# Structural-Schematic Approach in the Modelling of Dynamic Systems

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### Abstract
The structural-schematic approach is offered for the stage of construction of model of mathematical modeling process of the dynamic systems. The phase-space (state-space) method is utilized. The methods of construction of schemes and equations are illustrated for more simple cases, and ideology of construction of mathematical models of the multi-circuit systems is demonstrated.

### Keywords [1]
Differential equation systems, operator equations, phase space, control.

## 1. Introduction

The theory of mathematical modeling, as a separate scientific direction, appeared not so long ago and is only beginning to take shape. Although many interesting results are already known in various disciplinary fields: engineering [1-3], chemistry, chemical technology and production [4,5], biology [5-8], economics and ecology [9,10]. In all these works, it is shown how to use the knowledge of each specific field as much as possible to apply it to the construction of a mathematical model of a particular process. In principle, for an experienced scientist, building a simple mathematical model is not too difficult task, and the result can be ambiguous. It is much more difficult to choose one single mathematical model which, on the one hand-side, is quite simple to study, and on the other, most adequately describes the real process. But, as experience shows, for a beginner, a senior student, a graduate, the task of constructing a model turns out to be extremely difficult (this means the stage of transition from a humane description to a formal mathematical abstraction record). Modern textbooks [5,10-12], which are primarily focused on the specifics of each subject area, and scientific works in which the principles are clearly formulated and the mathematical modeling process is written step by step [13], can help solve this problem.

Recently, a group of modeling methods has been widely developed, which are based on a multi-theoretical approach to the description of the behavior of dynamic systems, and allow the most effective use of computer technology as a tool of engineering calculation practice [14-16].

Unfortunately, the construction of state equations of the studied systems is not a sufficiently formalized procedure, and any researcher needs to show considerable skill. This creative stage is actually zero-level, the initial stage in the modern theory of systems modeling [13,17,18].

One of the most common ways in engineering to construct the state equations of the studied system is to present it in the form of a structural diagram of the system in phase variables (state space variables). Such a structural scheme usually consists of integrators, amplifiers and adders, and other elementary links [12,14-16,18]. There are many varieties of modeling schemes for the same system [12,17-19], hence the ambiguity of the description of dynamic systems using state equations.

## 2. Formulation of the problem

In this work, we will mainly consider the methods ofconstruction the mathematical models of dynamic control systems with single input and single output (SISO) using structural diagrams in state variables, the dynamics of which can be described by an ordinary differential equation of the form.

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \ldots + b_0 u(t) \tag{1}$$

under initial conditions

$$y(0) = y_0, \; y^{(1)}(0) = y_0^{(1)}, \ldots, y^{(n-1)}(0) = y_0^{(n-1)}; \, m < n$$

or by a system of ordinary differential equations (ODE).

In the last part of our work, we will briefly show that the methods that are applicable for systems such as SISO also work successfully in the case of multi-circuit systems with multiple inputs and outputs (MIMO).

## 2.1.  The method of combining derivatives.

Let us first consider a simpler case when the initial conditions of equation (1) are zero. At the same time, using Laplace transformation [20], equation (1) in operator form will be rewritten as

$$A(p)Y(p) = B(p)U(p), \tag{2}$$

where

$$A(p) = p^n + \sum a_i p^i; \qquad B(p) = \sum b_i p^i \; .$$

Let's present equation (2) in the form

$$Y(p)/B(p) = U(p)/A(p)$$

We replace this equation with the following two:

$$Y(p)/B(p) = X(p), \quad U(p)/A(p) = X(p),$$

where $X(p)$ - auxiliary variable Laplace-image of $x(t)$.

