Application of Daubechies wavelet analysis in problems of acoustic detection of UAVs

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Abstract

One of the effective directions in the detection of UAVs is acoustic surveillance, the main advantage of which is the operation in passive mode, which ensures the secrecy of the applied means, and thus the safety of the operating personnel. Noise generated by the UAVs propulsion system and propeller is a significant de-masking feature. Creation and improvement of methods of detection, direction finding and recognition of small UAVs by receiving and processing of sound signals is an urgent task. When recognizing objects, the most important and problematic task is the selection of features of the acoustic signal. The selection of features affects the process of building a recognition algorithm, as well as the performance of the entire system and the quality of recognition. The use of spectral analysis allows to allocate the main features of the UAV quite effectively, such as: engine speed, the presence of harmonics of the speed, the nature of the behavior of the envelope of harmonics. A promising method for identifying the characteristic features of acoustic radiation of UAVs is Daubechies wavelet analysis. Wavelet spectrum analysis is a powerful tool for detecting and recognizing a specific type of UAV. The method provides much more informative data than simple Fourier spectral analysis. The main idea of Daubechies wavelet analysis is to decompose the studied acoustic signal by a system of Daubechies basis functions, which have special properties, in particular good localization in the time domain, which gives a significant advantage in the analysis of non-stationary acoustic signals.

Keywords

acoustic signal, UAV detection, spectrum analysis, wavelet transform, Fourier transform, Daubechies wavelet function, wavelet coefficients

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1. Introduction

Expansion of the application spheres of small unmanned aerial vehicles (UAVs) in various fields of human activity (military applications, meteorological observations, environmental protection, etc.) provides a significant economic effect. At the same time, the use of UAVs creates a number of problems associated with inadequate behavior of some UAV owners, unauthorized monitoring

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of objects and territories, etc. Accordingly, the task of UAV detection becomes relevant, which can be solved by means of active and passive radar, thermal location, video surveillance or acoustic observation systems [1, 2].

As follows from the results of studies, the total acoustic emission spectrum of a small UAV is due to harmonic random components. In known algorithms for UAV detection and direction finding, the problem is solved for a signal in a sufficiently narrow frequency band. However, the narrow-band processing of acoustic UAV signals does not allow to fully utilize the energy and information of the received signal. This becomes possible only with appropriate signal processing using wavelet analysis based on the Daubechies basis [3].

The application of spatial and temporal wavelet processing for acoustic signals of UAVs in the tasks of aircraft detection provides the expansion of the dynamic range of devices for receiving and processing signals and increasing noise immunity, which occurs due to adaptive suppression of interference in the bandwidth of the receiving device with minimal distortion of the useful signal [4]. The maximum number of suppressed interference increases, in-phase summation of acoustic signals in communication channels in the entire frequency band is provided, which allows to more fully utilize the energy of the acoustic signal of the UAV coming to the input and, consequently, allows to increase the signal-to-noise ratio at the output [5].

Thus, the implementation of wavelet algorithms for acoustic signal processing based on the Daubechies wavelet function opens up a wide range of possibilities to further improve UAV detection.

2. Literature review and problem statement

Fourier analysis is based on the statement that any 2π-periodic function can be decomposed into components, i.e., can be obtained by superposition of integer stretches of the basis function e^{ix} [6].

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx},$$

where c_n is Fourier coefficients

$$c_n = \frac{1}{2\pi} \int_0^{2\pi} f(x) e^{-inx} dx.$$

Fourier transform

$$\widehat{f}\,(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt,$$

gives spectral information $\widehat{f}\,(\omega)$ about the acoustic signal $f(t)$ and describes its behavior in the frequency domain ω, which is very important in acoustic UAV detection [7].

When moving to the Fourier frequency domain $\widehat{f}\,(\omega)$, time information is completely lost t, which makes the Fourier spectral analysis method unsuitable for processing non-stationary acoustic signals $f(t)$, in which the determining value is the moment in time t, at which the characteristic distortions in the acoustic signal emitted by the UAV occurred [8].

In contrast to the short-time Fourier transform

$$\widehat{f}\,(\omega, t) = \int_{-\infty}^{\infty} e^{-i\omega t} \left(f(t) \cdot W\,(t)\right) dt,$$

which provides a uniform grid (figure 1) in the frequency-time domain $\widehat{f}\,(\omega, t)$ through the use of the window function $W\,(t)$, the wavelet transform has non-uniform resolution, which allows the acoustic signal of the UAV $f(t)$ to be investigated both locally and completely [9].

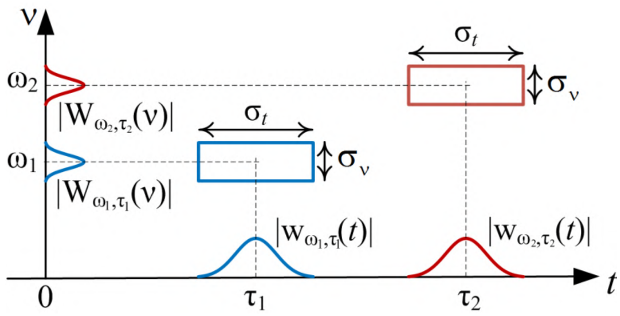


Figure 1: Time-frequency resolution of the Fourier transform.

Since the frequency ω is inversely proportional to the period T, i.e. $\omega = 1/T$, a narrower window $W\,(t)$ is required to localize the high-frequency component $\omega \to \max$ of the acoustic signal $f(t)$ and a wider window $W\,(t)$ for the low-frequency component $\omega \to \min$. The short-time Fourier transform $\widehat{f}\,(\omega, t)$ is acceptable for a signal with a relatively narrow bandwidth $\Delta\omega \to \min$, but acoustic signals $f(t)$ are not. For an acoustic signal it would be desirable to have a window $W\,(t)$, capable of changing its width with changing frequency ω [10].

Let us introduce a function $\phi \in L^2(R)$, satisfying the condition

$$\int_{-\infty}^{\infty} \frac{\left|\hat{\phi}(\omega)\right|^2}{|\omega|} d\omega < \infty,$$

and we'll call it the "base wavelet".

