A Case Study Using Large Language Models to Generate Metadata for Math Questions

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Abstract

Creating labels for assessment items, such as concept used, difficulty, or vocabulary used, can improve the quality and depth of research insights as well as targeting the right kinds of questions for students depending on their needs. However, traditional processes for metadata tagging are resource intensive in terms of labor, time, and cost, and these metadata become quickly outdated with any changes to the question content. Given thoughtful prompts, Large Language Models (LLMs) like GPT-3.5 and 4 can efficiently automate generation of assessment metadata and can help scale the process for larger volumes of questions as well as address any updates to question content that would otherwise have been tedious to reanalyze. With a human subject matter expert in-the-loop, recall and precision were analyzed for LLM generated tags for two metadata variables: problem context and math vocabulary. We conclude that LLMs like GPT-3.5 and 4 are highly reliable at generating assessment metadata, and make actionable recommendations for others intending to apply the technology to their own assessment items.

Keywords

Large Language Models, Assessments, Metadata, Human-in-the-loop

1. Introduction

Learning Sciences research often requires extensive metadata for assessment items in order to make meaningful insights about student learning. For example, if we know what concept each question targets, we can track a student's competency in that concept over the course of the school year [1]. We aimed to examine the role reading comprehension plays in math achievement [2,3], specifically in middle school algebra. This research direction required that we create a new suite of metadata for our assessment bank that gauged the various factors that might affect a student's reading comprehension. Some of these variables, such as word count, are easy to automate, but others, such as whether a question is set in a real world context like a baseball game, must be done manually. Manual metadata generation poses a number of logistical problems; it takes a lot of time and resources, it is tedious, and ones' efforts are quickly made irrelevant once any of the source content is updated or edited. Faced with multiple variables that required manual coding, we were motivated to find a way to automate as many as possible.

One such example is question context. Math problems that are contextualized in a real-world context require higher reading comprehension skills than math problems that are purely symbolic. A student who struggles with reading is much more likely to have their true understanding of the underlying math concepts obscured by their poor reading comprehension on a problem (See Table 1).

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A third category, labeled “School Math”, consists of symbolic math questions that are contextualized with a hypothetical student who is trying to solve the math problem. These require less reading comprehension than “Real-World” questions, but more reading comprehension than “Symbolic” questions. They can be used to assess metacognitive knowledge of math procedures or to assess a students’ ability to identify errors or misconceptions.

Table 1
Example math word problems set in real-world vs. symbolic contexts.

Real-world context	Symbolic	School Math
“These equations represent the number of bacteria in four different dishes as a function of time, t , in days. Which equation represents the population with the greatest growth factor?”	“What is the solution to 4(y - 3) + 19 = 8(2y + 3) + 7?”	-“Noah is solving an equation, and one of his moves is unacceptable. Here are the moves he made:...” -“What should Mai do next?”

The real-world context problems require the student to have familiarity with a greater number of vocabulary words, both math vocabulary like “function” and “equation”, but also real world vocabulary such as “bacteria”[4,5]. It also expects the student to be able to generate an accurate problem model of the relationships between the terms [6,7]. If a student struggles to read the problem or translate the problem into an accurate problem model [8], or if they are spending too much cognitive load deciphering the unfamiliar vocabulary [9], they may not be able to demonstrate their true understanding of the underlying concept: growth factors.

In order to research the role reading plays in math success, and in order to potentially intervene with appropriate support when students are struggling due to reading rather than computation, it is helpful to know whether a given question is contextualized in a real-world situation or if it is purely symbolic or in-between. Making such a judgment manually is relatively easy, but doing so for hundreds of questions takes time.

Luckily, LLMs like GPT-3.5 and GPT-4 are well-suited to making judgments like this. This paper documents the process we used to generate metadata using GPT-3.5 and GPT-4 for two variables: Context and Math Vocabulary. As part of our overall methodology, we include the prompts used, the errors generated, revisions to the prompts, our QA process, and accuracy, reliability, and precision analyses comparing the AI generated tags to those created by a human coder. We conclude with lessons learned and recommendations for others considering the use of LLMs to generate metadata for math content as well as other domains.

1.2 General Methods

We integrated OpenAI’s GPT-3.5 with Google Sheets using an extension available at GPTforWork.com [10]. In a column adjacent to the question text we queried using the formula =GPT(quesiton_text, “text of our request to GPT”). The formula could then be quickly repeated across rows for the whole column. Initial queries underwent a QA process, comparing a sample of GPT tags to the manual tags created by a human subject matter expert (SME). Recurrent errors were identified, and the prompts were iteratively refined to address the errors. For example, GPT model had a bad habit of trying to answer the math question in addition to responding to the prompt, so subsequent queries instructed it not to solve the math problem.