In the differential form, the last two equations will take the form:

$$\frac{d^n x(t)}{dt^n} + a_{n-1}\frac{d^{n-1}x(t)}{dt^n} + \ldots + a_0 x(t) = u(t) \tag{3}$$

$$b_m \frac{d^m x(t)}{dt^m} + b_{m-1}\frac{d^{m-1}x(t)}{dt^n} + \ldots + b_0 x(t) = y(t) \tag{4}$$

Thus, instead of solving equation (1), you can solve equations (3), (4). Let's build a structural diagram of the solution of equation (3), for which we will solve it with respect to the higher derivative

$$\frac{d^n x(t)}{dt^n} = -a_{n-1}\frac{d^{n-1}x(t)}{dt^n} - a_{n-1}\frac{d^{n-2}x(t)}{dt^{n-2}} - \ldots - a_0 x(t) + u(t) \tag{5}$$

Suppose that the $n$-th derivative of the function $x(t)$ is known, then, applying it to a chain of serially connected integrators, at the output of each subsequent integrator we will have derivatives of the function $x(t)$ of lower orders. If we multiply each of these derivatives by the corresponding coefficient $a_i$ and form their negative sum together with the function $u(t)$, then we get the right-hand side of equation (5), i.e., the $n$-derivative of the function $x(t)$, which was previously assumed to be known. The function $y(t)$, which is a solution of equation (1), is obtained by a linear combination of derivatives of $x(t)$, taken with coefficients $b_1$ in accordance with equation (4). The structural diagram of the solution of equations (3), (4) is shown in Fig. 1

If the initial conditions of equation (1) are non-zero, they must be converted into the initial conditions of equation (3). Assuming that equations (3), (4) are equivalent to (1) and with non-zero initial conditions, we rewrite these equations in operator form:

$$A(p)Y(p)+Y_0(p,a)=B(p)U(p),$$
$$A(p)X(p)+X_0(p,a)=U(p), \qquad (6)$$
$$B(p)X(p)+X_0(p,b)=Y(p),$$

where

$Y_0(p,a),\ X_0(p,a),\ X_0(p,b)$ - polynomials of the initial conditions.

Substituting $U(p)$ and $Y(p)$ from the last two equations (6) into the first, we get

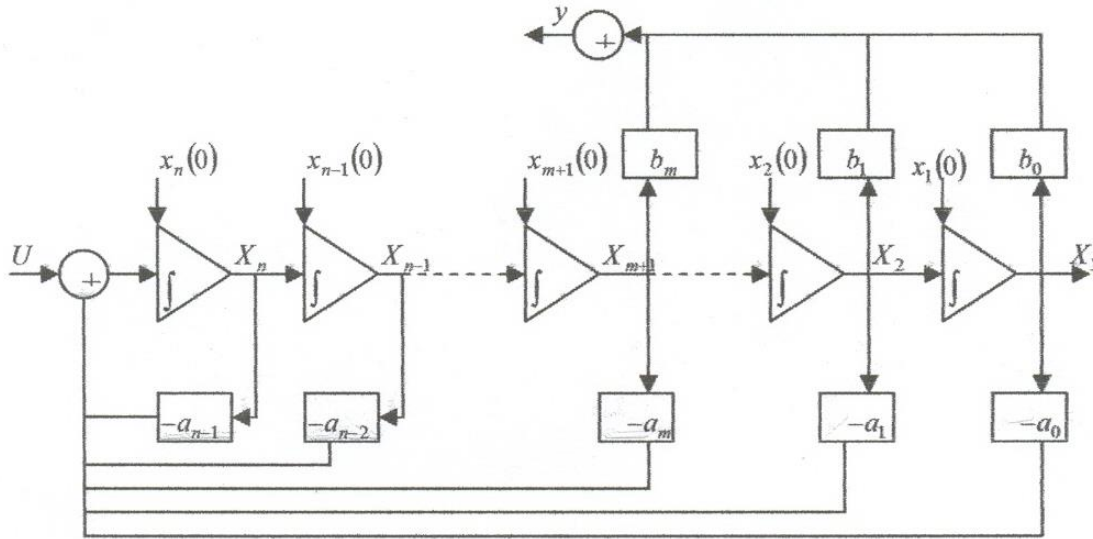$$A(p)[B(p)X(p)+X_0(p,b)]+Y_0(p,a)=B(p)[A(p)X(p)+X_0(p,a)] \qquad (7)$$



**Figure 1:** Structural diagram in state variables for method of combining derivatives

By equating the coefficients with the same degrees of variable $p$ of this equality, we find the initial conditions of the integrators of the structural scheme of Fig. 1.

