Upper Bound of Buffer Content Distribution for Self-Similar Traffic Models

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Abstract

One of the most valuable results of traffic over the last years is the property of self-similarity in packet networks. In this paper, a self-similar traffic model driven by FBM is investigated. The estimation of upper bound probability buffer overflow for the traffic model is obtained. We also consider traffic that enters as input on some time-invariant linear system and find the estimation of buffer content distribution taking into account input traffic and response of the stochastic system. Finally, the obtained results in one particular case are applied.

Keywords

Fractional Brownian Motion, self-similarity, traffic, backlog

1. Introduction

The information content has increased seriously with regard to the steady development of communications and telecommunications and the growth of new types of services. It's already proven that classical distributions are not always adequate to explain the existing flows in advanced networks. Therefore, new ways and types of distributions are used to understand traffic characteristics, and their study sometimes cannot be studied analytically. The experimental and numerical research carried out in the last decades show that the traffic in many telecommunication and multimedia networks has a fractal structure. This traffic has a particular property that is preserved during scaling. It differs from ordinary traffic in a number of specific characteristics: it has the properties of self-similarity and long-term dependence.

In the development of telecommunications networks, their intelligent dataand methods of statistical modeling are often used. The modeling method is well suited for problems that cannot be solved by classical mathematical methods. Statistical modeling is used to collect characteristics and parameters that reflect the behavior of complex systems, also taking into account the influence of possible external factors. Multifractal characteristics of traffic have been intensively studied. These studies were started in [1-2]. The fractal properties of traffic make it possible to develop a series of traffic models based on fractal stochastic processes [3-13]. The discovery of the self-similarity property of traffic made it possible to reconsider the probabilistic-temporal characteristics of such networks. Fractal or self-similar traffic models include such phenomena such as long-term dependence (the effect of the value of the number of packets that arrived some time ago on the number of packets at a given time) and traffic self-similarity. One of the main parameters of multifractal traffic is the Hurst index which can be taken to measure the degree of long-term dependence (the rate of decrease of the correlation function).

Different methods and approaches are used to estimate the Hurst index [14-17]. The fractal traffic model is usually based on random variables and processes with heavy-tailed distributions. The use of fractal stochastic processes for modeling telecommunication traffic is based on fractional Brownian motion. FBM properties and their practical applications were investigated in [18-20]. So, for example,

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in medicine [18], to detect DDoS attacks [19], to analyze financial processes [20], to analyze emotions and the human condition [21-22].

One of the important aspects of using a simulation model is to assess the accuracy and reliability of the results obtained. Methods for statistical modeling of Gaussian random processes with a given accuracy and reliability were studied in [23-27]. Thus, this paper is devoted to self-similar traffic models that can be described by FBM. The article examines the methods of estimating the Hurst index and modeling self-similar traffic, which was carried out by the authors in [14, 28-29].

Since this process is self-similar and has stationary increments it can be used to simulate traffic in high-speed networks. In Section 2 general aspects of stochastic models in telecommunications are studied. Especially attention paid to understanding of traffic self-similarity. In Section 3 the arrival and backlog processes connected with FBM are defined and the main properties are studied. The estimation of probability buffer overflow is obtained. We also analyze traffic that enters as input on some time-invariant linear systems [30]. To provide the connection of input traffic and the response (output) of such a system a quadratic form built on these two processes is considered shown in Section 4. Section 5 is devoted to the upper bound of buffer content distribution taking into account input and output traffic. In Section 6 a particular case of self-similar traffic model is observed and the dependence of model parameters is studied.

2. Stochastic models in telecommunication

Stochastic models are widely adopted in the telecommunications industry. Discrete and fluid queueing models played a major role in the development of computer and communication networks. There are several branches of telecommunications that use stochastic models, Let's focus in networking systems and other stochastic systems that aide high-performance networking.

There are some important scenarios in the Internet and other networks where stochastic modeling is applicable. Now the networks carry a huge variety of traffic (Data, Voice, Video, etc) and In the future the it will only be increased, as the users will demand very high quality from the networks. Therefore, it is vital to ensure that certain types of performance factors, known as quality of service (QoS), are taken into account. There are some commonly known end-to-end QoS metrics, for example, loss probability, delay, delay-jitter, and bandwidth. Let us briefly describe them. When messages flow from a source to a destination (end-to-end) through a network, parts of a message or the whole message may be dropped due to unavailable resources (buffer capacity) to store the messages. The probability of delivering a message with some data loss is termed loss probability.