With respect to each basis wavelet, the wavelet transform is defined as

$$\left(\Psi_\phi f\right)(\tau, s) = |s|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \overline{\phi\left(\frac{t-\tau}{s}\right)} dt,$$

where s and τ are the scaling and shifting parameters $s, \tau \in R$; $a \neq 0$.

Then we denote

$$\phi_{\tau;s}(t) = |s|^{-\frac{1}{2}}\, \phi\left(\frac{t-\tau}{s}\right),$$

and the transformation will take the form

$$\left(\Psi_\phi f\right)(\tau, s) = \langle f, \phi_{\tau;s} \rangle\,.$$

If the center and radius of the window function ϕ, respectively, are equal to t^* and Δ_ϕ, then $\phi_{\tau;s}(t)$ is a window function with center $\tau + st^*$ and radius $s\Delta_\phi$. Hence, the wavelet transform localizes the signal in the time window (figure 2) [11]

$$[\tau + st^* - s\Delta_\phi, \tau + st^* + s\Delta_\phi] .$$

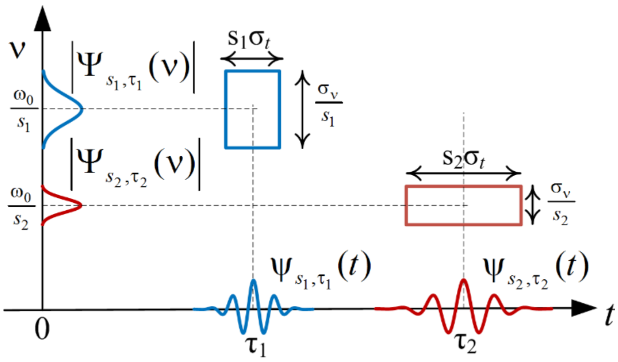


Figure 2: Time-frequency resolution of the Daubechies wavelet transform.

Thus it was shown not only that the Fourier transform is uninformative in the problems of analyzing non-stationary signals, which are acoustic signals, but also the fact that the basis of wavelet decomposition plays a major role in the effectiveness of using wavelet analysis in the task of acoustic detection of UAVs, so in this research work it is proposed to use the decomposition in Daubechies series, since this function is an orthogonal wavelet with a compact carrier computed iteratively.

3. Daubechies wavelet analysis of acoustic signals

To calculate the coefficients of the generating Daubechies wavelet filter n-th order, we need to specify only the number of zero moments of the wavelet function N, i.e., the order of the function is determined by the number of zero moments, hence $N = n$ [12].

Then the calculation of the generating Daubechies wavelet filter implies finding the coefficients of the polynomial

$$P_k = \frac{\prod_{i=-N+1}^{N} \left(\frac{1}{2} - i\right)}{\prod_{i=-N+1}^{N} (k - i)}, k = 1, \ \ldots, \ N,$$

which for all values of $k \neq i$ form a vector

$$P = \begin{pmatrix} P_N & 0 & P_{N-1} & 0 & \ldots & 0 & P_1 & 1 & P_1 & 0 & P_2 & 0 & \ldots & 0 & P_N \end{pmatrix}, \quad (1)$$

length $4N - 1$.

In case $N = 1$, then all values of coefficients of polynomial $P_{1,\,\ldots,\,4N-1}$ satisfying the condition $P_{1,\,\ldots,\,4N-1} < 1$ define the vector

$$P = \begin{pmatrix} P_1 & \ldots & P_{2N} \end{pmatrix}, \quad (2)$$

length $2N$, whose values correspond to the coefficients of the Daubechies wavelet filter of the 1st order.

If $N > 1$ is required to compute the roots of the coefficients of the polynomial P given by the vector (1).

Then the vector of coefficients of the polynomial P is transformed into the following form

$$P = \begin{pmatrix} \frac{P_2}{P_1} & \frac{P_3}{P_1} & \dots & \frac{P_{4N-1}}{P_1} \end{pmatrix},$$

length $L = 4N - 2$.

Let's form a square matrix A of order L

$$A_L = \begin{pmatrix} -P_1 & -P_2 & \dots & -P_{L-1} & -P_L \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix},$$

where the first row of the matrix A_L defines the coefficients of the characteristic equation, which has the form

$$\lambda^L - P_1\lambda^{L-1} - P_2\lambda^{L-2} - \ \dots \ - P_{L-1}\lambda - P_L = 0, \tag{3}$$

where the roots $\lambda_{1,\ \dots,\ L}$ of this equation are the eigenvalues of the matrix A_L. The order of the square matrix A_L is always a multiple of two since $L = 4N - 2$.

Solving the equation (3) by one of the numerical methods (by the method of half division, combined, iterations, etc.), we find the roots $\lambda_{1,\ \dots,\ L}$ of this equation and thus the vector λ of eigenvalues of the matrix A_L is formed.

$$\lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_L \end{pmatrix}.$$

The values of the vector λ should be arranged in ascending order

$$\lambda = \begin{pmatrix} \lambda_{\min} & \dots & \lambda_{\max} \end{pmatrix},$$

observing the condition $|\lambda_{1,\ \dots,\ L} + 1|$, and select only those values that match the condition of the expression

$$\lambda = \begin{pmatrix} \lambda_{K+2} & \dots & \lambda_{2K} \end{pmatrix},$$

where $K = 2N - 1$, then the length of the vector λ is equal to $M = 2K - (K + 2) + 1$ values.

Let's rearrange the values of the vector λ in ascending order

$$\lambda = \begin{pmatrix} \lambda_{\min} & \dots & \lambda_{\max} \end{pmatrix},$$

complying with the condition $|\lambda_{1,\ \dots,\ M}|$.

Thus we obtain a vector λ of length M, which includes the values of the roots of $\lambda_{1,\ \dots,\ M}$ arranged in ascending order

$$\lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_M \end{pmatrix}.$$

Then all values of the roots of $\lambda_{1,\ \dots,\ M}$ satisfying the condition $|\lambda_{1,\ \dots,\ M}| < 1$ define the vector

$$\lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_H \end{pmatrix},$$

where H depends on the condition $|\lambda_{1,\ \dots,\ M}| > 1$, i.e., how many values of $\lambda_{1,\ \dots,\ M}$ are modulo greater than one.