The application of this method for constructing state equations does not require transformation of equation (1). It is possible to make a scheme in state variables directly according to the form of equation (1), because its coefficients are the coefficients of the structural scheme in state variables. Recalculation of the initial conditions is carried out according to equation (7).

## 2.2. The method of successive integration.

Let's rewrite, using Laplace-transform [20], equation (1) in operator form with zero initial conditions in the form

$$p^n Y(p)+\sum_{i=0}^{n-1} a_i p^i Y(p)=\sum_{i=0}^{m} b_i p^i U(p)$$

from where we have

$$Y(p)=\sum_{i=0}^{n-1}\frac{1}{p^{n-i}}[b_i U(p)-a_i Y(p)]; \quad b_i=0 \text{ if } i>m \qquad (8)$$

Let's make a chain of **n** serially connected integrators. We will take the signal at the output of the rightmost integrator as $y(t)$. The fulfillment of equality (8) is ensured if the difference $b_0 u(t)-a_o y(t)$ is applied to the input of the leftmost integrator, the difference $b_1 u(t)-a_1 y(t)$ and the output of the previous integrator are applied to the input of the next one, etc. The structural diagram of the solution of equation (8) will then take the form shown in Fig. 2.

If we again choose as state variables $x(t)$ the outputs of the integrators of the structural diagram of Fig. 2, then we will get a system of equations

$$x_1'(t) = x_2(t) - a_{n-1}x_1(t);$$

$$x_2'(t) = x_3(t) - a_{n-2}x_1(t);$$

$$\dots\dots\dots\dots\dots\dots\dots\dots \qquad (9)$$

$$x_{n-1}'(t) = x_{n-m+1}(t) - a_m(t)x_1(t) + b_m u(t);$$

$$x_n'(t) = -a_0 x_1(t) + b_0 u(t);$$
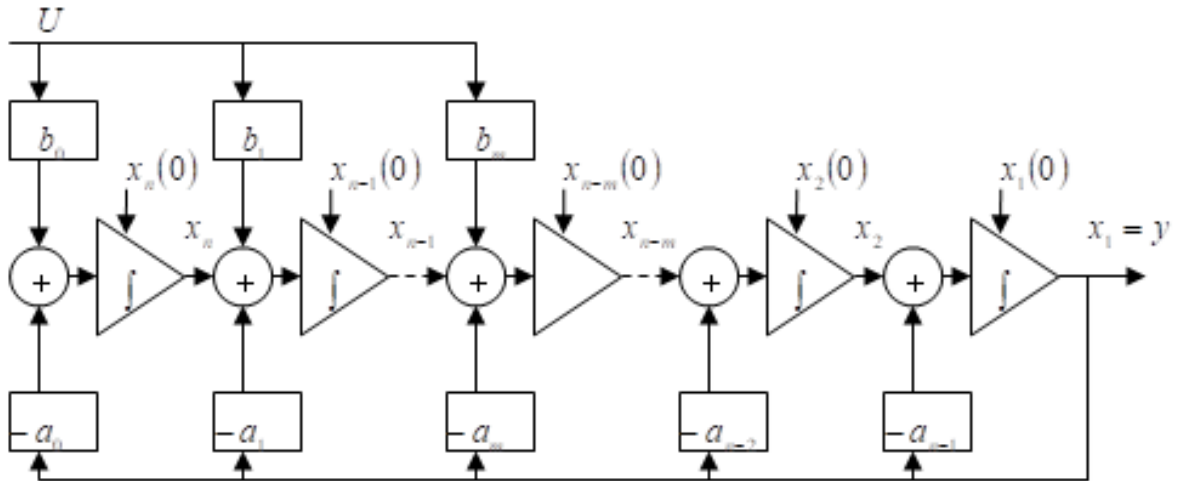
$$y(t) = x_1(t). \qquad (10)$$



**Figure 2:** Scheme in state variables for the method of successive integration

With non-zero initial conditions of equation (1), the initial conditions of the integrators of the structural diagram are determined by the ratio [18]

$$x_i(0) = \sum_{k=0}^{i} a_{n-k} y_0^{(i-k)}$$

As for the method of combining derivatives, the coefficients of equation (1) are simultaneously the coefficients of the structural scheme in the state variables.

