stable semantics is defined by generalizing the definition in [1].

Definition 4 (EAAF Semantics). *Let $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}\}$ be a semantics and W a world view of EAAF Δ. Then, W is a σ-world view for Δ if $\forall \Delta' \sqsubseteq \Delta$ the following conditions hold:*

(i) *every $S \in W_{\downarrow \Delta'}$ is a $W_{\downarrow \Delta'}$-σ set, and*

(ii) *there is no world view W^* for Δ' such that $W_{\downarrow \Delta'} \subset W^*$ and every $S^* \in W^*$ is W^*-σ for Δ'.*

Table 1
σ-world view for each EAAF $\Delta^* \sqsubseteq \Delta$ in Figure 3.

Δ^*	$\mathtt{gr}(\Delta^*)$	$\mathtt{co}(\Delta^*)$	$\mathtt{pr}(\Delta^*)=\mathtt{st}(\Delta^*)$
Δ'	$\{\emptyset\}$	$\{\emptyset, \{\mathtt{a}\}, \{\mathtt{b}\}\}$	$\{\{\mathtt{a}\}, \{\mathtt{b}\}\}$
Δ''	$\{\emptyset\}$	$\{\{\mathtt{c}\}, \{\mathtt{a},\mathtt{c}\}, \{\mathtt{b},\mathtt{c}\}\}$	$\{\{\mathtt{a},\mathtt{c}\}, \{\mathtt{b},\mathtt{c}\}\}$
Δ'''	$\{\emptyset\}$	$\{\emptyset, \{\mathtt{f}\}, \{\mathtt{a},\mathtt{f}\}, \{\mathtt{b}\}, \{\mathtt{b},\mathtt{e}\}, \{\mathtt{b},\mathtt{f}\}\}$	$\{\{\mathtt{a},\mathtt{f}\}, \{\mathtt{b},\mathtt{e}\}, \{\mathtt{b},\mathtt{f}\}\}$
Δ	$\{\emptyset\}$	$\{\{\mathtt{c}\}, \{\mathtt{c},\mathtt{f}\}, \{\mathtt{a},\mathtt{c},\mathtt{f}\}, \{\mathtt{b},\mathtt{c}\}, \{\mathtt{b},\mathtt{c},\mathtt{e}\}, \{\mathtt{b},\mathtt{c},\mathtt{f}\}\}$	$\{\{\mathtt{a},\mathtt{c},\mathtt{f}\}, \{\mathtt{b},\mathtt{c},\mathtt{e}\}, \{\mathtt{b},\mathtt{c},\mathtt{f}\}\}$

We now explain Definition 4. Given a semantics σ, a W-σ set intuitively represents a candidate set of σ-extensions for an EAAF. Then, such a set turns out to actually be a set of extensions if the conditions in Definition 4 hold, whose rationale is as follows. Given a world view W of an EAAF Δ, we check that for all sub-frameworks Δ', every element $S \in W' = W_{\downarrow\Delta'}$ is a W'-σ set (condition i) and W' is maximal (condition ii). Intuitively, the first condition ensures that the status of an argument is confirmed in all sub-frameworks considered. The second condition of Definition 4 ensures that, if there is a larger σ-world view for which condition i) holds, then we prefer to take it. That is, intuitively, we aim at having the whole set of extensions. In [1], it is shown that this set is unique under grounded, complete and preferred semantics. Finally, as shown below in Example 9, checking that the above-mentioned conditions hold for all sub-frameworks is important to avoid returning wrong conclusions (i.e., world views that contradict our intuition).

It is worth noting that whenever $\Psi = \Phi = \emptyset$, we have that the definitions of defeated and acceptable arguments coincide with the ones defined for AAF, that is $Def(\{S\}, S) = Def(S)$ and $Acc(\{S\}, S) = Acc(S)$. This lead to the following result that states that EAAF semantics extends that of AAF.

Proposition 1. *Let $\Delta = \langle A, \Omega, \Psi, \Phi\rangle$ be a well-formed EAAF with $\Psi = \Phi = \emptyset$, and $\Lambda = \langle A, \Omega\rangle$ the AAF corresponding to Δ. Then, if $\mathtt{st}(\Lambda) \neq \emptyset$ then $\mathtt{st}(\Lambda)$ is the only stable-world view of Δ.*

Clearly, as stable semantics is not guaranteed to exist in AAF, the same holds in EAAF. Indeed, as stated next, any well-formed EAAF has at most one stable world view.