Let's set the vector

$$O = \begin{pmatrix} O_1 & \dots & O_N \end{pmatrix},$$

where $O_{1,\ \dots,\ N} = -1$, since the values of the roots of $\lambda_{1,\ \dots,\ H}$ are complex numbers, the values of $O_{1,\ \dots,\ N}$ are converted to complex form, hence $O_{1,\ \dots,\ N} = -1.0000 + 0.0000i$.

As a result, we obtain the vector

$$\lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_H & O_1 & \dots & O_N \end{pmatrix}, \tag{4}$$

defined by the root values $\lambda_{1,\ \dots,\ H}$ and unit vectors $O_{1,\ \dots,\ N}$, of length $J = H + N$.

Then let us represent the vector λ in the form

$$\lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_J \end{pmatrix}, \tag{5}$$

equating the values of vector (4) to the notations (5).

So, having a pre-formed vector of values of roots of $\lambda_{1,\ \dots,\ \mathrm{J}}$ polynomial, let us calculate the vector of values of coefficients of this polynomial according to the expression

$$P_k = P_k - \lambda_j P_i, \tag{6}$$

where in cases where $j = 1, \ \dots, \ J$, then $k = 2, \ \dots, \ j+1$, $i = 1, \ \dots, \ j$, and the initial values of the coefficients correspond to the vector

$$P = \begin{pmatrix} P_1 & P_2 & \dots & P_{J+1} \end{pmatrix},$$

length $J + 1 = 2N$, where $P_1 = 1$, $P_{2,\ \dots,\ J+1} = 0$, since the values of the roots $\lambda_{1,\ \dots,\ J}$ of the polynomial are complex numbers, the values of the coefficients $P_{1,\ \dots,\ J+1}$ are converted to complex form, and thus $P_1 = 1.0000 + 0.0000i$, $P_{2,\ \dots,\ J+1} = 0.0000 + 0.0000i$.

Let us explain the recursive algorithm of expression (6) in more detail [13].

So if $j = 1, \ \dots, \ J$, where $J = 5$
we have

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

at $j = 1$

$$k = 2, \ \dots, \ j+1 = 2, \ \dots, \ 2$$

$$i = 1, \ \dots, \ j = 1, \ \dots, \ 1$$

then

$$P = \begin{pmatrix} 1 & P_2 - \lambda_1 P_1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

at $j = 2$

$$k = 2, \ \dots, \ j+1 = 2, \ \dots, \ 3$$

$$i = 1, \ \ldots, \ j = 1, \ \ldots, \ 2$$

then

$$P = \begin{pmatrix} 1 & P_2 - \lambda_2 P_1 & P_3 - \lambda_2 P_2 & 0 & 0 & 0 \end{pmatrix}$$

at $j = 3$

$$k = 2, \ \ldots, \ j + 1 = 2, \ \ldots, \ 4$$

$$i = 1, \ \ldots, \ j = 1, \ \ldots, \ 3$$

then

$$P = \begin{pmatrix} 1 & P_2 - \lambda_3 P_1 & P_3 - \lambda_3 P_2 & P_4 - \lambda_3 P_3 & 0 & 0 \end{pmatrix}$$

at $j = 4$

$$k = 2, \ \ldots, \ j + 1 = 2, \ \ldots, \ 5$$

$$i = 1, \ \ldots, \ j = 1, \ \ldots, \ 4$$

then

$$P = \begin{pmatrix} 1 & P_2 - \lambda_4 P_1 & P_3 - \lambda_4 P_2 & P_4 - \lambda_4 P_3 & P_5 - \lambda_4 P_4 & 0 \end{pmatrix}$$

at $j = 5$

$$k = 2, \ \ldots, \ j + 1 = 2, \ \ldots, \ 6$$

$$i = 1, \ \ldots, \ j = 1, \ \ldots, \ 5$$

then

$$P = \begin{pmatrix} 1 & P_2 - \lambda_5 P_1 & P_3 - \lambda_5 P_2 & P_4 - \lambda_5 P_3 & P_5 - \lambda_5 P_4 & P_6 - \lambda_5 P_5 \end{pmatrix}.$$

Thus, according to expression (6) we obtain a vector of complex values of polynomial coefficients from which it is required to leave only the real part, and to discard the imaginary part, which will form the vector

$$P = \begin{pmatrix} P_1 & \ldots & P_{2N} \end{pmatrix}, \tag{7}$$

length $2N$, whose values correspond to the coefficients of the Daubechies wavelet filter n-th order.

Normalization of coefficients of the Daubechies wavelet filter n-th order is carried out as follows

$$P_k = S_P \frac{P_k}{\sum_{\mathrm{k=1}}^{\mathrm{2N}} \mathrm{P_k}}, \tag{8}$$

where $k = 1, \ \ldots, \ 2N$, forming the resulting vector of normalized coefficients

$$P = \begin{pmatrix} P_1 & \ldots & P_{2N} \end{pmatrix}, \tag{9}$$

such that the sum of the coefficients of $\sum_{k=1}^{2N} P_k$ will equal S_P, i.e., if $S_P = 1$, then $\sum_{k=1}^{2N} P_k = 1$ (figure 3).