The time between the source sending a message and the destination receiving it is called latency or delay. Typically real-time or multimedia traffic (such as live video conference) can be tolerated to some loss but have very strong delay requirements. However, data traffic such as emails, fax, file transfers, etc can tolerate some delay but almost zero loss. The other QoS measures are delay-jitter (which is a measure of the variation in the delay) and bandwidth (which is the rate at which messages are processed).[27]

Further areas of application of stochastic processes in communications involve coding theory, signal processing, image processing, pattern recognition, speech recognition, etc.

Broadly there are three types of telecommunication networks – telephony (telephone networks for voice calls, fax, and also dial-up connections), cable-TV networks (cable, web-TV, etc), and high-speed networks such as the Internet. We focus on high-speed networks.

Traffic that flows through the networks can be divided into several types. Two of the most common traffic types are ethernet packets/frames and ATM cells. Depending on the network domain, all messages are devided into either packets or cells. The length or size of an ethernet packet ranges anywhere from 60 bytes to 1500 bytes and generally follows a bimodal distribution. The length of ATM cells is fixed at 53 bytes. Therefore, the network traffic comprises of millions and billions of these little packets or cells! One of the most important tasks before evaluating the performance of telecommunication networks is to fit appropriate models for traffic to capture their stochastic nature. Data can be obtained by using "sniffers" on the network and analyzing a "dump" of all the packets or cells that were generated during the time the sniffer was used. The information that can be obtained about each packet or cell by sniffing include: its arrival time, its source, its destination, its length, its type, etc. To fit traffic models, only the time of arrival and packet size are sufficient. [27].

Telecommunication networks are typically hierarchical in nature. Appropriate traffic models can be used depending on the levels being considered. Although, different researchers prefer to use different traffic models, the models can be broadly classified into two parts, discrete models and fluid models. In the discrete model each packet or cell is assumed to be a discrete entity that can be of varying sizes. In the fluid models it is assumed that the packets or cells are packed so close to each other that the traffic can be assumed to be a fluid flowing across a pipe as a stochastic process with continuous time parameters, maybe at different rates.[27]

3. Self-similarity in traffic models

One of the most important findings of traffic measurement studies over the last years is the observed self-similarity in packet network traffic. A lot of research has focused on the origins of this self-similarity, and the network engineering significance of this phenomenon, see for example [25].

In the case of data networks, high time-resolution packet level traffic measurements are generally recorded from the physical link over which the data is sent, by copying either an initial part of each packet (i.e., the packet header) or every single bit of each packet (i.e., header plus payload) over to a high-performance storage device. The last decade has seen an enormous increase in empirical studies of high-quality and high-volume data sets of traffic measurements from a variety of different data networks, but especially from different links within the global Internet. These studies typically describe pertinent statistical characteristics of the temporal dynamics of the “packet” or bit rate processes (i.e., the time series representing the number of packets or bits per time unit, over a certain time interval) as seen on a link within the network. They provide a piece of evidence that measured packet traffic exhibits extended temporal correlations [i.e., long-range dependence (LRD)], and hence when viewed within some range of (sufficiently large) time scales, the traffic appears to be fractal-like or self-similar, in the sense that a segment of the traffic measured at some time scale looks or behaves just like an appropriately scaled version of the traffic measured over a different time scale. In effect, this empirically-based effort toward describing actual data network traffic has demonstrated that self-similarity provides an elegant and compact mathematical framework for capturing the essence behind the wide range of observed traffic traces (see[29] for further references). The observed self-similar behavior of measured traffic was in sharp contrast to what the conventional models for data traffic predicted, models that in general lacked validation against measured traffic traces. A hallmark of these traditional voice-based data traffic models is a correlation function which decays exponentially fast [i.e., short-range dependence (SRD)], implying that time-aggregation quickly results in white noise traffic characterized by the absence of any significant temporal correlations, and capable only of reproducing the observed bursty behavior of measured traffic over a narrow range of time scales.[25]

A good starting point in understanding the impact of self-similarity was provided by Norros [1-2], who developed a formula that can be used to estimate buffer overflow probabilities at network switches and routers. The Norros results showed that the queueing backlogs were in general worse with self-similar traffic, in the sense that the buffer sizes to achieve a certain loss objective could be significantly greater. This agrees with the common intuition that the presence of positive correlations in the incident traffic can only aggravate queueing delays. The long-range correlations provide themselves in extended periods of time over which the incident traffic exceeds link capacity, leading to heavy queueing backlogs. In contrast, a small amount of buffering is sufficient to smooth out the peaks and valleys in SRD traffic. The large deviations principles that underpin the Norros’ formula, and its refinements [24,28,31-34], directly relate the traffic characteristics (e.g., distribution of arrival counts) to performance measures (e.g., queue length distributions, loss rates). They indicate that to obtain the performance estimates necessary to accomplish even basic network engineering tasks, one must in principle characterize an infinite family of distributions of the arrival counts. This is clearly impossible to do in practice. In theory (i.e., assuming an idealized Gaussian setting), it enables the representation of the infinite family of distributions by three parameters over the entire scaling region—an enormous reduction in the description complexity. These three parameters are the mean and the variability of the traffic process, and the self-similarity or Hurst parameter [25].

