Revising Weighted Knowledge Bases Using FH-Conditioning

Omar Et-Targuy[1,2,3,4,†], Ahlame Begdouri[2,†], Salem Benferhat[1,†] and Carole Delenne[3,4,†]

[1]CRIL, CNRS UMR 8188, University of Artois, France.
[2]Université Sidi Mohamed Ben Abdellah (USMBA), Fez, Morocco.
[3]HSM, University of Montpellier, CNRS, IRD, Montpellier, France
[4]Inria, Team Lemon, Montpellier, France

Abstract

Conditioning is an important task for updating and revising uncertain information when new information, often considered reliable, is added. This paper deals with the so-called Fagin and Halpern (FH-)conditioning within the framework of possibility theory. We discuss in particular the computation of FH-conditioning when it is applied to weighted knowledge bases. We also compare FH-conditioning with the two forms of standard possibilistic conditioning (min-based conditioning and product-based conditioning).

Keywords

Conditioning, possibility theory, weighted knowledge bases.

1. Introduction

Belief revision [1] is a fundamental problem in knowledge representation. It consists in revising a set of beliefs of an agent in the light of new information, considered completely reliable. This problem has been widely studied in the literature both from the rational postulates point of view and from a computational point of view. A large number of belief revision operators have been proposed; in particular within the framework of propositional logic (and its extensions).

Within the frameworks of uncertainty theories, the process of belief revision is realized through the concept of conditioning. A large number of conditioning operators have been defined: Bayesian conditioning (in probability theory), Dempster's rule of conditioning (in belief functions theory [2]), min-based and product-based possibilistic conditioning (in possibility theory [3]), different forms of conditioning in ordinal conditional functions (OCF) [4], etc. These "standard" conditioning modes have been extensively studied in the literature from a semantic point of view but also from a computational point of view; in particular for the propagation of the uncertainty of beliefs in the presence of new observations.

This paper focuses on Fagin and Halpern conditioning (denoted by FH-conditioning), initially defined within the framework of belief function theory in [5]. This conditioning was proposed in order to have a better characterization of belief functions in terms of particular families of probability distributions (see [5] for more details). We are interested in the study of FH-conditioning within the framework of possibility theory; a particular framework of belief function theory. The possibilistic counterpart of FH-conditioning has already been discussed only from a semantic point of view [6]. This paper is interested in the revision of the weighted belief bases which is in full agreement with the possibilistic FH-conditioning. A weighted belief base is represented by a set of pairs (ϕ_i, α_i) where ϕ_i is a propositional logic formula, and α_i is a degree of certainty (a degree of necessity) attached to the formula ϕ_i.

The rest of the paper is organized as follows. We first recall the basic elements of possibility theory. Next, we summarize the syntactic computation of FH-conditioning of the weighted knowledge bases that we recently developed in [7]. Section 4 briefly positions the computation of possibilistic FH-conditioning in relation to the two standard forms of possibilistic conditioning (min-based and product-based possibilistic conditioning). Section 5 concludes the paper.

2. Weighted Knowledge Bases and Possibility Distributions

We place ourselves within the framework of propositional logic. We will denote $\mathcal{L}$ the set of propositional logic for-

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† Contact authors:
ettarguy@cril.fr (O. Et-Targuy); ahlame.begdouri@usmba.ac.ma (A. Begdouri); benferhat@cril.fr (S. Benferhat); carole.delenne@umontpellier.fr (C. Delenne)

mulas and Ω the set of interpretations. A possibility distribution π is a mapping from the set of propositional logic interpretations Ω to the unit interval $[0,1]$. $\pi(\omega)$ represents the degree of compatibility or consistency of the interpretation ω with respect to the set available knowledge. A possibility distribution is said to be normalized if there exists an interpretation ω which is fully possible (i.e., $\pi(\omega)=1$).

Given a possibility distribution π, we can define two measures over the set of formulas:

- The degree of consistency (or possibility):
  $$\Pi(\phi)=\max\{\pi(\omega)|\omega \models \phi\}$$
  which evaluates to what extend the propositional logic formula ϕ is consistent with the available knowledge expressed by π.
- The degree of certainty (or of necessity):
  $$N(\phi)=1-\Pi(\neg\phi)$$
  which is measures to what extent a proposition the propositional logic formula ϕ is entailed by the knowledge expressed by π.

A possibilistic weighted knowledge base is a finite set of weighted formulas, denoted as $\Sigma=\{(\phi_i,\alpha_i), i=1,...,n\}$, where $\alpha_i \in]0,1]$ serves as the weight assigned to each formula. This weight is treated as a lower bound for the degree of necessity $N(\phi_i)$.