Thus, at the output of the above transformations, at $N = 1$ we obtain the vector of values of coefficients of the Daubechies wavelet filter of the 1st order according to (2), and at $N > 1$ we obtain the vector of values of coefficients of the Daubechies wavelet filter of the n-order

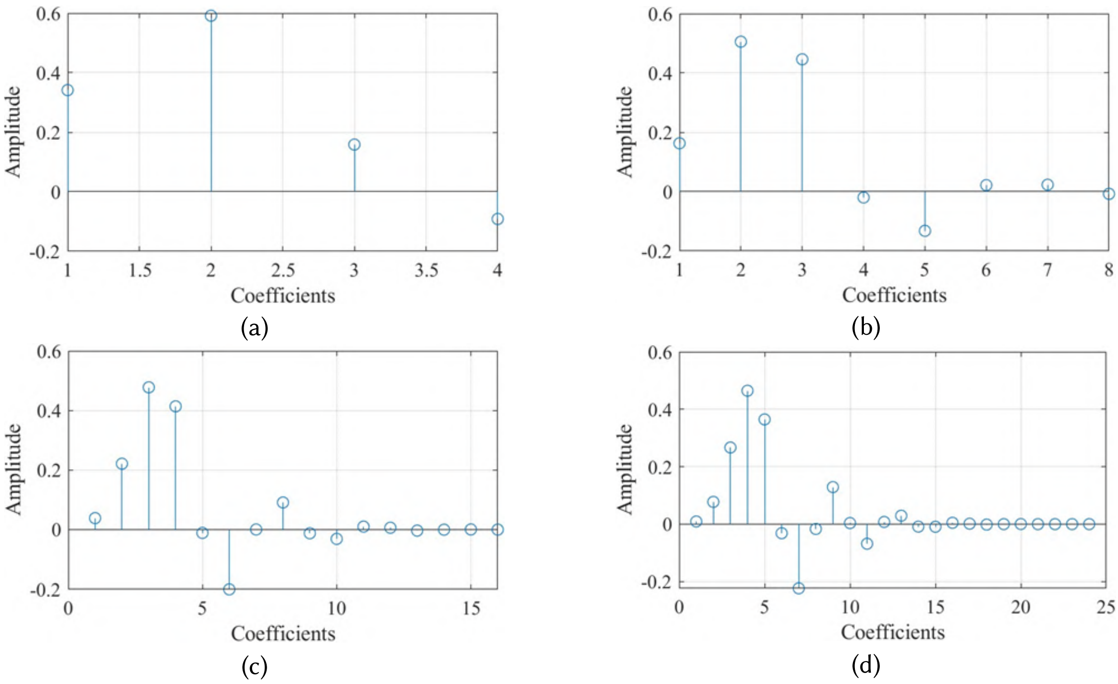


Figure 3: Coefficients of generating Daubechies wavelet filters of the 2nd (a), 4th (b), 8th (c) and 12th (d) orders.

according to (7), where in both cases the procedure of normalization of coefficients according to (8) is applied, which as a result forms the vector (9) [14].

The coefficients of the generating Daubechies wavelet filters of the 2nd, 4th, 8th and 12th orders found by the above algorithm are shown below (figure 3).

Let us calculate the coefficients of the orthogonal wavelet filters on the basis of the values of the coefficients of the generating Daubechies wavelet filter n-th order found earlier according to (9).

Thus, the coefficients of the orthogonal low-pass wavelet filter for the inverse discrete wavelet transform are defined as follows

$$R = \sqrt{2} \begin{pmatrix} P_1 & \dots & P_{2N} \end{pmatrix},$$

forming a vector

$$R = \begin{pmatrix} R_1 & \dots & R_{2N} \end{pmatrix}, \tag{10}$$

length $2N$, then the coefficients of the orthogonal low-pass wavelet filter for the direct discrete wavelet transform are defined by

$$D = \begin{pmatrix} R_{2N} & \dots & R_1 \end{pmatrix},$$

which corresponds to the inversion (10), forming the vector

$$D = \begin{pmatrix} D_1 & \dots & D_{2N} \end{pmatrix}. \tag{11}$$

The coefficients of the orthogonal high-pass wavelet filter for the inverse discrete wavelet transform are determined by computing the quadrature-mirror filter as follows

$$W = \begin{pmatrix} R_{2N} & -R_{2N-1} & R_{2N-2} & -R_{2N-3} & \dots & -R_4 & R_3 & -R_2 & R_1 \end{pmatrix},$$

forming a vector

$$W = \begin{pmatrix} W_1 & \dots & W_{2N} \end{pmatrix}, \tag{12}$$

then the coefficients of the orthogonal high-pass wavelet filter for the direct discrete wavelet transform are determined by

$$V = \begin{pmatrix} W_{2N} & \dots & W_1 \end{pmatrix},$$

which corresponds to the inversion (12), forming the vector

$$V = \begin{pmatrix} V_1 & \dots & V_{2N} \end{pmatrix}. \tag{13}$$

Thus we obtained vectors of values D and V, as well as R and W, which correspond to the coefficients of orthogonal wavelet filters of low and high frequencies for forward and inverse discrete wavelet transform, respectively [15].

As an example, let's show the coefficients of orthogonal wavelet filters based on the 8th-order Daubechies generating wavelet filter found by the above method (figure 4).