There are three fundamental conditions that should be satisfied for a Gaussian self-similar traffic, corresponding to the observation of long-range dependence in traffic, to apply:

1) the network traffic should be sufficiently aggregated so that the marginal distributions of counts are at least approximately Gaussian (due to Central Limit Theorem);
2) the long-range dependent scaling region should span the engineering time scales of interest;
3) the impact of network controls on the traffic flows must not be significant over the engineering time scales of interest.

These three conditions together suggest a feasibility regime for the standard self-similar traffic model based on fractional Brownian motion (FBM). It is delineated by: moderate to heavy traffic (so that the aggregation levels are sufficient for a second-order description to be valid), aggregation from a large number of low-activity sources (so that no one source is dominant), and moderate to large buffer sizes (so that the scaling region covers the time scales of interest). [25]

4. Arrival and backlog processes

To describe the character of the observed properties of traffic data more precisely, we introduce a second order stochastic process.

Let (Ω, B, P) is a standard probability space.

Definition 1. We say that stochastic process $B_\alpha(t),\ t \in [0,1]$, is called the generalized Wiener process (fractional Brownian motion, FBM) with the Hurst index $\alpha \in (0,1)$ if the following conditions hold true:

1. it's Gaussian stochastic process;
2. starts at zero, $B_\alpha(0) = 0$,
3. with zero expectation $EB_\alpha(t) = 0$ and
4. it has a covariance function $R_\alpha(t,s) = \frac{1}{2}\left(|t|^{2\alpha} + |s|^{2\alpha} - |t-s|^{2\alpha}\right)$.

The self-similarity parameter $\alpha \in (0,1)$, Hurst index, has the following role. If $\alpha \neq \frac{1}{2}$ then the process $B_\alpha(t)$, is a process with dependent increments. There are $\frac{1}{2}$-self-similar processes with independent increments. In the case $\alpha < \frac{1}{2}$ the increments of FBM are negatively correlated. In contrast, when $\alpha > \frac{1}{2}$ the increments of FBM is positively correlated and the increments of the process $B_\alpha(t)$ are long-range dependent. The case $\alpha < \frac{1}{2}$ corresponds to short-range dependence.

When studying self-similar traffic, as usual, $\alpha > \frac{1}{2}$ is considered.

Under $A(t)$ let us denote the the arrival traffic, e.i. amount of traffic coming to the network over a period of time $[0,T]$. The increment will be denoted as $A(s,t) = A(t) - A(s), \quad t > s > 0$. In [11] it's shown that input traffic ca be presented as

$$A(t) = mt + \sqrt{am}B_\alpha(t),$$

where m is an average traffic rate, $B_\alpha(t)$ is FBM with Hurst index α, a is some constant.

If the network has one service device with the rate $C > m$, the backlog process can be defined as [26]

$$Q(t) \cong \sup_{s \le t}\left(A(s,t) - C(t-s)\right).$$

The system with n independent identically service devices provide the backlog processes as

$$Q_n(t) \cong \sup_{s\le t}\left(\sum_{i=1}^{n} A_i(s,t) - nC(t-s)\right).$$

Here, the symbol $\cong$ means identically distributed quantity.

Study now the probability of overloading by threshold b of $Q(t)$ on time interval $[0,T]$. Let

$$Q \cong \sup_{t\in[0,T]}\left(Q(t)\right), \quad \pi(b) = P\{Q \ge b\}.$$

We are interested in the upper bound for buffer content distribution

$$P\{Q \ge b\} \le P\left\{\sup_{t\in[0,T]}\left(Q(t)\right) > b\right\}.$$

Really, in [24] it was shown that

$$P\{Q \ge b\} \le P\left\{\sup_{t\in[0,T]}\left(\left|B_\alpha(t)\right|\right) > \frac{b+T(C-m)}{2\sqrt{am}}\right\}$$

Let us denote $x = \frac{b+T(C-m)}{2\sqrt{am}}$, then the following theorem is fulfilled.