## 2.3. Method of decomposition of the transfer function into elementary fractions.

A rather promising approach for the synthesis of systems using the state space method is the construction of a structural scheme in state variables by decomposing the transfer function into elementary fractions. Its idea is as follows. Equation (1) is presented in the form of a transfer function

$$\frac{Y(p)}{U(p)} = \frac{b_m p^m + b_{m-1}p^{m-1} + \dots + b_0}{p^n + a_{n-1}p^{n-1} + \dots + a_0} \qquad (11)$$

Let's decompose this transfer function into elementary fractions

$$\frac{Y(p)}{U(p)} = d + \frac{c_1}{(p - \lambda_1)} + \frac{c_2}{(p - \lambda_2)} + \dots + \frac{c_n}{(p - \lambda_n)},$$

hence

$$Y(p) = dU(p) + \left[ \frac{c_1}{(p - \lambda_1)} + \frac{c_2}{(p - \lambda_2)} + \dots + \frac{c_n}{(p - \lambda_n)} \right] U(p). \qquad (12)$$

The coefficient $d$ will be different from zero only when $m = n$. The structural scheme of the state variables of equation (12) is shown in Fig. 3.
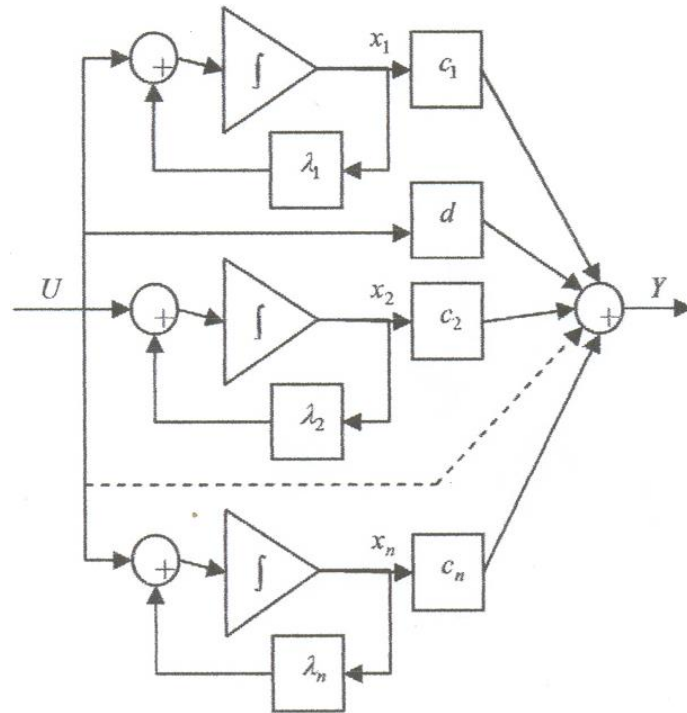
**Figure 3:** Scheme in state variables for the method of decomposition of the transfer function into elementary fractions

The state equations have the form

$$
\begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u; \tag{13}
$$

$$
y = c_1 x_1 + c_2 x_2 + \dots + c_n x_n + du. \tag{14}
$$

## 3. Equation of state of multi-circuit systems.

The advantages of the state space method are most evident when building models of complex multidimensional and multi-circuit systems.

It is appropriate to use the following steps of constructing state equations for modeling complex systems. Usually, a functional scheme of the system is given, the analysis of which must be performed. From the functional scheme by determining the dynamics equations (transfer functions) of the elements of this system, you can go to the algorithmic scheme. Such a scheme makes it possible to preserve the mutual relations of the elements of the functional scheme of the system and at the same time obtain a mathematical description of its dynamics. (This step is demonstrated in Example 2). Then, for all elements of the system, structural schemes are drawn up in state variables, which are combined with each other in the same way as the elements of the system under study. This method of constructing state equations has an important advantage. It consists in the fact that some state variables will correspond to the real variables of the system, which makes it possible to trace their behavior over time after solving the state equations. Let us consider the methodology for determining the equations of state for a multidimensional system of the MIMO type using a specific example.