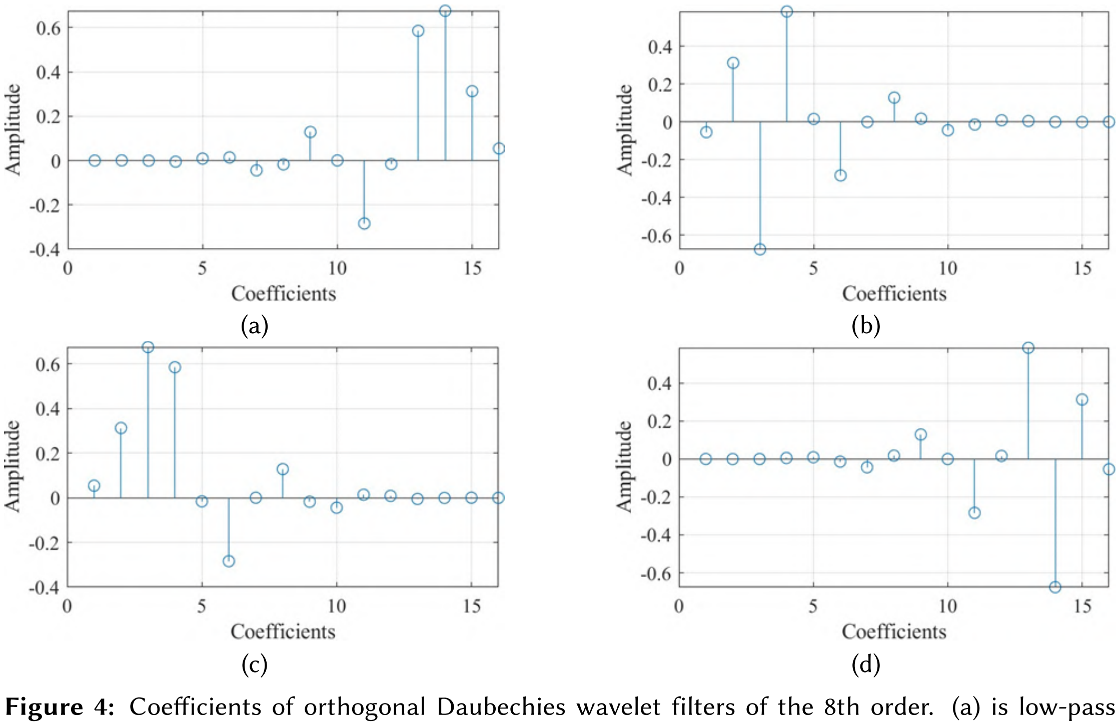


Figure 4: Coefficients of orthogonal Daubechies wavelet filters of the 8th order. (a) is low-pass decomposition filter D, (b) is high-pass decomposition filter V, (c) is low-pass reconstruction filter R, (d) is high-pass reconstruction filter W.

Then the direct discrete wavelet transform is nothing but a mathematical convolution of the values of the studied vector

$$X = \begin{pmatrix} X_1 & \dots & X_L \end{pmatrix}, \tag{14}$$

length L, with previously found vectors of coefficient values of orthogonal wavelet filters of low and high frequencies D (11) and V (13), respectively, followed by twofold thinning of $\downarrow 2$ coefficients obtained after convolution operation, thus obtaining a vector of coefficient values Z containing the low-frequency component and a vector of values Y corresponding to the high-frequency component of the studied vector X, where the formed vectors of coefficient values Z, Y are the result of this transformation [16].

Thus, the operation of mathematical convolution of the values of the investigated vector X (14) with the values of coefficients of the orthogonal low-pass wavelet filter D (11) is defined by the following expression

$$Z_k = \sum_{j=\max(1,\, k+1-2N)}^{\min(k,\, L)} \mathrm{X_j} D_i, \tag{15}$$

where $k = 1, \ \dots, \ L + 2N - 1$, $i = k + 1 - j$,
forming a vector of values

$$Z = \begin{pmatrix} Z_1 & \dots & Z_K \end{pmatrix},$$

where $K = L + 2N - 1$.

Let us explain expression (15) in more detail.

So, we have the vector under study

$$X = \begin{pmatrix} X_1 & \dots & X_L \end{pmatrix},$$

length $L = 8$, as well as the vector of values of coefficients of the orthogonal wavelet filter of low frequencies

$$D = \begin{pmatrix} D_1 & \dots & D_{2N} \end{pmatrix},$$

length $2N = 4$,
from where

$$k = 1, \ \dots, \ L + 2N - 1 = 1, \ \dots, \ 11$$

then according to (15)
at $k = 1$, $j = 1, \ \dots, \ 1$, $i = 1, \ \dots, \ 1$

$$Z_1 = X_1 D_1$$

at $k = 2$, $j = 1, \ \dots, \ 2$, $i = 2, \ \dots, \ 1$

$$Z_2 = X_1 D_2 + X_2 D_1$$

at $k = 3$, $j = 1, \ \dots, \ 3$, $i = 3, \ \dots, \ 1$

$$Z_3 = X_1 D_3 + X_2 D_2 + X_3 D_1$$

at $k = 4, j = 1, \ \ldots, \ 4, i = 4, \ \ldots, \ 1$

$$Z_4 = X_1D_4 + X_2D_3 + X_3D_2 + X_4D_1$$

at $k = 5, j = 2, \ \ldots, \ 5, i = 4, \ \ldots, \ 1$

$$Z_5 = X_2D_4 + X_3D_3 + X_4D_2 + X_5D_1$$

at $k = 6, j = 3, \ \ldots, \ 6, i = 4, \ \ldots, \ 1$

$$Z_6 = X_3D_4 + X_4D_3 + X_5D_2 + X_6D_1$$

at $k = 7, j = 4, \ \ldots, \ 7, i = 4, \ \ldots, \ 1$

$$Z_7 = X_4D_4 + X_5D_3 + X_6D_2 + X_7D_1$$

at $k = 8, j = 5, \ \ldots, \ 8, i = 4, \ \ldots, \ 1$

$$Z_8 = X_5D_4 + X_6D_3 + X_7D_2 + X_8D_1$$

at $k = 9, j = 6, \ \ldots, \ 8, i = 4, \ \ldots, \ 2$

$$Z_9 = X_6D_4 + X_7D_3 + X_8D_2$$

at $k = 10, j = 7, \ \ldots, \ 8, i = 4, \ \ldots, \ 3$

$$Z_{10} = X_7D_4 + X_8D_3$$

at $k = 11, j = 8, \ \ldots, \ 8, i = 4, \ \ldots, \ 4$

$$Z_{11} = X_8D_4.$$

Thus the vector of values is formed

$$Z = \begin{pmatrix} Z_1 & \ldots & Z_K \end{pmatrix},$$

where $K = L + 2N - 1 = 11$.

Then having calculated the values of convolution coefficients $Z_{1,\ \ldots,\ K}$ according to (15), it is necessary to perform the operation of double thinning $\downarrow 2$, according to expressions

$$Z = \begin{pmatrix} Z_2 & Z_4 & Z_6 & \ldots & Z_K \end{pmatrix},$$

when K is a multiple of two, and when K is not a multiple of two

$$Z = \begin{pmatrix} Z_2 & Z_4 & Z_6 & \ldots & Z_{K-1} \end{pmatrix},$$

which in turn forms the vector

$$Z = \begin{pmatrix} Z_1 & \ldots & Z_Q \end{pmatrix}, \tag{16}$$

length $Q = \frac{K}{2}$ or $Q = \frac{K-1}{2}$ depending on the multiple of two K.