Theorem 1.[24] For $x \ge D$ we have

$$P\left\{\sup_{t\in[0,T]}\left(\left|B_\alpha(t)\right|\right) > x\right\} \le 2\exp\left\{-\frac{(x-D)^2}{2V}\right\} \tag{1}$$

where

$$D = \sqrt{2}\left(T^\alpha \ln^{\frac{1}{2}}\left(2^{\frac{1}{\alpha}}+1\right) + 4\int_0^{\frac{T^\alpha}{4}} \ln^{\frac{1}{2}}\left(\frac{1}{u^{\frac{1}{\alpha}}}+1\right)du\right),$$

$V = 4T^{2\alpha}$.

Remark 1. Using the estimates obtained, we can determine what the threshold should be, so that the probability of being exceeded is less than given.

Thus,

$$P\{Q \ge b\} \le P\left\{\sup_{t\in[0,T]}\left(Q(t)\right) > b\right\} \le P\left\{\sup_{t\in[0,T]}\left(\left|B_\alpha(t)\right|\right) > x\right\} \le \varepsilon_b, \text{ if}$$

$$x \ge D \text{ and } 2\exp\left\{-\frac{(x-D)^2}{2V}\right\} \le \varepsilon_b.$$

And the threshold should be $b \ge 2D\sqrt{am} - T(C-m)$.

The value ε_b one can interpretate as significance level.

The linear system with input-output signals is often used in practice. Therefore, there is an interesting task, how to estimate the buffer overflow, taking into account the input and output data.

5. Square-Gaussian stochastic processes

To connect the input traffic signal of some linear system and the response (output) of this system we should, on the one hand, find out a such functional relation that depends on input and output signals. On the other hand, such relationship should be easily investigated. It's natural to consider a quadratic form of input and output process. That's why we introduce a quadratic form on Gaussian distributions. Let's give the definitions and some properties of Square-Gaussian random variables and stochastic processes.

Assume that (T,ρ) is some compact metric space with metric ρ.

Definition 2. Let $\Xi = \{\xi_t, t \in T\}$ be a family of zero-mean joint Gaussian random variables. A space $SG_\Xi(\Omega)$ is a space of Square-Gaussian random variables if any element of this space $\eta \in SG_\Xi(\Omega)$ can written as

$$\eta = \zeta A \zeta^T - E\zeta A \zeta^T,$$

where $\zeta = (\xi_1, \xi_2, ..., \xi_n)$, $\xi_k \in \Xi$, $k = \overline{1,n}$, A is a real-valued matrix or an element, $\eta \in SG_\Xi(\Omega)$ is a square mean limit of the sequence

$\eta = \underset{n\to\infty}{\mathrm{l.i.m}}(\zeta_n A \zeta_n{}^T - E\zeta_n A \zeta_n{}^T).$

Remark 2. The space $SG_\Xi(\Omega)$ is a Banach space with respect to the norm $\|\xi\| = \sqrt{E\xi^2}$.

Definition 3. A stochastic process $\xi(t), t \in [0,T]$, is called Square-Gaussian if for any fixed t ϵ [0,T] each random variable $\xi(t)$ belongs to the space $SG_\Xi(\Omega)$ and $\sup_{t\in[0,T]} |\xi(t)| < \infty$.

We will use the following theorem on the tail distribution of the supremum of Square-Gaussian stochastic process. The proof of the theorem can be found in [28].

Theorem 2. [29] Assume that $\xi(t), t \in [0,T]$, is a separable Square-Gaussian stochastic process and

$$\sup_{|t-s|<h} \sqrt{D(\xi(t)-\xi(s))} \le \sigma(h) = kh^\beta, \ \alpha \in (0,1], \tag{2}$$

where k is some constant. Then for x such that

$$x > \frac{2\sqrt{2}\max\{\delta_0, k(T/2)^\beta\}}{\beta},$$

the inequality

$$P\left\{\sup_{t\in[0,T]} |\xi(t)| > x\right\} < 4e^{\frac{3}{\beta}} \exp\left\{-\frac{x}{2\sqrt{2}\delta_0}\right\} \times \left(\frac{x\beta}{2\sqrt{2}\delta_0}\right)^{2/\beta} \left(1 + \frac{2x}{\sqrt{2}\delta_0}\right)^{1/2}$$

holds true where $\delta_0 = \sup_{t\in[0,T]} (D(\xi(t)))^{1/2}$.

6. Upper bound of buffer content distribution taking into account input and output traffics of stochastic system

In [24] it's shown that the distribution of backlog buffer can be estimated by the distribution of suprema for FBM.

In the case of traffic signal transfer on some system not only input process but also the response (output) of system should be taken into account. For these purposes the distribution of suprema of Square-Gaussian processes can be used.