Theorem 1. *Any well-formed EAAF admits at most one $\mathtt{st}$-world view.*

For any (well-formed) EAAF Δ and semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}\}$ we use $\sigma(\Delta)$ to denote the σ-world view of Δ, and will often call its elements σ-extensions.

Example 9. Continuing with Example 7, Table 1 reports the σ-world view for each EAAF $\Delta^* \sqsubseteq \Delta$ in Figure 3 and $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}\}$.

Now, consider the EAAF Δ'' (shown in Figure 3), the world view $W = \{S = \{\mathtt{a}\}\}$, and the stable semantics. If in Definition 4 we had only focused on the given EAAF Δ'' without looking at its sub-frameworks, as S is a W-stable set and W is maximal (i.e., both conditions (i) and (ii) of Definition 4 are satisfied if focusing on Δ'' only), we would have concluded that $\mathtt{c}$ is defeated. However, we had expected that $\mathtt{c}$ would have been accepted. Indeed, according to Definition 4, the only stable-world view of Δ'' is $W'' = \{\{\mathtt{a},\mathtt{c}\}, \{\mathtt{b},\mathtt{c}\}\}$ (cf. Table 1). In fact, considering the sub-framework Δ' (cf. Figure 3) obtained from Δ'' by deleting the epistemic argument $\mathtt{c}$,

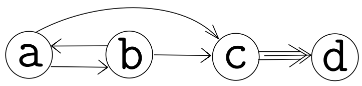

Figure 4: EAAF Δ of Example 10.

the only stable-world view of Δ' is $W' = W''_{\downarrow\Delta'} = \{\{\mathtt{a}\}, \{\mathtt{b}\}\}$, which using Definition 4 allows us to discard $W = \{\{\mathtt{a}\}\}$ from being a stable-world view of Δ''. □

According to the proposed EAAF semantics, epistemic arguments are deterministic, that is, they have the same "truth assignment" in a world view, that in turn depends on either the credulous or skeptical acceptance of its attackers.

Proposition 2. *Let $\Delta = \langle A, \Omega, \Psi, \Phi \rangle$ be an EAAF, and W the* st*-world view of Δ. Then, any epistemic argument $x \in A$ is deterministic, that is, one of the following three conditions hold:*
i) $\forall\, S \in W\,.\;\; x \in Acc(W, S)$;
ii) $\forall\, S \in W\,.\;\; x \in Def(W, S)$;
iii) $\forall\, S \in W\,.\;\; x \notin (Acc(W, S) \cup Def(W, S))$.

An alternative way to define stable extensions for EAAF could be that of choosing among complete extensions those that are total, as it is done for AAF. More in detail, given an EAAF Δ and its complete-world view $W = \mathtt{co}(\Delta)$, we could have defined the stable-world view for Δ as $\mathtt{st}(\Delta) = \{S \in W \mid S \cup Def(W, S) = A\}$). This is different from what is done in Definition 4 where to define a st-world view we start with a world view W that is not necessarily $\mathtt{co}(\Delta)$. However, the above-mentioned alternative way to define stable extensions for EAAF may lead to counter-intuitive solutions, as shown in the following example.

Example 10. Consider the EAAF $\Delta = \langle \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{c}), (\mathtt{b}, \mathtt{c})\}, \{(\mathtt{c}, \mathtt{d})\}, \emptyset \rangle$, shown in Figure 4, and the stable semantics. Intuitively, the strong epistemic attack states that d is accepted if c is skeptically rejected. The stable extensions of Δ, that is, the elements in its st-world view are $\{\mathtt{a}, \mathtt{d}\}$ and $\{\mathtt{b}, \mathtt{d}\}$. Thus, we obtain that c is skeptically defeated and, consequently, d is accepted.

However, if we start with the complete-world view $\mathtt{co}(\Delta)$, we have that there are three complete extensions $S_1 = \emptyset$, $S_2 = \{\mathtt{a}\}$ (with b and c defeated and d undecided) and $S_3 = \{\mathtt{b}\}$ (with a and c defeated and d undecided). As there are no total sets in $\mathtt{co}(\Delta)$, we conclude that under the above-mentioned "alternative" stable semantics there is no stable status for d and c , contradicting our intuition. □

As stated next, differently from AAF, stable extensions are not guaranteed to be complete extensions of EAAF. Related to this, even in AAF credulous and skeptical acceptance may give different results under different semantics.