Thus, the found vector of coefficient values Z (16) defines the low-frequency component of the direct discrete wavelet transform of the investigated vector X.

Then to find the high-frequency component Y of the direct discrete wavelet transform of the investigated vector X, it is required to repeat the given mathematical operations (2.30 - 2.34), but respectively, for the values of the coefficients of the orthogonal high-frequency wavelet filter V (13) [17].

Thus, the operation of mathematical convolution of the values of the investigated vector X (14) with the values of coefficients of the orthogonal wavelet filter of high frequencies V (13) is defined by the following expression

$$Y_k = \sum_{j=\max(1,\, k+1-2N)}^{\min(k,\, L)} \mathrm{X_j} V_i, \tag{17}$$

where $k = 1, \ \ldots, \ L + 2N - 1$, $i = k + 1 - j$,
forming a vector of values

$$Y = \begin{pmatrix} Y_1 & \ldots & Y_K \end{pmatrix},$$

where $K = L + 2N - 1$.

Then, having calculated the values of convolution coefficients $Y_{1,\, \ldots,\, K}$ according to (17), it is necessary to perform the operation of two-fold thinning $\downarrow 2$, according to expressions

$$Y = \begin{pmatrix} Y_2 & Y_4 & Y_6 & \ldots & Y_K \end{pmatrix},$$

when K is a multiple of two, and when K is not a multiple of two.

$$Y = \begin{pmatrix} Y_2 & Y_4 & Y_6 & \ldots & Y_{K-1} \end{pmatrix},$$

which in turn forms the vector

$$Y = \begin{pmatrix} Y_1 & \ldots & Y_Q \end{pmatrix}, \tag{18}$$

length $Q = \frac{K}{2}$ or $Q = \frac{K-1}{2}$ depending on the multiple of two K [18].

Then the vectors of coefficient values Z (16) and Y (18) are the result of one level of direct discrete wavelet transform, which can be written in the following form

$$\Omega = \begin{pmatrix} Z_1 & \ldots & Z_Q & Y_1 & \ldots & Y_Q \end{pmatrix},$$

then

$$\Omega = \begin{pmatrix} \Omega_1 & \ldots & \Omega_{2Q} \end{pmatrix},$$

length $2Q$.

To reconstruct the studied vector X (14) by the values of wavelet coefficients $Z_{1,\, \ldots,\, Q}$ (16) and $Y_{1,\, \ldots,\, Q}$ (18), it is required to perform the operation of doubling $\uparrow 2$ coefficients, according to the expressions

$$Z = \begin{pmatrix} Z_1 & 0 & Z_2 & 0 & Z_3 & 0 & \ldots & 0 & Z_{2Q-1} \end{pmatrix},$$

$$Y = \begin{pmatrix} Y_1 & 0 & Y_2 & 0 & Y_3 & 0 & \ldots & 0 & Y_{2Q-1} \end{pmatrix},$$

forming vectors

$$Z = \begin{pmatrix} Z_1 & \ldots & Z_{2Q-1} \end{pmatrix}, \tag{19}$$

$$Y = \begin{pmatrix} Y_1 & \ldots & Y_{2Q-1} \end{pmatrix}, \tag{20}$$

length $2Q-1$.

Then the inverse discrete wavelet transform is defined according to the expression

$$X_k = \sum_{j=\max(1,\, k+1-2N)}^{\min(k,\, 2Q-1)} \mathrm{Z_j} R_i + \sum_{j=\max(1,\, k+1-2N)}^{\min(k,\, 2Q-1)} \mathrm{Y_j} W_i, \tag{21}$$

where $k = 1,\ \ldots,\ 2Q-1+2N-1$, $i = k+1-j$,
forming a vector of values

$$X = \begin{pmatrix} X_1 & \ldots & X_K \end{pmatrix},$$

where $K = 2Q-1+2N-1$.

Expression (21) can be characterized as the sum of two mathematical convolution of the wavelet coefficient values of $Z_{1,\ \ldots,\ 2Q-1}$ (19) and $Y_{1,\ \ldots,\ 2Q-1}$ (20) with the coefficients of the orthogonal lowpass and highpass wavelet filters $R_{1,\ \ldots,\ 2N}$ (10) and $W_{1,\ \ldots,\ 2N}$ (12), respectively [19].

From where we determine the required values $X_{1,\ \ldots,\ L}$ according to the expression

$$X = \begin{pmatrix} X_{2N-1} & \ldots & X_{2N-2+L} \end{pmatrix},$$

then we obtain the vector

$$X = \begin{pmatrix} X_1 & \ldots & X_L \end{pmatrix}, \tag{22}$$

length L, which is the result of the inverse discrete wavelet transform, i.e., the values of the vector X (22) are the result of the process of reconstructing the values of the studied vector X (14) by the values of the wavelet coefficients $Z_{1,\ \ldots,\ Q}$ (16) and $Y_{1,\ \ldots,\ Q}$ (18) [20].

4. Simulation results

The disadvantages of the Fourier transform are demonstrated in figure 5a, 5b and figure 6a, 6b.

In figure 5a and figure 6a show two harmonic components $S_1(t) = A_1 \cdot \sin(\omega_1 t)$ and $S_2(t) = A_2 \cdot \sin(\omega_2 t)$, with angular frequencies $\omega_1 = 63$ rad/s and $\omega_2 = 252$ rad/s.

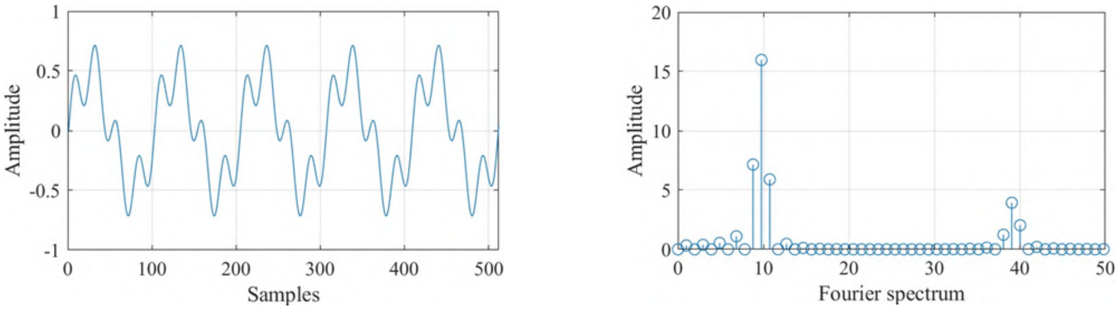
The angular frequency ω in rad/s is expressed through the frequency f in Hz, as $\omega = 2\pi f$ and $f = \frac{\omega}{2\pi}$. Based on this, $f_1 = 10$ is Hz, and $f_2 = 40$ is Hz.