Consider a time-invariant linear system with a real-valued square integrable impulse response function $H(\tau)$ which is defined on a finite domain $\tau \in [0,T]$. This means that the response of the system to an input signal $X(t)$ which is observed on $[-T,T]$ has the following form

$$Y(t) = \int_0^T H(\tau)\, X(t-\tau)\, d\tau, \quad t \in [0,T] \tag{3}$$

and $H \in L_2([0,T])$.

Some properties and estimators of impulse response function can be found in [29-30].

In [35] we can find that Fractional Brownian Motion can be shown in the form of random series

$$B_\alpha(t) = X_\alpha(t) = \sum_{k=1}^{\infty} \left(a_k \sin(x_k t) X_k + b_k (1 - \cos(y_k t)) Y_k\right)$$

where $\{X_k, Y_k\}$ are uncorrelated standard Gaussian random variables,

$\{x_k\}$ are zeros of Bessel function $J_{-\alpha}(x)$ and
$\{y_k\}$ zeros of Bessel function $J_{1-\alpha}(x)$,

$$a_k = \frac{\pi^\alpha \sqrt{2C}}{x_k^{\alpha+1} J_{1-\alpha}(x_k)},\ b_k = \frac{\pi^\alpha \sqrt{2C}}{y_k^{\alpha+1} J_{-\alpha}(y_k)},\ C = \frac{\Gamma(2\alpha+1)\sin(\pi\alpha)}{\pi^{2\alpha+1}}.$$

Suppose that the impulse response function is known. We also suggest that the input signal in system (3) is FBM with Hurst index α. From (3) follows that the response of the system (output) $Y(\mathrm{t})$ can be presented as

$$Y(t) = Y_\alpha(\mathrm{t}) = \sum_{k=1}^{\infty} (\xi_k \cdot \mathrm{c}_k(t) + \eta_k \cdot \mathrm{s}_k(t)), \tag{4}$$

where the functions $c_k(\mathrm{t})$, $\mathrm{s}_k(\mathrm{t})$ equal

$$\begin{aligned} \mathrm{c}_k(\mathrm{t}) &= b_k \int_0^T H(\tau)(1-\cos(\mathrm{y}_k(\mathrm{t}-\tau)))\,\mathrm{d}\,\tau, \\ s_k(\mathrm{t}) &= a_k \int_0^T H(\tau)\sin(\mathrm{x}_k(\mathrm{t}-\tau))\,\mathrm{d}\,\tau. \end{aligned} \tag{5}$$

In this section we investigate the backlog of system with input signal FBM $X_\alpha(\mathrm{t})$, taking into account the output of the system $Y(\mathrm{t})$. To perform such results, we use the theory of Square-Gaussian random variables and processes. Sometimes the input process isn't known and it's possible to construct the model of the process and then to estimate the probability of overflow. The results on simulation of Gaussian process were obtained in [28,29].

Under $\xi(t)$ we denote the sum of square $X_\alpha(\mathrm{t})$ and $Y_\alpha(\mathrm{t})$

$$\xi(t) = (X_\alpha(\mathrm{t}))^2 + (Y_\alpha(\mathrm{t}))^2.$$

Making the same manipulation as in [24] the probability of backlog can be estimated as

$$P\{Q \ge b\} \le P\left\{\sup_{t\in[0,T]} \left(\sqrt{X^2_\alpha(t) + Y^2_\alpha(t)}\right) > \frac{b+T(C-m)}{2\sqrt{am}}\right\} =$$

$$= P\left\{\sup_{t\in[0,T]} \left(X^2_\alpha(t) + Y^2_\alpha(t)\right) > \left(\frac{b+T(C-m)}{2\sqrt{am}}\right)^2\right\} =$$

$$= P\left\{\sup_{t\in[0,T]} |\xi(t) - E\xi(t)| > x_0\right\},$$

where $x_0 = \left(\frac{b+T(C-m)}{2\sqrt{am}}\right)^2 + \sup_{t\in[0,T]} E\xi(t)$.

It's easy to shown that zero-mean process $\xi(t) - E\xi(t)$ is Square-Gaussian. So, Theorem 2 can be applied in this case.