Proposition 3. *There exists an EAAF Δ such that $S \in \mathtt{st}(\Delta)$ and $S \notin \mathtt{co}(\Delta)$.*

Particularly, consider the EAAF $\Delta = \langle \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}\}, \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{a}, \mathtt{c}), (\mathtt{b}, \mathtt{c}), (\mathtt{c}, \mathtt{d})\}, \{(\mathtt{d}, \mathtt{e})\ (\mathtt{e}, \mathtt{f})\}, \emptyset \rangle$. With a little effort, it ca be checked that $\mathtt{st}(\Delta) = \{S_1 = \{\mathtt{a}, \mathtt{d}, \mathtt{f}\}, S_2 = \{\mathtt{b}, \mathtt{d}, \mathtt{f}\}\}$ and $\mathtt{co}(\Delta) = \{\emptyset, \{\mathtt{a}, \mathtt{d}\}, \{\mathtt{b}, \mathtt{d}\}\}$, and thus neither $S_1 \in \mathtt{co}(\Delta)$ nor $S_2 \in \mathtt{co}(\Delta)$.

Finally, stable semantics coincides with preferred semantics in case of odd-cycle free EAAFs.

Proposition 4. *Let Δ be a well-formed, odd-cycle free EAAF. Then, it holds that* $\mathtt{st}(\Delta) = \mathtt{pr}(\Delta)$.

4. Complexity

We investigate the complexity of two fundamental reasoning problems for EAAF under stable semantics. In particular, we study the *existence* and *credulous/skeptical* acceptance problems, that are often considered for analyzing the complexity of argumentation frameworks.

We recall the main complexity classes used in this section and, in particular, the definition of the classes P, Σ_h^p, Π_h^p and Δ_h^p, with $h \geq 0$ (see e.g. [27]). For $h > 0$: $\Sigma_0^p = \Pi_0^p = \Delta_0^p = P$; $\Sigma_1^p = NP$ and $\Pi_1^p = coNP$; $\Delta_h^p = P^{\Sigma_{h-1}^p}$; $\Sigma_h^p = NP^{\Sigma_{h-1}^p}$, and $\Pi_h^p = co\Sigma_h^p$. Herein, P^C (resp. NP^C) denotes the class of problems that can be solved in polynomial time using an oracle in the class C by a deterministic (resp. non-deterministic) Turing machine. The class $\Theta_h^p = \Delta_h^p[log\ n]$ denotes the subclass of Δ_h^p consisting of the problems that can be solved in polynomial time by a deterministic Turing machine performing $O(log\ n)$ calls to an oracle in the class Σ_{h-1}^p. Under the standard complexity-theoretic assumptions, we have that $\Sigma_h^p \subset \Theta_{h+1}^p \subset \Delta_{h+1}^p \subset \Sigma_{h+1}^p \subseteq PSPACE$ and $\Pi_h^p \subset \Theta_{h+1}^p \subset \Delta_{h+1}^p \subset \Pi_{h+1}^p \subseteq PSPACE$. A decision problem is in D_h^p iff it is the conjunction of a decision problem in Σ_h^p and a decision problem in Π_h^p. Hence, D_1^p (or simply *DP*) denotes the class of the problems that are a conjunction of a problem in NP and one in $coNP$. Under the standard complexity-theoretic assumptions, we have that $NP \subset$ *DP*, $coNP \subset$ *DP*, and *DP*$\subset \Theta_2^p$.

Given an EAAF $\Delta = \langle A, \Omega, \Psi, \Phi \rangle$ and a semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{st}\}$:

- the *existence* (resp. *non-empty existence*) problem for EAAF, denoted as Ex_σ (resp. $Ex_\sigma^{\neg\emptyset}$) consists in deciding whether there exists at least one (resp. at least one non-empty) σ-extension S for Δ;
- the *credulous* (resp. *skeptical*) acceptance problem, denoted as CA_σ (resp. SA_σ), consists in deciding whether a given goal argument $g \in A$ belongs to any (resp. every) σ-extension of Δ.

Observe that if argument g is epistemic, credulous and skeptical acceptance problems coincide (cf. Proposition 2). Therefore, we call this problem *epistemic acceptance* and denote it as EA_σ.