The process shown in figure 5a, is an adaptive combination of two sinusoids [21] $S_1(t)$ and $S_2(t)$

$$U_1(t) = S_1(t) + S_2(t),\ t \in (0, T],$$

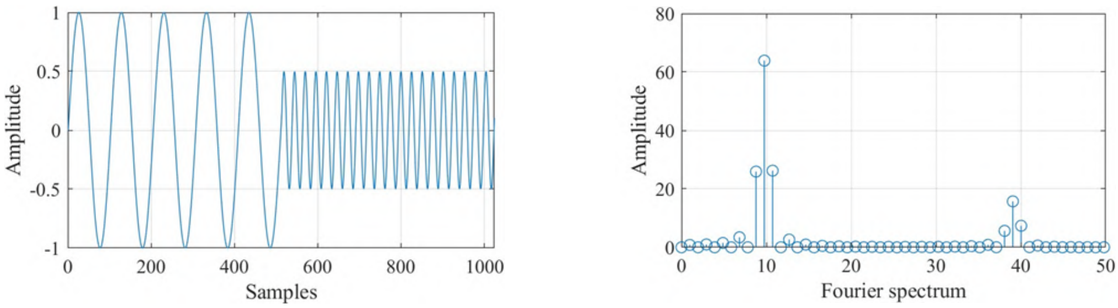
where $A_1 = 0.5$, $A_2 = 0.25$, respectively, and $T = 512$, and the process shown in figure 6a is described as follows

$$U_2(t) = \begin{cases} S_1(t),\ t \in (0, t_0], \\ S_2(t),\ t \in (t_0, T], \end{cases}$$



(a) (b)

Figure 5: Fourier transform of the signal $U_1(t)$. (a) is signal $U_1(t)$; (b) is Fourier transform spectrum of the signal $U_1(t)$.



(a) (b)

Figure 6: Fourier transform of the signal $U_2(t)$. (a) is signal$U_2(t)$; (b) is Fourier transform spectrum of the signal $U_2(t)$.

where $A_1 = 0.5$, $A_2 = 0.25$ respectively and $t_0 = 512$ where $T = 1024$.

Outside the interval $(0, T]$, the functions $U_1(t)$ and $U_2(t)$ are 0.

As a result of the Fourier transform of the signals $U_1(t)$ and $U_2(t)$, we obtained poorly distinguishable spectral images, which are shown in figure 5b and figure 6b.

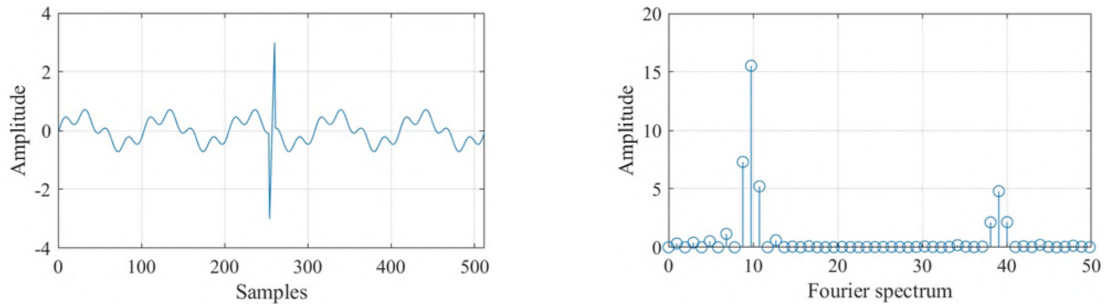
The following example also shows the low information content of the Fourier transform. The signal presented in figure 7a, the signal $U_3(t)$ in the vicinity of $t = 253 : 260$ contains a short pulse $I(t)$ (anomaly) [22], where $t \in (-3, 3]$

$$U_3(t) = \begin{cases} U_1(t), & t \in (0, t_1], \\ I(t), & t \in (t_1, t_2], \\ U_1(t), & t \in (t_2, T], \end{cases}$$

where $t_1 = 253$, $t_2 = 260$.

The Fourier transform made it possible to clearly distinguish two harmonic components of the signal, and the spectral components of the anomaly, as expected, were distributed along the entire frequency axis.

In figure 5, figure 6, figure 7 showed specific examples of the disadvantages of the Fourier transform that can be overcome by using the wavelet transform.

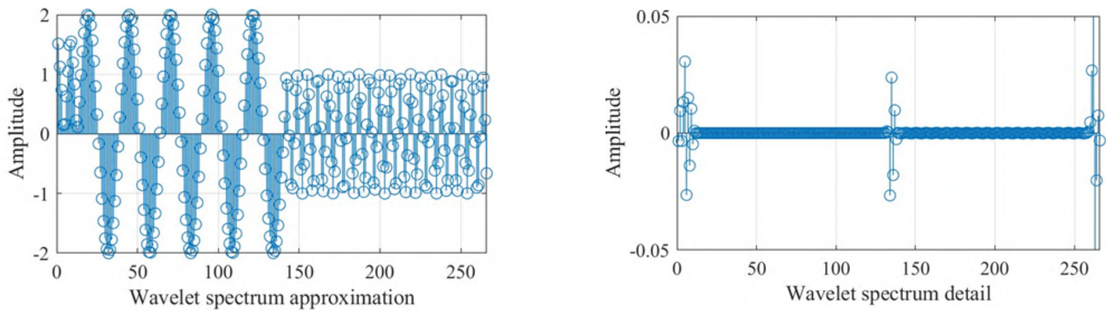


(a) (b)

Figure 7: Fourier transform of the signal $U_3(t)$. (a) is signal $U_3(t)$; (b) is Fourier transform spectrum of the signal $U_3(t)$.