Let's make the following notation:

$$\phi^1_{kl} = \phi^1_{kl}(\mathrm{t}) = \mathrm{b}_k\, a_l(1-\cos(\mathrm{y}_k\, t))(1-\cos(\mathrm{y}_l\, t)) + c_k(\mathrm{t})\,\mathrm{c}_l(\mathrm{t});$$
$$\phi^2_{kl} = \phi^2_{kl}(\mathrm{t}) = 2(\mathrm{b}_k\, a_l(1-\cos(\mathrm{y}_k\, t))\sin(\mathrm{x}_l\, t) + c_k(\mathrm{t})\,\mathrm{s}_l(\mathrm{t}));$$
$$\phi^3_{kl} = \phi^3_{kl}(\mathrm{t}) = a_k a_l \sin(\mathrm{x}_k\, t)\sin(\mathrm{x}_l\, t) + s_k(\mathrm{t})\,\mathrm{s}_l(\mathrm{t}).$$

Then by (3), (4) we have that quadratic form $\xi(t)$ can be written as

$$\xi(\mathrm{t}) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (\phi^1_{kl}(\mathrm{t})\xi_k\xi_l + \phi^2_{kl}(\mathrm{t})\xi_k\eta_l + \phi^3_{kl}(\mathrm{t})\eta_k\eta_l).$$

Let us also denote the increments of the functions

$$\Delta\phi_{kl}^1 = \phi_{kl}^1(\mathrm{t}) - \phi_{kl}^1(\mathrm{s}); \quad \Delta\phi_{kl}^2 = \phi_{kl}^2(\mathrm{t}) - \phi_{kl}^2(\mathrm{s});$$
$$\Delta\phi_{kl}^3 = \phi_{kl}^3(\mathrm{t}) - \phi_{kl}^3(\mathrm{s}).$$

At first, present the auxiliary relationships concerning mean, variance and variance of increments for the process $\xi(t)$.

Lemma 1. The mean, variance and variance for the increments of stochastic form $\xi(\mathrm{t})$ equal:

$$E\xi(\mathrm{t}) = \sum_{k=1}^{\infty} (\phi_{kk}^1(\mathrm{t}) + \phi_{kk}^3(\mathrm{t}));$$

$$D\xi(\mathrm{t}) = \sum_{k,l=1}^{\infty} \left(2(\phi_{kl}^1(\mathrm{t}))^2 + (\phi_{kl}^2(\mathrm{t}))^2 + 2(\phi_{kl}^3(\mathrm{t}))^2 \right);$$

$$D(\xi(\mathrm{t}) - \xi(\mathrm{s})) = \sum_{k,l=1}^{\infty} \left(2(\Delta\phi_{kl}^1)^2 + (\Delta\phi_{kl}^2)^2 + 2(\Delta\phi_{kl}^3)^2 \right).$$

Proof.

Since $\xi_k, \eta_l, k \ge 0, l \ge 0$, are jointly independent Gaussian with mean 0 and variance 1 then the average of the process is

$$E\xi(\mathrm{t}) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (\phi_{kl}^1(\mathrm{t})\,\mathrm{E}\,\xi_k\xi_l + \phi_{kl}^2(\mathrm{t})\,\mathrm{E}\,\xi_k\eta_l + \phi_{kl}^3(\mathrm{t})\,\mathrm{E}\,\eta_k\eta_l) = \sum_{k=1}^{\infty} (\phi_{kk}^1(\mathrm{t}) + \phi_{kk}^3(\mathrm{t}))$$

Deriving the variance of the process $\xi(t)$ we should first find the second moment

$$E(\xi(\mathrm{t}))^2 = E\left(\sum_{k,l=1}^{\infty} (\phi_{kl}^1(\mathrm{t})\,\mathrm{E}\,\xi_k\xi_l + \phi_{kl}^2(\mathrm{t})\,\mathrm{E}\,\xi_k\eta_l + \phi_{kl}^3(\mathrm{t})\,\mathrm{E}\,\eta_k\eta_l) \right)^2.$$

By Isserlis formulas we can compute the moment of the forth order for standard Gaussian random variables:

$$EX_1X_2X_3X_4 = EX_1X_2EX_3X_4 + EX_1X_3EX_2X_4 + EX_1X_4EX_2X_3.$$

Then we obtain

$$E(\xi(\mathrm{t}))^2 = E\sum_{k=1}^{\infty}\sum_{l=1}^{\infty} (\phi_{kk}^1(\mathrm{t})\phi_{ll}^1(\mathrm{t}) + 2(\phi_{kl}^1(\mathrm{t}))^2 + (\phi_{kl}^2(\mathrm{t}))^2 + \phi_{kk}^3(\mathrm{t})\phi_{ll}^3(\mathrm{t}) + 2(\phi_{kl}^3(\mathrm{t}))^2 + 2\phi_{kk}^1(\mathrm{t})\phi_{ll}^3(\mathrm{t})).$$

Therefore, the variance of the process $\xi(t)$ should be

$$D\xi(\mathrm{t}) = E(\xi(\mathrm{t}))^2 - (\mathrm{E}\,\xi(\mathrm{t}))^2 = \sum_{k,l=1}^{\infty} \left(2(\phi_{kl}^1(\mathrm{t}))^2 + (\phi_{kl}^2(\mathrm{t}))^2 + 2(\phi_{kl}^3(\mathrm{t}))^2 \right).$$

Similarly, it can be proved the formula for variance of process increments $\xi(\mathrm{t}) - \xi(\mathrm{s})$.