The following fact states that the epistemic acceptance problem captures the credulous and skeptical acceptance problems also for non-epistemic arguments under stable semantics.

Fact 1. *Let $\Delta = \langle A, \Omega, \Psi, \Phi \rangle$ be an EAAF, $g \in A$ any of its non-epistemic arguments. Then:*

- $CA_{\mathtt{st}}(\Delta, g) = EA_{\mathtt{st}}(\Delta', g'')$ *with* $\Delta' = \langle A \cup \{g', g''\}, \Omega \cup \{(g, g')\}, \Psi \cup \{(g', g'')\}, \Phi \rangle$
- $SA_{\mathtt{st}}(\Delta, g) = EA_{\mathtt{st}}(\Delta', g'')$ *with* $\Delta' = \langle A \cup \{g', g''\}, \Omega \cup \{(g, g')\}, \Psi, \Phi \cup \{(g', g'')\} \rangle$.

Thus, asking for the credulous and skeptical acceptance of an argument g w.r.t. an EAAF Δ is equivalent to asking for the epistemic acceptance of a fresh epistemic argument g'' w.r.t. an EAAF Δ', that is obtained from Δ by adding only a pair of attacks.

For this reason and for the fact that epistemic arguments are deterministic (Proposition 2), w.l.o.g. we study the complexity of existence and epistemic acceptance problems in EAAFs (without considering credulous and skeptical acceptance that, as shown above, can be immediately reduced to epistemic acceptance).

Table 2
Complexity of the credulous acceptance (CA_σ), skeptical acceptance (SA_σ), existence (Ex_σ), non-empty existence ($Ex_\sigma^{\neg\emptyset}$), and determinism problems for AAF, and of the epistemic acceptance (EA_σ), existence (Ex_σ), and non-empty existence ($Ex_\sigma^{\neg\emptyset}$) problems for EAAF. For any complexity class C, C-c (resp. C-h) means C-complete (resp. C-hard); an interval C-h, C' means C-hard and in C'. The results for $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}\}$ have been presented in [1], while those for $\mathtt{st}$ are new.

	AAF					EAAF		
σ	CA_σ	SA_σ	Ex_σ	$Ex_\sigma^{\neg\emptyset}$	DS_σ	EA_σ	Ex_σ	$Ex_\sigma^{\neg\emptyset}$
$\mathtt{gr}$	P	P	trivial	P	trivial	P	trivial	P
$\mathtt{co}$	NP-c	P	trivial	NP-c	coNP-c	Θ_2^p-h, Δ_2^P	trivial	NP-c
$\mathtt{st}$	NP-c	coNP-c	NP-c	NP-c	DP-c	DP-h	NP-h	NP-h
$\mathtt{pr}$	NP-c	Π_2^P-c	trivial	NP-c	Π_2^P-c	Π_2^P-h, Δ_3^P	trivial	NP-c

The next theorem states the complexity of epistemic acceptance under stable semantics.

Theorem 2. *$EA_{\mathtt{st}}$ is DP-hard.*

The following corollary states that for EAAF the existence of at least one extension is not always guaranteed, as for the case of AAF.

Corollary 1. *$Ex_{\mathtt{st}}$ coincides with $Ex_{\mathtt{st}}^{\neg\emptyset}$ and it is NP-hard.*

The results of this section, along with some related complexity results for AAF, are summarized in Table 2. We have also reported the results for $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}\}$ which are from [1]; those for $\mathtt{st}$ are new. We found that the complexity generally increases w.r.t. that of AAF for the acceptance problems under stable semantics. This particularly holds if we compare the complexity of $EA_{\mathtt{st}}$ for EAAF with that of $CA_{\mathtt{st}}$ and $SA_{\mathtt{st}}$ for AAF . Finally, deciding acceptance (resp. existence) in EAAF under stable semantics is at least hard as checking determinism (resp. existence) in AAF. For future work we plan to close the complexity gap related to the complexity of acceptance problems in EAAF under the different semantics.

5. Conclusion and Future Work

Several proposals have been made to extend Dung's framework with the aim of better modeling the knowledge to be represented. The extensions include Bipolar AAF [28, 29], AAF with recursive attacks and supports [30, 31, 32], Dialectical framework [33], Abstract Reasoning Framework [34], AAF with preferences [35, 36] and constraints [22, 23], as well extensions for representing uncertain information, e.g. incomplete AAF [37] and probabilistic AAF [38, 39, 40, 41, 42, 43, 44].