Once we were satisfied with the prompt, we cleaned the data so that answer formatting was consistent. GPT would sporadically put a period at the end of a response, at times it would explain its reasoning after giving a response, and despite instructions it would still occasionally include the solution to the math problem. These inconsistencies were resolved and unrequested additions were removed so that responses could be compared statistically.

Once responses were cleaned, that LLM generated metadata were compared against the SME generated metadata and evaluated on recall and precision.

2. Context Extraction Methods

Whether a math question is contextualized with a real-world application can greatly increase the role that reading plays in a students' ability to solve a math problem. We labeled questions like this "Real-World". Questions that use purely symbolic math were labeled "Symbolic".

We had a total of 339 quetions in our assessment bank. 109 were identified by SME as having a real world context, 224 were identified as being purely symbolic, and 6 were identified as "School Math". In our initial experimentation with this idea using the ChatGPT interface, we'd already established that GPT-3.5 had trouble distinguishing School Math from Symbolic questions. Thus, the first prompt for GPT-3.5 was "Is the math problem set in a real-world context, or is it symbolic math? If it is set in a real-world context, say Real-World, if it does not have a real world context say Symbolic". Upon inspection, GPT-3.5 routinely made the following errors:

1. Solving the math question in addition to answering the prompt.
2. Considering a graph to be a real-world context, even if the math was symbolic

Our prompt was updated to: *"Is the math problem set in a real-world context, or is it symbolic math? If it is contextualized in a real-world setting, say Real-World, if it is not applying math in a real-world environment, say Symbolic. Do not solve the math problem."*. The types of errors made by this prompt were artifacts of our question format; question images and answer options for multiple choice questions were not included in the question text given to GPT. For problems that relied heavily on either images or answer option text (e.g. "What is true about the following diagram?" or "Which rule can describe the table below"), GPT-3.5 would either respond that the question was in both categories (n=9) or it would say that it needed more information (n=11). A total of 20 questions out of 339 questions were removed from the analysis for this variable.

We then created a different prompt on just the symbolic questions to separate the School Math questions. This prompt read: *"Does the math question contain a person's name? If the answer is yes, respond with School Math, if the answer is no respond with Symbolic. Do not solve the math problem"*. This prompt did not capture our sampled cases, so we revised it to: *"Is this math question about a hypothetical person trying to solve a symbolic math problem? Does the question text contain a person's name? If the answer to either is yes, respond with School Math, if the answer is no respond with Symbolic. Do not solve the math problem"*. This also tapped into the LLM's named-entity recognition abilities [11]. The "Real-World" labels for the remaining questions were combined with the resulting list.

2.1. Context Evaluation Results

Manual Tags and GPT-3.5 tags generated on our 319 items from our assessment bank were compared on recall (0.92), and precision (0.92). Results can be seen in Table 2.

Table 2
Evaluation results for the GPT-3.5 generated context metadata with the SME created metadata.

Metric	Result
Recall	0.92
Precision	0.92

Our results suggest that GPT was highly reliable at distinguishing question context, with a recall of 0.92. In 22 out of 25 cases of an incongruity, GPT coded a symbolic problem as a real-world problem. Almost all of these made reference to graphs or tables, indicating that, while the update to our prompt improved this issue in the sample we reviewed, the error of interpreting a graph as a context persisted to some extent. In 2 out of 25 cases, GPT coded a Real-World problem as Symbolic. It is a mystery as to why; the first referenced bacteria growing in a dish and the second compared three runners in a 400 meter race. In just one instance did GPT-3.5 categorize a School Math Question as Symbolic.

3. Math Vocabulary Extraction Methods

We identified "math vocabulary" as a second variable that would be a good candidate for using GPT rather than a human coder. Words like "linear" and "quadratic" are key to understanding Algebra questions; if a student needs support on vocabulary, these words pose the biggest barrier to their ability to demonstrate understanding. Real-World vocabulary, like the "bacteria" example described in the introduction, is a separate variable.

The first prompt for GPT-3.5 was framed as *"List the math vocabulary words in the question text. Do not answer the math question."*. The errors that resulted fell into two categories. Primarily, GPT-3.5 would sometimes identify the mathematical expressions in the question as math vocabulary. For example, for the question text *"Which equation is equivalent to the equation 6x + 9 = 12?"*, it provided *"Equation, 6x + 9 = 12"* in response to our prompt. Secondly, GPT would only sometimes identify *"graph"* as math vocabulary, whereas the human coder always considered *"graph"* to be math vocabulary. For multi-word phrases, such as *"linear relationship"*, GPT will sometimes return the whole phrase but at other times will only return part of the phrase, such as *"linear"*; however, the human coder could occasionally be inconsistent about this as well.

At times GPT-3.5 could be considered *more* accurate than the human coder. For example, the human coder did not consider "*data"* to be math vocabulary, whereas GPT did. This can be seen as a benefit to using LLMs for this task; the risks of a "false positive" in this case are minimal. If GPT identifies additional words for which a student might need support, that is a benefit rather than a drawback.