If we put $d_{kl} = \sup_{t\in[0,\mathrm{T}]} (2(\phi_{kl}^1(\mathrm{t}))^2 + (\phi_{kl}^2(\mathrm{t}))^2 + 2(\phi_{kl}^3(\mathrm{t}))^2)$. Then $\sqrt{D\xi(\mathrm{t})} \le \left(\sum_{k,l=1}^{\infty} d_{kl} \right)^{1/2} := \delta_0$.

Under some conditions it could be shown that $(D(\xi(\mathrm{t}) - \xi(\mathrm{s})))^{1/2} \le K \cdot |\,\mathrm{t} - \mathrm{s}\,|^{\beta}, \quad \beta \in (0,1]$,

where K is a some constant.

The following theorem gives the upper bound of the backlog buffer content of the system with respect to input and output process providing equal weights for them.

Theorem 3. Suppose that the input traffic of system (3) is FBM $X_{\alpha}(t)$. If

$$x_0 > \frac{2\sqrt{2}\max\{\delta_0, K(\mathrm{T}/2)^{\beta}\}}{\beta}, \text{ then}$$

the inequality

$$P\{Q > b\} < 4e^{\frac{3}{\beta}} \exp\left\{ -\frac{x_0}{2\sqrt{2}\delta_0} \right\} \left(\frac{x\beta}{2\sqrt{2}\delta_0} \right)^{2/\beta} \left(1 + \frac{2x_0}{\sqrt{2}\delta_0} \right)^{1/2}.$$

Using the obtained results, it is possible to estimate the required buffer size in the framework of different measurement options. The estimation is performed with given accuracy and reliability.

7. Case study

In this section we will investigate the dependence of the Hurst index and the threshold value in backlog process, the dependence of the service characteristics such as rate and the value of the threshold.

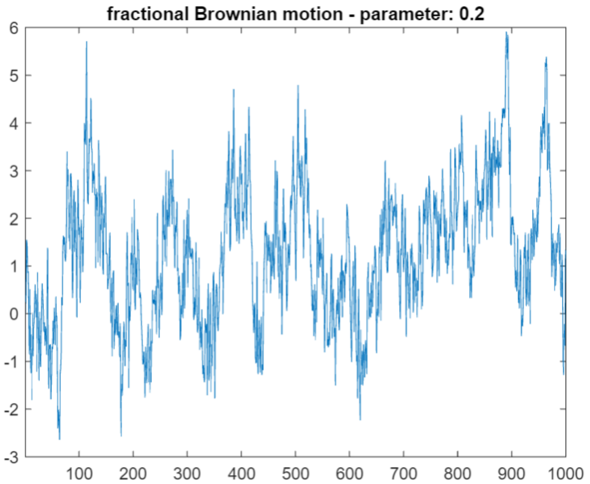

Figure 1: The sample of FBM with Hurst index 0.2

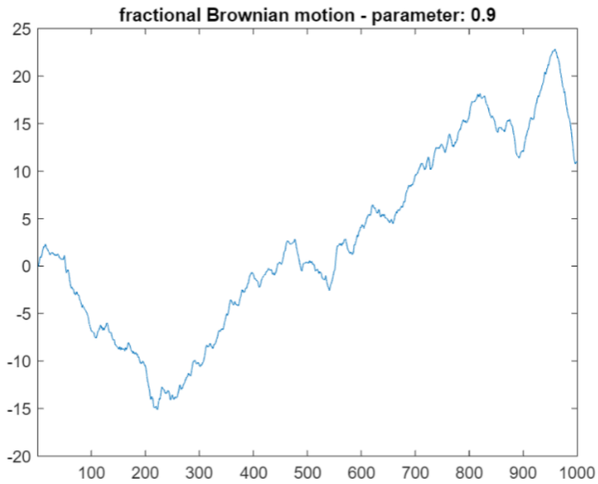

Figure 2: The sample of FBM with Hurst index 0.9

At first, we present the results of simulation of FBM with different Hurst indices. According to the value of the Hurst parameter (α) FBM detects a long dependence and a short dependence. Let's demonstrate each of these two cases on the graphs. As it can be seen in Fig. 1 and in Fig. 2, the higher the Hurst index, the smoother the curve will be.