Each possibilistic weighted knowledge base Σ induces a unique possibility distribution [8], denoted by π_Σ, defined by :

$$\pi_\Sigma(\omega)=\begin{cases}1, & \text{if } \forall(\phi_i,\alpha_i)\in\Sigma, \omega\models\phi_i \\ 1-\max\{\alpha_i:(\phi_i,\alpha_i)\in\Sigma,\omega\not\models\phi_i\} & \\ & \text{otherwise}\end{cases} \tag{1}$$

3. Syntactic Computation of Possibilistic FH-Condiotioning

At the semantic level, possibilistic conditioning consists in transforming a priori possibility distribution π and a certain information, represented here by a propositional logic formula ψ, into a new possibility distribution (a posteriori) denoted by $\pi(.|\psi)$.

Several methods exist to define $\pi(. \mid \psi)$ (as discussed in [9]). The two major definitions of possibilistic conditioning are [10]:

- Min-based conditioning :
  $$\begin{aligned}\pi(\omega \mid_m \phi) &= 1 \text{ if } \pi(\omega)=\Pi(\phi) \text{ and } \omega\models\phi \\ &= \pi(\omega) \text{ if } \pi(\omega)<\Pi(\phi) \text{ and } \omega\models\phi \\ &= 0 \text{ Otherwise .}\end{aligned} \tag{2}$$
- Product-based conditioning (also known as Dempster rule of conditioning [2]) where we assume that $\Pi(\phi)>0$:
  $$\begin{aligned}\pi(\omega \mid . \phi) &= \tfrac{\pi(\omega)}{\Pi(\phi)} \text{ if } \omega\models\phi \\ &= 0 \quad \text{otherwise}\end{aligned} \tag{3}$$

An alternative to these two definitions of possibilistic conditioning is the FH-conditioning proposed by Fagin and Halpern[11], which was originally introduced within the context of belief functions. Since possibility theory can be seen as a special case of belief functions (i.e. can be represented by consonant belief functions where elements with positive mass are nested), the FH-conditioning was then adapted to the possibility theory framework as follows [6]:

$$\Pi(\phi \mid_{FH} \psi)=\frac{\Pi(\phi\wedge\psi)}{\Pi(\phi\wedge\psi)+N(\neg\phi\wedge\psi)} \tag{4}$$

Where $N(\phi)=1-\Pi(\neg\phi)$.

For justifications of FH-conditioning and a discussion of its various interpretations see for example [11, 12, 13].

When we restrict the definitions $\Pi(. \mid_{FH} \psi)$ to interpretations, we get the definition of FH-conditioning defined on possibility distributions:

$$\pi(\omega \mid_{FH} \psi)=\begin{cases}\max\left(\pi(\omega), \frac{\pi(\omega)}{\pi(\omega)+N(\psi)}\right) & if \omega\models\psi \\ 0 & if \omega\not\models\psi\end{cases} \tag{5}$$

The interesting question is how to compute the FH-conditioning, in an equivalent way, from the weighted knowledge bases. More specifically, given the initial weighted knowledge base Σ and the fully certain information $(\psi,1)$, how to compute a novel weighted knowledge base, denoted by Σ_{FH}, such that:

$$\forall\omega\in\Omega, \pi_{\Sigma_{FH}}(\omega)=\pi_\Sigma(\omega \mid_{FH} \psi). \tag{6}$$

where $\pi_{\Sigma_{FH}}$ (resp. π_Σ) is the possibility distribution associated with Σ_{FH} (resp. Σ) as defined by Equation 1.

In [7], a positive answer was obtained to this question. To compute the knowledge base Σ_{FH}, a reformulation of the semantic definition of FH-conditioning, as a sequence of three transformation operations of possibility distributions, has been first proposed. For each of these semantic transformation operations, an equivalent characterization on the weighted belief bases has been defined. At the end of the third operation, the following final weighted knowledge base was obtained (see [7] for more details):

$$\Sigma_{FH}=\{(\psi,1)\}\cup\Sigma_2. \tag{7}$$

with,
$\Sigma_2=\{(\phi_i,\min(\alpha_i,1-\frac{1-\alpha_i}{1-\alpha_i+N(\psi)})),(\phi_i,\alpha_i)\in\Sigma\}$.

Example 1. *Let*

$$\Sigma = \{(\neg q \vee s, 0.72), (q \vee \neg s, 0.65), (\neg q \vee \neg r, 0.03), (q \vee s, 0.41)\}$$

be a weighted knowledge base. Assume that the new piece of information is:

$$\psi = q \vee r \vee s.$$

Table 1 gives the possibility distribution $\pi_\Sigma(\omega)$ obtained from Σ using Equation 1. Table 1 also gives the result of FH-conditioning ($\pi(\omega \mid_{FH} \psi)$) of $\pi_\Sigma(\omega)$ with ψ:

Table 1

Example of a possibility distribution $\pi_\Sigma(\omega)$ and the result of its conditioning $\pi(\omega \mid_{FH} \psi)$

q	r	s	$\pi_\Sigma(\omega)$	$\pi(\omega \mid_{FH} \psi)(\omega)$
1	1	1	0.97	0.97
1	1	0	0.28	0.406
1	0	1	1	1
1	0	0	0.28	0.406
0	1	1	0.35	0.460
0	1	0	0.59	0.59
0	0	1	0.35	0.460
0	0	0	0.59	0

In the possibility distribution $\pi_\Sigma(\omega)$, the interpretation $q\neg rs$ is the most preferred one since it is the only one which is consistent with Σ. Hence, their possibility degree is $\pi_\Sigma(q\neg rs) = 1$. The interpretation qrs gets the possibility degree $\pi_\Sigma(q\neg rs) = 0.97$ because it falsifies the least certain belief in Σ; namely $(\neg q \vee \neg r, 0.03)$.