**Example 1.** Figure 4 shows the algorithmic diagram of a multi-circuit system (MIMO). It has three inputs *u1*, *u2*, *u3* and two outputs *y1*, *y2*. The presence of many internal connections between its elements makes it difficult to analyze this system using classical methods of automatic control theory. Let's consider a method for obtaining equations of state for this system. Figure 5 shows a circuit-scheme in state variables, and each element of the algorithmic circuit-scheme has its own scheme in state

variables and they are connected to each other in the same way as the elements of the algorithmic circuit-scheme.
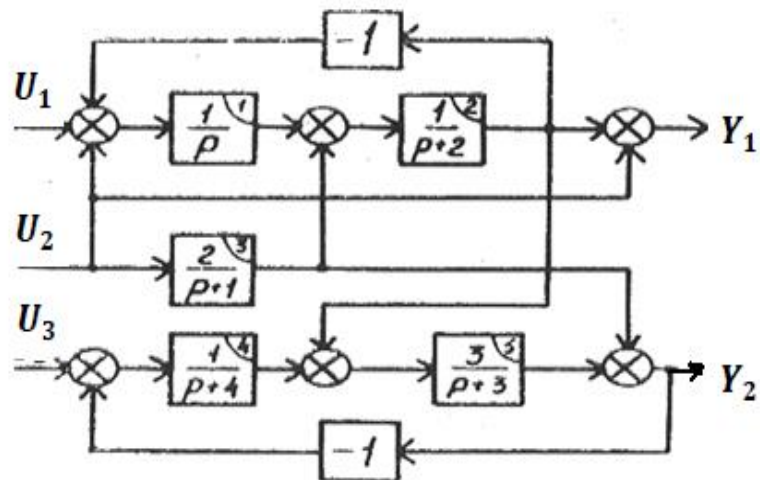


**Figure 4:** Algorithmic scheme of a multi-circuit system

The equations connecting the state variables $(x_1 - x_5)$ with each other, as well as the input $(u_1 - u_3)$ and output $(y_1, y_2)$, are obtained directly from the diagram in the state variables:
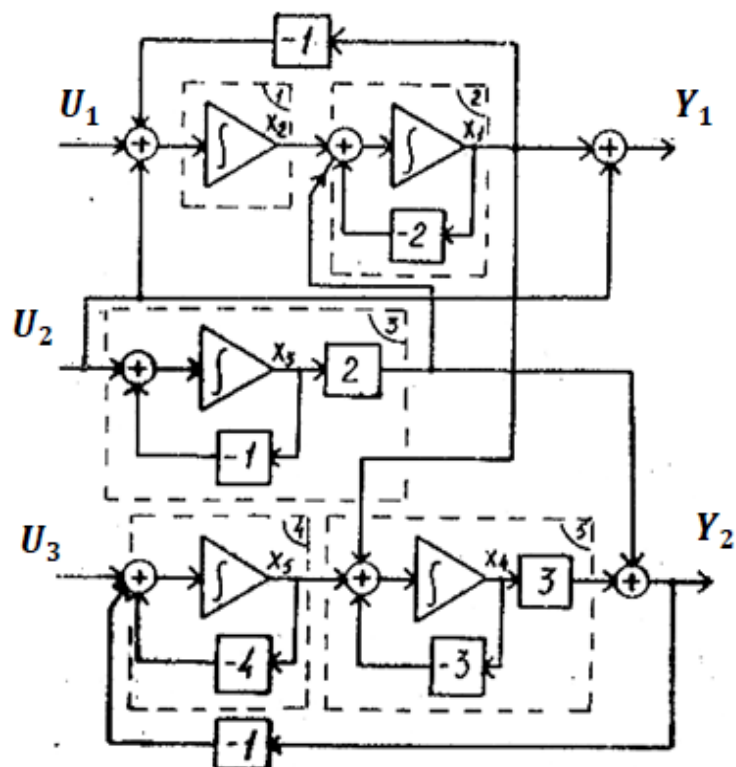


**Figure 5:** Scheme in state variables of a multi-circuit MIMO system

$$\begin{cases} \dot{x}_1 = -2x_1 + x_2 + 2x_3 \\ \dot{x}_2 = -x_1 + u_1 + u_2 \\ \dot{x}_3 = -x_3 + u_2 \\ \dot{x}_4 = x_1 - 3x_4 + x_5 \\ \dot{x}_5 = -2x_3 - 3x_4 - 4x_5 + u_3 \end{cases}$$

$$y_1 = x_1 + u_2 \quad y_2 = 3x_4 + 2x_3$$

Finally, we want to show one simple example, of which confirms the practical value of the results of the presented article.