We have presented the stable semantics for Epistemic Abstract Argumentation Framework, a generalization of Dung's framework where epistemic attacks and arguments can be expressed. We also provided complexity bounds for the existence and acceptance problems in EAAF under the well-known stable argumentation semantics. Our complexity analysis shows that the epistemic elements (i.e., epistemic attacks/arguments) impact on the complexity of some of the problems considered. In general, it turns out that EAAF is more expressive than AAF.

The idea of extending logic with epistemic constructs has been investigated also in the field of Answer Set Programming (ASP) [45, 46, 47]. Epistemic logic programs, firstly proposed in [45], extend disjunctive logic programs under the stable model semantics with modal constructs called subjective literals [46, 48, 49, 47]. The introduction of this extension was originally motivated to correctly represent incomplete information in programs that have several stable models. Using subjective literals, it is possible to check whether a literal is true in every or some stable model of the program. These models in this context are also called belief sets, being collected in a set called world view. The main idea was to expand the syntax and semantics of Answer Set Programming by modal operators **K** and **M** where $\mathbf{K}\varphi$ holds if φ is true in all answer sets of a program and $\mathbf{M}\varphi$ holds if φ is true in at least one answer set. Using this notation, $not\ \mathbf{K}p \wedge not\ \mathbf{K} -p$ would correspond to "the truth value of p is unknown" even in the presence of multiple answer sets. In such a context, several problems are still open and they regard the support required by stable models, as well as splitting properties that are satisfied by classical ASP semantics, but not satisfied by epistemic ASP-based semantics [50, 49, 51].

Although our focus is on argumentation, we believe that our results could be of interest to the logic community. In fact, by exploiting the correspondence between AF and Logic Programming [52], the proposed EAAF semantics could be seen as an alternative semantics for a special class of epistemic logic programs whose complexity and computation can be characterized by using our results.

Future work will be devoted to considering other argumentation semantics such as the semi-stable semantics. Another interesting direction for future work is exploring EAF in a dynamic setting [53, 54, 55, 56, 57, 58, 59], where objective evidence (underlying AF) and subjective beliefs (epistemic formulae) may change over time.

Acknowledgments

We acknowledge the support of the PNRR project FAIR - Future AI Research (PE00000013), Spoke 9 - Green-aware AI, under the NRRP MUR program funded by the NextGenerationEU. This work was also funded by the Next Generation EU - Italian NRRP, Mission 4, Component 2, Investment 1.5, call for the creation and strengthening of 'Innovation Ecosystems', building 'Territorial R&D Leaders' (Directorial Decree n. 2021/3277) - project Tech4You - Technologies for climate change adaptation and quality of life improvement, n. ECS0000009. This work reflects only the authors' views and opinions, neither the Ministry for University and Research nor the European Commission can be considered responsible for them.

References

[1] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Epistemic abstract argumentation framework: Formal foundations, computation and complexity, in: Proc. of AAMAS, 2023, pp. 409–417.

[2] T. Bench-Capon, P. E. Dunne, Argumentation in artificial intelligence, Artif. Intell. 171 (2007) 619 – 641.