Our second, revised prompt read *"Excluding numbers, variables, mathematical expressions and equations, list only the math vocab words/phrases in the question text. Do not answer the question"*. This phrasing successfully captured multi-word phrases in the sample we reviewed, and the instances of numbers and expressions being included were reduced (but not eliminated). The new phrasing did not reduce the instances of inconsistently considering words like “graph” to be math vocabulary. As we did not know why this inconsistency happened, we could not think of a way to address it in our prompt.

The process was repeated using GPT-4, starting with the prompt *“Excluding numbers, variables, mathematical expressions and equations, list only the math vocab words/phrases in the question text. Do not answer the question”*. The resulting errors suggested that GPT-4 was much more liberal in what it considered to be “mathematical vocabulary”. It included most of the real-world vocabulary (e.g. *“softball team”* and *“landscaping company”*) in its response to the prompt. We changed the prompt to say *“"List the math vocabulary words in the selected question text. Only include mathematical vocabulary, do not include vocabulary without a mathematical definition. Do not answer the math question."*. The removed many cases of including real-world context vocabulary, but many still remained. Some, such as “nickles”, one could make an argument for having a “mathematical definition”; however for others, such as “scarves”, it is harder to make an argument for why GPT-4 considered the term to be mathematical vocabulary.

The problems seen in our previous use of GPT-3.5, such as inconsistent formatting for the response, and including expressions and variables in the response, persisted with GPT-4. For our cleaning process we made a second column and, referencing the list of vocabulary GPT-4 had just created, provided this prompt: *“Reformat the text in this cell as a list separated by commas. Remove all periods, mathematical expressions, solitary letters, and variables representing numbers”.*

GPT-4 also introduced a new problem, in that in some cases it considered the text from the prompt in its response. This led to the inclusion of the phrase “mathematical vocabulary” in dozens of responses, despite the phrase never appearing in the question bank text. Language from the prompt was removed manually before analysis.

3.1 Math Vocabulary Evaluation Results

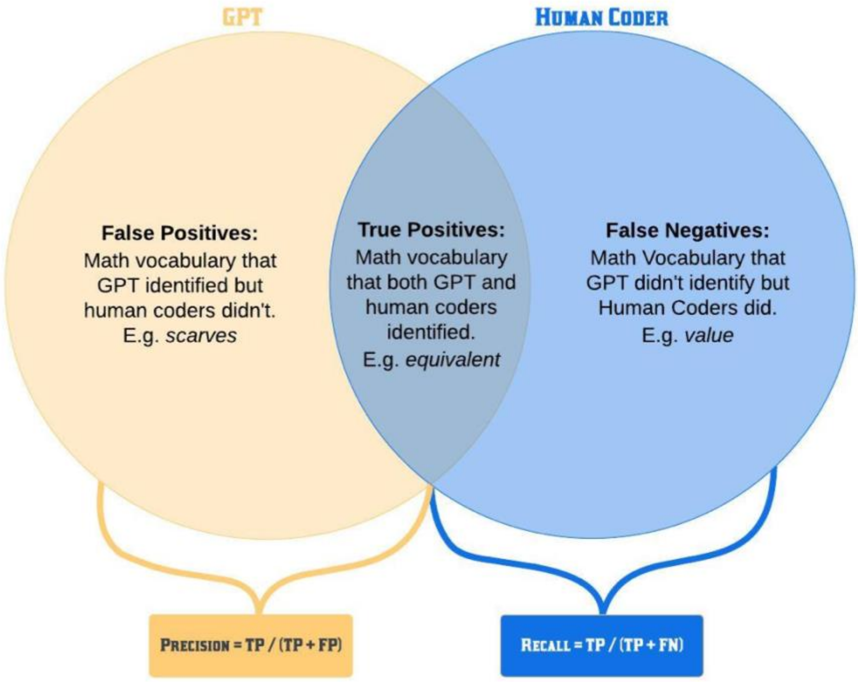
In cases like this, where the consequences of the AI identifying a case where a human did not (false positive) are null, a recall is a more useful indicator of success. Recall was calculated by dividing the number of correctly suggested words generated by GPT (true positives) by the total number of words identified by the human coder for each question (see Figure 1). This was averaged across questions, resulting in a mean recall of 0.75; GPT-3.5 successfully identified the vocabulary the human coder did in 75% of cases (See Table 3).

Table 3
Evaluation results for the GPT 3.5 generated math vocabulary metadata with the SME created metadata.