**Example 2.** Let's consider a simplified diagram of the operation of a radar station (Fig. 6). Namely, we are interested in modeling the tracking system, which receives as input the angle of extinction of the observed target, and its precise processing by the radar. In Fig. 7 shows a functional scheme of a power tracking system, containing as a measuring-converting element an SD-SP synchro pair, a phase discriminator FD, a magnetic amplifier MU, an electric machine power amplifier EMU, a DC motor DW, a gearbox Red, a working mechanism RM, a tach generator TG in the circuit local feedback. The algorithmic scheme of this system is shown in Fig. 8. The corresponding scheme in state variables is shown in Fig. 9. For clarity of the circuit in state variables, individual elements are constructed using different methods. (See for detail [18]). System of equations for the scheme in Fig. 9 looks like

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = x_3 \\
\dot{x}_3 = -\dfrac{1}{T^2} x_2 - \dfrac{2\xi}{T} x_3 + \dfrac{K_{DW}}{T^2} x_4 + \dfrac{K_{DW}}{T^2} x_5 \\
\dot{x}_4 = -\dfrac{1}{T_1} x_4 + \dfrac{K_{EMU}}{(T_1 - T_2)} \dfrac{K_{MU}}{T_{MU}} x_6 \\
\dot{x}_5 = -\dfrac{1}{T_2} x_5 + \dfrac{K_{EMU}}{(T_1 - T_2)} \dfrac{K_{MU}}{T_{MU}} x_6 \\
\dot{x}_6 = -\left(K_P + \dfrac{K_{TG}}{T_{TG}}\right) x_1 - \dfrac{1}{T_{MU}} x_6 + \dfrac{1}{K_{TG}} x_7 + K_P \alpha \\
\dot{x}_7 = \dfrac{K_{TG}}{T_{TG}} x_1 - \dfrac{1}{T_{TG}} x_7 \\
\beta = x_1
\end{cases}
$$

The all obtaining results are constructive from the point of view of performing computational experiments. All presented analytics are currently implemented without problems in a fairly wide range of IT, for example, one of the most common is the SIMULINK toolkit of the MATLAB package [14-16]. In the future, they can be extended to the case of multi-circuit systems, which is described in terms of functional-differential equations (FDE), taking into account the factor of argument deviation.



**Figure 6:** System of Radar stations

$$\alpha(t) = \alpha_0 = 5^o$$
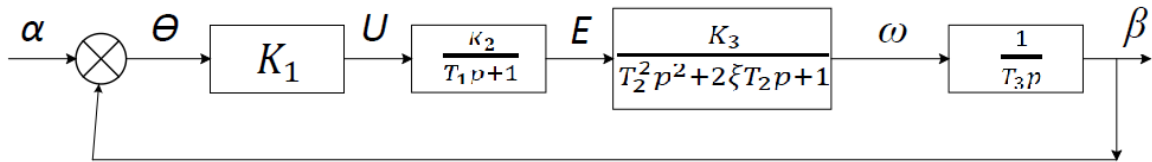
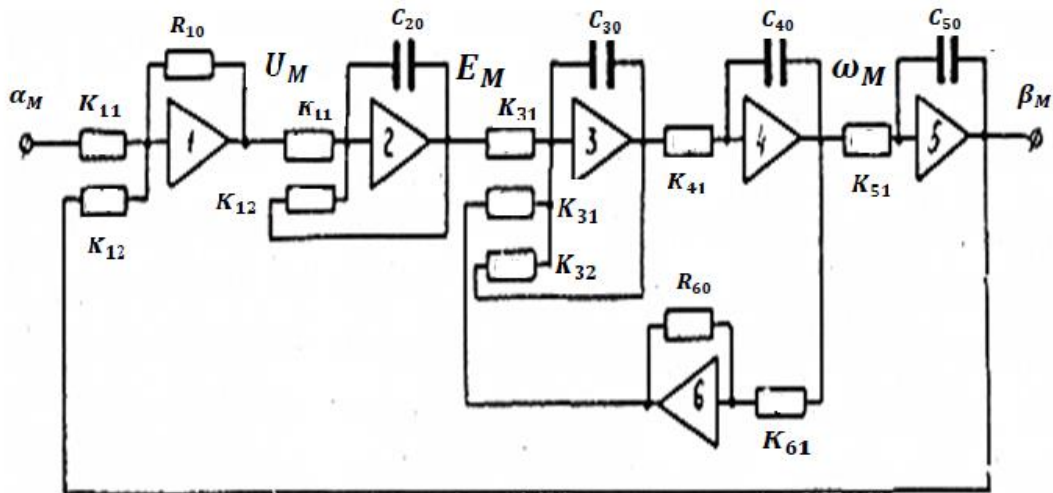**Figure 7:** Functional scheme of a power tracking system