[3] G. R. Simari, I. Rahwan (Eds.), Argumentation in Artificial Intelligence, Springer, 2009.
[4] K. Atkinson, P. Baroni, M. Giacomin, A. Hunter, H. Prakken, C. Reed, G. R. Simari, M. Thimm, S. Villata, Towards artificial argumentation, Artificial Intelligence Magazine 38 (2017) 25–36.
[5] Y. Shoham, K. Leyton-Brown, Multiagent Systems - Algorithmic, Game-Theoretic, and Logical Foundations, Cambridge University Press, 2009.
[6] L. Amgoud, Y. Dimopoulos, P. Moraitis, A unified and general framework for argumentation-based negotiation, in: Proc. of AAMAS, 2007, p. 158.
[7] Y. Dimopoulos, J. Mailly, P. Moraitis, Argumentation-based negotiation with incomplete opponent profiles, in: Proc. AAMAS, 2019, pp. 1252–1260.
[8] H. Prakken, Models of persuasion dialogue, in: Argumentation in Artificial Intelligence, 2009, pp. 281–300.
[9] L. Amgoud, M. Serrurier, Agents that argue and explain classifications, Auton. Agents Multi Agent Syst. 16 (2008) 187–209.
[10] Á. Carrera, C. A. Iglesias, A systematic review of argumentation techniques for multi-agent systems research, Artif. Intell. Rev. 44 (2015) 509–535.
[11] S. Ontañón, E. Plaza, An argumentation-based framework for deliberation in multi-agent systems, in: Int. Workshop, ArgMAS, volume 4946, 2007, pp. 178–196.
[12] S. Parsons, C. Sierra, N. R. Jennings, Agents that reason and negotiate by arguing, J. Log. Comput. 8 (1998) 261–292.
[13] X. Fan, F. Toni, Agent strategies for aba-based information-seeking and inquiry dialogues, in: Proc. of ECAI, volume 242, 2012, pp. 324–329.
[14] P. McBurney, S. Parsons, Dialogue games for agent argumentation, in: Argumentation in Artificial Intelligence, Springer, 2009, pp. 261–280.
[15] H. Prakken, G. Sartor, Modelling reasoning with precedents in a formal dialogue game, Artif. Intell. Law 6 (1998) 231–287.
[16] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, Artif. Intell. 77 (1995) 321–358.
[17] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Incomplete argumentation frameworks: Properties and complexity, in: Proc. of AAAI, 2022, pp. 5451–5460.
[18] P. Baroni, M. Giacomin, G. Guida, Scc-recursiveness: a general schema for argumentation semantics, Artificial Intelligence 168 (2005) 162–210.
[19] P. Baroni, M. Giacomin, On principle-based evaluation of extension-based argumentation semantics, Artif. Intell. 171 (2007) 675–700.
[20] W. Dvorák, P. E. Dunne, Computational problems in formal argumentation and their complexity, FLAP 4 (2017).
[21] C. Sakama, T. C. Son, Epistemic argumentation framework: Theory and computation, J. Artif. Intell. Res. 69 (2020) 1103–1126.
[22] S. Coste-Marquis, C. Devred, P. Marquis, Constrained argumentation frameworks, in: Proc. of (KR), 2006, pp. 112–122.
[23] O. Arieli, Conflict-free and conflict-tolerant semantics for constrained argumentation frameworks, J. Appl. Log. 13 (2015) 582–604.
[24] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Abstract argumentation framework with conditional preferences, in: Proc. of AAAI, 2023, pp. 6218–6227.

[25] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Preferences and constraints in abstract argumentation, in: Proceedings of International Joint Conference on Artificial Intelligence (IJCAI), 2023.
[26] M. Bernreiter, W. Dvorák, S. Woltran, Abstract argumentation with conditional preferences, in: Proc. of COMMA, 2022, pp. 92–103.
[27] C. H. Papadimitriou, Computational complexity, Addison-Wesley, 1994.
[28] F. Nouioua, Afs with necessities: Further semantics and labelling characterization, in: Proc. of SUM, 2013, pp. 120–133.
[29] S. Villata, G. Boella, D. M. Gabbay, L. W. N. van der Torre, Modelling defeasible and prioritized support in bipolar argumentation, Ann. Math. Artif. Intell. 66 (2012).
[30] A. Cohen, S. Gottifredi, A. J. Garcia, G. R. Simari, An approach to abstract argumentation with recursive attack and support, J. Appl. Log. 13 (2015) 509–533.
[31] S. Gottifredi, A. Cohen, A. J. Garcia, G. R. Simari, Characterizing acceptability semantics of argumentation frameworks with recursive attack and support relations, Artif. Intell. 262 (2018) 336–368.
[32] C. Cayrol, J. Fandinno, L. F. del Cerro, M. Lagasquie-Schiex, Structure-based semantics of argumentation frameworks with higher-order attacks and supports, in: Proc. of COMMA, 2018, pp. 29–36.
[33] G. Brewka, H. Strass, S. Ellmauthaler, J. P. Wallner, S. Woltran, Abstract dialectical frameworks revisited, in: Proc. of IJCAI, 2013, pp. 803–809.
[34] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On acceptance conditions in abstract argumentation frameworks, Information Sciences 625 (2023) 757–779.
[35] L. Amgoud, C. Cayrol, Inferring from inconsistency in preference-based argumentation frameworks, J. Autom. Reason. 29 (2002) 125–169.
[36] S. Modgil, H. Prakken, A general account of argumentation with preferences, Artif. Intell. 195 (2013) 361–397.
[37] D. Baumeister, M. Järvisalo, D. Neugebauer, A. Niskanen, J. Rothe, Acceptance in incomplete argumentation frameworks, Artif. Intell. (2021) 103470.
[38] P. M. Dung, P. M. Thang, Towards (probabilistic) argumentation for jury-based dispute resolution, in: Proc. of COMMA, 2010, pp. 171–182.
[39] H. Li, N. Oren, T. J. Norman, Probabilistic argumentation frameworks, in: Proc. of TAFA, 2011, pp. 1–16.
[40] A. Hunter, Some foundations for probabilistic abstract argumentation, in: Proc. of COMMA, 2012, pp. 117–128.
[41] G. Alfano, M. Calautti, S. Greco, F. Parisi, I. Trubitsyna, Explainable acceptance in probabilistic abstract argumentation: Complexity and approximation, in: Proc. of KR, 2020, pp. 33–43.
[42] B. Fazzinga, S. Flesca, F. Furfaro, Complexity of fundamental problems in probabilistic abstract argumentation: Beyond independence (extended abstract), in: Proc. of IJCAI, 2019, pp. 6362–6366.
[43] B. Fazzinga, S. Flesca, F. Furfaro, Complexity of fundamental problems in probabilistic abstract argumentation: Beyond independence, Artif. Intell. 268 (2019) 1–29.
[44] G. Alfano, M. Calautti, S. Greco, F. Parisi, I. Trubitsyna, Explainable acceptance in probabilistic and incomplete abstract argumentation frameworks, Artificial Intelligence (2023)