Let's consider the dependencies of the upper bound of buffer content and simulation on different traffic characteristics. Let us remind the notations that were used in this paper: m is the average traffic rate, C is the service rate (it is necessary that C>m), α is the Hurst index, T is the time period over which the service process is considered, and is a certain traffic coefficient, the probability of traffic buffer overflow is less then a given ε_b that can be considered as significance level.

Applying Theorem 1 and substituting the values of above described parameters: $\varepsilon_b = 0.05$; C = 100; m = 90; a = 1; T = 1; $\alpha = 0.9$, we obtain the upper bound of traffic backlog threshold should be b >= 62,6. For parameters $\varepsilon_b = 0.05$; C = 100; m = 90; a = 1; T = 1; $\alpha = 0.2$ it follows that b >= 131.2.

Table 1

The dependence of threshold level on the Hurst index under unchanged other parameters

№	Significance level, ε_b	Service rate, C	Traffic rate, m	Time period, T	Hurst index, H	Threshold, b
1	0.05	100	90	1	0.1	189.4
2	0.05	100	90	1	0.2	131.2
3	0.05	100	90	1	0.3	106.0
4	0.05	100	90	1	0.4	91.4
5	0.05	100	90	1	0.5	81.8
6	0.05	100	90	1	0.6	74.9
7	0.05	100	90	1	0.7	69.8
8	0.05	100	90	1	0.8	65.8
9	0.05	100	90	1	0.9	62.6

Thus, changing the value of the Hurst index as an input argument (and not changing the value of the service rate *C* and the average rate of arrival traffic *m*), we have the following dependence, shown in Fig. 3. It's seen that the larger Hurst index is the smaller threshold value is.

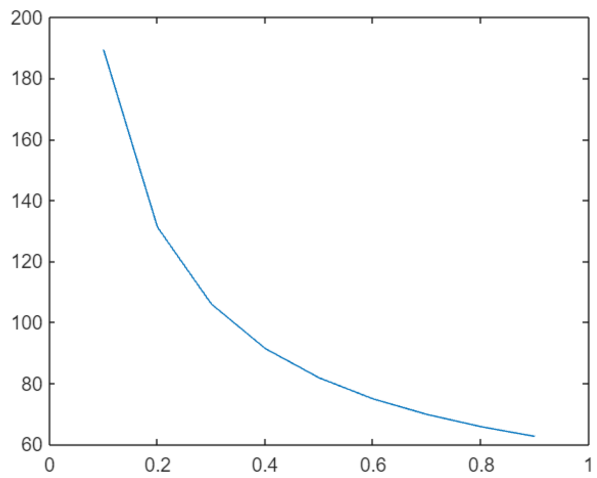

Figure 3: The functional relation of threshold and Hurst index

Table 2
The dependence of threshold level on average traffic rate under unchanged other parameters

№	Significance level, ε_b	Service rate, C	Traffic rate, m	Time period, T	Hurst index, H	Threshold, b
1	0.05	100	99	1	0.9	75.1
2	0.05	100	90	1	0.9	62.6
3	0.05	100	80	1	0.9	48.4
4	0.05	100	70	1	0.9	34.0
5	0.05	100	60	1	0.9	19.3
6	0.05	100	50	1	0.9	4.1

The inequality in Theorem 1 can be analyzed in different ways. Table 2 shows how the changes in average traffic rate influences on the level of threshold. It is clear that such a dependence should be reversed, which is confirmed by obtained results

8. Conclusions

One of the most important findings of traffic studies over the last years is the property of self-similarity in packet network. This paper investigated self-similar traffic model driven by FBM. We defined the arrival and backlog processes of stochastic network and obtained estimation of upper bound probability buffer overflow.

We also explored a traffic that enters as input on some time-invariant linear system and found the estimation of buffer content distribution taking into account input traffic and a response of stochastic system. Finally, we examined the obtained results in one particular case and showed how the threshold level of buffer overflow depends on such parameters of stochastic network as Hurst index and arrival traffic rate.

We see further research of the problem in the study of generalized FBM to consider traffic data. The generalization of FBM can be guided in several directions. It is possible to generalize the type of covariance function for Gaussian processes, while the stationary increments property can be lost. The other way to generalize FBM is to go away from Gaussian distributions allowing to include the distribution with heavy tails that is more attractive in case of network framework.

The algorithms for statistical modeling of self-similar traffic allows to use of computational experiment methods for research and analysis of traffic in telecommunication networks.

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