Metric	Result
Recall	0.75
Precision	0.63

Precision (0.63) was calculated as true positives divided by total words identified by GPT for each question, averaged across questions (see Figure 1). Precision was lower than recall, suggesting that false positives were numerous enough to negatively impact precision. However, as discussed previously, the consequences of false positives are minimal (and may even be beneficial), so this result should carry less weight than the rate of recall. Interpreting these results is not as straightforward as the interpretation for context, as each question could have multiple potential matches. In many ways, if GPT looks at a math question and identifies 4 out of the 5 vocabulary words a human would identify, that is still a relatively successful application of this technology despite the slight decrease in recall it represents. The question is now whether 75% recall is high enough to warrant relying on AI-generated tags in place of human-generated ones.

Figure 1.
Illustration of how Recall and Precision are calculated.



We repeated our math vocabulary tagging process using GPT-4 to see if it improved recall (see Table 4). GPT-4 was indeed better at identifying vocabulary, with a recall 0.82, indicating that GPT-4 agreed with the human coder on 82% of the identified vocabulary words. Precision only moderately increased (from 0.63 to 0.66), indicating that false positives are still high.

Table 4.
Evaluation results for the GPT 4 generated math vocabulary metadata with the SME created metadata.

Metric	Result
Recall	0.82
Precision	0.66

These results (82% recall) get closer to the margin of error we would expect between two human coders, especially for a subjective, multi-class labeling task [12,13].

Inspection of the most common (i.e., identified more than once) false positives made by GPT-4 reveal four categories (2x2) of error varying along the dimensions of whether or not the words had a mathematical definition and constituted challenging vocabulary (Table 4).

Table 5.

Features of the typical errors made by GPT-4 in identifying math vocabulary words

		Mathematical Definition	
		Yes	***No***
Challenging Vocabulary	***Yes***	Cases where the word *does* have a mathematical definition *and* may be challenging vocabulary for some students. This includes things like "represent" and "completing the square", as well as things like a greater-than-or-less-than symbol (≥), which is indeed a type of math vocabulary for which a student may need support. This represented 88 of the 273 false positive cases analyzed.	Cases where the word *does not* have a mathematical definition, but may still be challenging vocabulary, e.g. "bacteria", "automatic feeder". This represented 14 of the 273 false positive cases analyzed.
	No	Cases where the word *does* have a mathematical definition, but is not the kind of word that would increase the difficulty of the problem for an Algebra I student, e.g. "time", "dollar", and "number". This represented 121 of the 273 false positive cases analyzed.	Cases where the word *does not* have a mathematical definition, and is a word an Algebra I student is likely to already know, e.g. "swimming pool" and "ticket". This represented 50 of the 273 false positive cases analyzed.

When viewed another way, 209 of the false positives made by GPT were successful identifications of math vocabulary, and 102 of the false positives made by GPT were successful identifications of words that might be challenging for a middle-school student. Only 50 of the 273 words were truly inaccurate. Seen through this lens, the risks of false positives seem minimal, and lower precision seems to come with benefits.

We intend to expand our metadata tags to include real-world vocabulary like "bacteria" as a separate variable. As many of the false positives are cases of real-world context vocabulary being included despite our attempts to remove them with our prompt experimentation, a promising next step would be to repeat this process for the real-world vocabulary, then undergo a cleaning process that removes redundant words from the math vocabulary generated by GPT-4.

4 Discussion

In the comparison between GPT-3.5 and a human coder on creating metadata for math, specifically algebra questions, the results were mixed. Context was a success, with GPT correctly labeling the problems with 92% recall and precision. Vocabulary was less successful, with 75% recall. GPT-4 was more successful at the vocabulary task, raising the recall to 82%. Overall we conclude that using AI to generate metadata for algebra questions shows promise and should be explored further. For many applications, the time, cost, and labor saved is worth the modest decreases to accuracy, particularly for cases where false positives carry few consequences.

4.1 Recommendations

We learned much in the process of automating the metadata generation of math assessments and wanted to share our learnings with other researchers, publishers, and/or edtech developers who are in the process of leveraging LLMs for similar use cases. Based on our experience, we recommend the following strategies:

1. GPT will try to give a narrative response explaining the answer it provides. You should set *creativity* to "precise" and *max response size* to "short" in the settings for the GPT Sheets API integration, and you should specify the format in which you want the responses (e.g. what labels to use, how to deliminate terms in a list).
2. You must prompt GPT to not solve the assessment item in your prompt. It will naturally want to consider the answer to the question in its response.
3. You will likely need to go through a manual cleaning process to remove predictable errors such as inconsistent use of periods or elaborations on/justifications for the response.
4. If a single prompt does not produce satisfactory results, a two-step prompt may provide a solution. In the case of context, we were able to distinguish School Math and Symbolic math with a second, unrelated prompt focusing on different problem features.

There are a number of creative ways in which LLMs can augment or improve educational research and impact student learning outcomes. Applications such as the one described in this paper represent a fruitful direction for such exploration with minimal risk.

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