103967.
[45] M. Gelfond, Strong introspection, in: T. L. Dean, K. R. McKeown (Eds.), Proc AAAI Conf., 1991, pp. 386–391.
[46] M. Gelfond, New semantics for epistemic specifications, in: Proc. of LPNMR Conf., volume 6645, 2011, pp. 260–265.
[47] J. Fandinno, W. Faber, M. Gelfond, Thirty years of epistemic specifications, Theory Pract. Log. Program. 22 (2022) 1043–1083.
[48] P. Cabalar, J. Fandinno, L. F. del Cerro, Autoepistemic answer set programming, Artif. Intell. 289 (2020) 103382.
[49] P. Cabalar, J. Fandinno, L. F. del Cerro, Splitting epistemic logic programs, Theory Pract. Log. Program. 21 (2021) 296–316.
[50] Y. Shen, T. Eiter, Considering constraint monotonicity and foundedness in answer set programming, in: L. D. Raedt (Ed.), Proc. of IJCAI, 2022, pp. 2741–2747.
[51] A. Herzig, A. Yuste-Ginel, On the epistemic logic of incomplete argumentation frameworks, in: Proc. of KR, 2021, pp. 681–685.
[52] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On the semantics of abstract argumentation frameworks: A logic programming approach, Theory Pract. Log. Program. 20 (2020) 703–718.
[53] G. Alfano, S. Greco, F. Parisi, Computing stable and preferred extensions of dynamic bipolar argumentation frameworks, in: Proc. of the 1st Workshop on Advances In Argumentation In Artificial Intelligence AI^3, 2017, pp. 28–42.
[54] G. Alfano, S. Greco, F. Parisi, G. I. Simari, G. R. Simari, Incremental computation for structured argumentation over dynamic delp knowledge bases, Artif. Intell. 300 (2021) 103553.
[55] G. Alfano, S. Greco, F. Parisi, Computing extensions of dynamic abstract argumentation frameworks with second-order attacks, in: Proc. of the 22nd International Database Engineering & Applications Symposium (IDEAS), 2018, pp. 183–192.
[56] G. Alfano, S. Greco, F. Parisi, On scaling the enumeration of the preferred extensions of abstract argumentation frameworks, in: Proceedings of ACM/SIGAPP Symposium on Applied Computing (SAC), 2019, pp. 1147–1153.
[57] G. Alfano, S. Greco, F. Parisi, Incremental computation in dynamic argumentation frameworks, IEEE Intell. Syst. 36 (2021) 80–86.
[58] A. Niskanen, M. Järvisalo, μ-toksia: An efficient abstract argumentation reasoner, in: Proc. ofKR, 2020, pp. 800–804.
[59] G. Alfano, S. Greco, Incremental skeptical preferred acceptance in dynamic argumentation frameworks, IEEE Intell. Syst. 36 (2021) 6–12.