**Figure 8:** Algorithmic scheme in elementary chain elements of a power tracking system
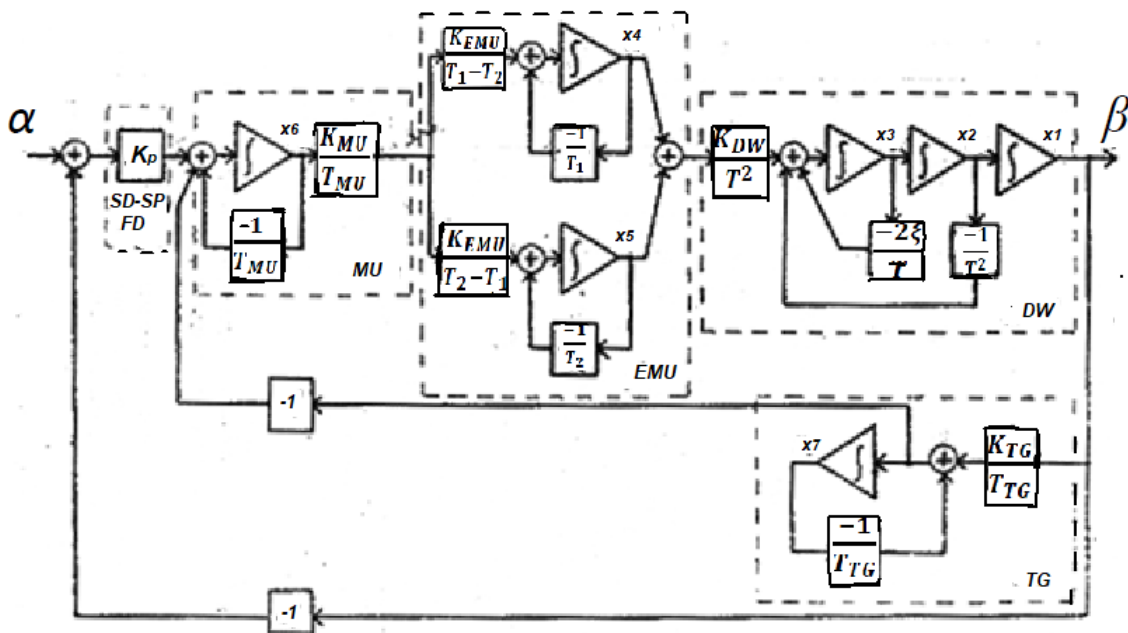


**Figure 9:** Scheme in state variables of a power tracking system

## 4. Conclusion

The following results were achieved in this work:

- Using a fairly simple approach, we showed three methods for constructing systems of equations of state in phase space for the case when the type of input and output of the system (SISO) is known, but its internal structure is not known.

- We have demonstrated the possibility of using state space methods in the case of multi-circuit systems MIMO, that allow a certain decomposition into SISO systems.

## 5. Acknowledgements

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