At the syntactic level, using Equation 7, we get:

$\Sigma_{FH} = \{(q \vee r \vee s, 1)\} \cup \{(\neg q \vee s, 0.594), (q \vee \neg s, 0.540), (\neg q \vee \neg r, 0.03), (q \vee s, 0.41)\}$.

Finally, one can check that computing the possibility distribution $\pi_{\Sigma_{FH}}$ associated with the weighted knowledge base Σ_{FH}, using Equation 1, gives exactly the same distribution $\pi(\omega \mid_{FH} \psi)$ given in the table above when applying the semantic FH-conditioning with $\psi = q \vee r \vee s$.

4. Min-based and Product-based Conditioning vs FH-conditioning

This section compares FH-conditioning with the two standard forms of possibilistic conditioning: min-based conditioning and product-based conditioning.

At the semantic level, FH-conditioning shares the following four properties with min-based and product-based conditioning (where $|_\diamond$ stands for possibilitic conditioning operator):

- $\pi(. \mid_\diamond \psi)$ is normalized (or consistent).
- $\forall \omega \in \Omega$, if $\omega \not\models \psi$ then $\pi(\omega \mid_\diamond \psi) = 0$.
  This property confirms that the new information ψ is completely certain and therefore any countermodel of ψ is considered impossible after the conditioning operation.
- $\forall \omega \in \Omega, \forall \omega' \in \Omega$ such that $\omega \models \psi, \omega' \models \psi$, we have: $\pi(\omega) > \pi(\omega')$ iff $\pi(\omega \mid_\diamond \psi) > \pi(\omega' \mid_\diamond \psi)$.
  This property means that the conditioning does not alter the relative order between the models of the new information ψ
- $\forall \omega \in \Omega$ if $\pi(\omega) = 0$ then $\pi(\omega \mid_\diamond \psi) = 0$
  This property means that a priori impossible conditional interpretations will remain so after conditioning.

The three possibilistic conditioning (min-based conditioning, product-based conditioning and FH-conditioning) satisfy the above four properties.

There remains however the following property which is satisfied by two standard possibilistic conditioning (min-based and product-based conditioning) but which is not satisfied by the FH-conditioning

- if $N(\psi) > 0$ then $\forall \omega \in \Omega$ such that $\omega \models \psi$, we have: $\pi(\omega) = \pi(\omega \mid_\diamond \psi)$.

This property means that if ψ is a priori accepted (expressed by $N(\psi) > 0$ or by $\Pi(\psi) > \Pi(\neg\psi)$) then the degrees of possibilities on the models remain unchanged. This property is not satisfied with possibilistic FH-conditioning where the possibility degree of a model of ψ can be modified, depending on the a priori degree of beliefs of ψ.

Moreover, it is easy to see in the equation 7 that when $N(\psi) = 0$ then the revised base is simply equal to the new information $\{(\psi, 1)\}$. This behavior is different with min-based conditioning and product-based conditioning which retains part of the initial information even if the new information was not initially accepted.

At the syntactic level, the computational complexity of performing the FH-conditioning of a weighted knowledge base is the same as that of the standard possibilistic conditioning (min-based and product-based possibilistic conditioning) given in [14]. For the three possibilistic conditioning operators, the spatial complexity is linear in with respect to the size of the initial base Σ. As for the time complexity, the difficult task when performing the FH conditioning is to compute the necessity degree of $N(\psi)$ from the initial weighted knowledge base. With min-based and product-based possibilistic conditioning, the most difficult task concerns the computation of inconsistency degrees of a weighted knowledge base. These two tasks (computing the degree of necessity of ψ or computing the degree of inconsistency of a weighted knowledge base) have the same level of computational complexity. Both tasks need $\log_2(n)$ calls to the propositional logic satisfiability test, where n is the number of different degrees in the weighted knowledge base Σ.

5. Conclusions

In this paper, we presented the computation of the FH-conditioning when it is defined on the weighted knowledge bases. This syntactic computation is in full agreement with the semantics of FH-conditioning defined at the level of possibility distributions.

Possibilistic FH-conditioning shares several properties with the two standard forms of possibilistic conditioning. They also differ on the revision to adopt if the new information is already accepted or not. The combination of product-based conditioning with FH-conditioning, to take into account the a priori status of the new information, will be studied in a future work. We also plan to apply different forms of conditioning for the revision of geographic information systems associated with wastewater networks.

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