It should be noted that the above Fourier transform spectra contain all the information about the input signals. This information is distributed in the phase and amplitude values of all spectral components. The input acoustic signals can be fully recovered after the inverse Fourier transform.

The advantages of the wavelet transform are demonstrated in figure 8a, 8b, and figure 9a, 9b.

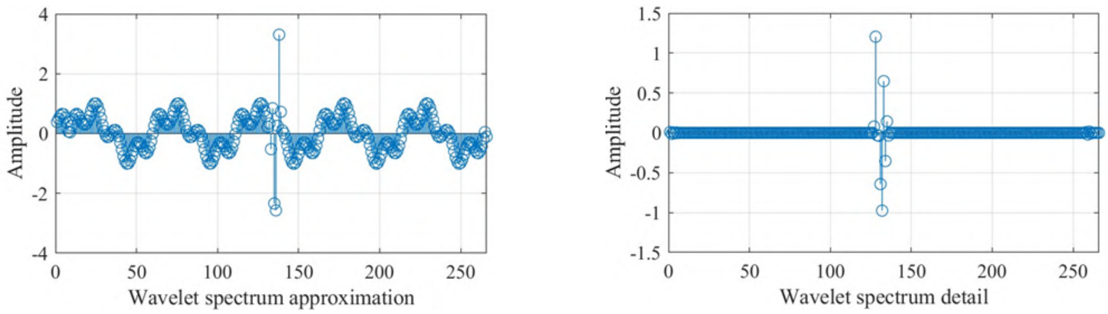


(a) (b)

Figure 8: Wavelet transform of the signal $U_2(t)$. (a) is approximation coefficients of the wavelet transform of the signal $U_2(t)$; (b) is detail coefficients of the wavelet transform of the signal $U_2(t)$.

5. Discussion

As a spectral analysis of a noisy acoustic signal, it was proposed to use a wavelet transform based on the Daubechies wavelet function. This transformation has advantages over the Fourier transform, as it is adaptive to obtain a set of informative acoustic features for UAV recognition, which will keep the classification at a sufficiently high level. As a result of the first step of the wavelet transform, the time resolution is halved, since only half of the samples characterize the entire acoustic signal. However, the frequency resolution is doubled, as the signal now occupies half the frequency band and the uncertainty is reduced. This procedure, known as subband coding, is repeated further and the wavelet coefficients at the output of the low-pass filter are



(a) (b)

Figure 9: Wavelet transform of the signal $U_3(t)$. (a) is approximation coefficients of the wavelet transform of the signal $U_3(t)$; (b) is detail coefficients of the wavelet transform of the signal $U_3(t)$.

fed to the same processing circuit, and the wavelet coefficients at the output of the high-pass filter are considered the resultant wavelet coefficients.

The most significant frequencies of the input acoustic signal will be displayed as large amplitudes of wavelet coefficients that characterize the corresponding frequency range. Small values of wavelet coefficients mean low energy of the corresponding frequency bands in the acoustic signal. These coefficients can be set to zero without significant signal distortion, which is very promising in the formation of acoustic signal recognition features for UAV detection.

A wavelet transform is a decomposition of an acoustic signal into a system of wavelet functions, each of which is a shifted and scaled copy of one function - the parent wavelet. Usually, the parameter that determines the choice of the type of mother wavelet is the external similarity of the signal under study and the transformation function. Based on this, it is advisable to use Daubechies wavelets as the mother wavelet function for processing acoustic signals.

This is one of the most famous wavelets and its main properties are as follows:

1) the functions have a finite number of zero values, i.e., the Daubechies wavelet system has the properties of smoothness and moment exclusion;
2) the functions have the properties of carrier compactness (rapidly increasing and rapidly decreasing) and orthogonality, which makes it possible to accurately restore the acoustic signal;
3) wavelets have both a wavelet function and a scaling function, which makes it possible to perform multiple-scale and fast wavelet analysis.

Functions on the same scale and on different scales are orthogonal. Note that the property of orthogonality allows us to obtain independent information at different scales, and normalization ensures that the value of information is preserved at different stages of the transformation. Among the disadvantages is the asymmetry of the Daubechies wavelet.

In acoustic signal processing tasks for UAV detection by noise, due to the unique sound characteristics of UAVs, the requirements imposed on the shape of wavelet function spectra are quite high, which leads to the use of a large number of zero moments (10-15 zero moments). Daubechies wavelets of length L have $= L/2$ zero moments. However, it should be remembered that the number of zero moments determines the length of wavelet functions and, therefore, the speed of the algorithm for calculating the wavelet transform. In the classical Daubechies design, the length of the filters is $L = 2$, where M is the number of zero moments. All Daubechies wavelet functions have a compact carrier.

It is easy to see that the smoothness of wavelets increases as their order increases. At the same time, the frequency of oscillations increases. These wavelets have a characteristic asymmetry, namely the rise of the function is stretched compared to the decay.

The main problem when working with a wavelet transform is the problem of choosing the most appropriate wavelet. The choice of a particular family of wavelets is dictated by the application tasks and the type of information about the signal that needs to be maximally detected (recognized). There are no hard and fast rules, but it is best to choose a wavelet so that it belongs to the same class of functions as the signal being analyzed. If the original function can be approximated by a polynomial, then the number of zero moments of the wavelet should be approximately equal to the degree of the polynomial. The number of zero moments is more important to achieve higher information content of wavelet coefficients, which increases with a large number of zero moments.

6. Conclusion

This research paper is devoted to the wavelet analysis of acoustic signals of UAVs, which can improve the efficiency of aircraft detection algorithms. The problem of spatial and temporal wavelet processing of the received UAV acoustic signal by the criterion of maximum useful signal-to-noise ratio on the basis of Daubechies wavelet basis is considered.

The necessary mathematical relations determining the sequence of processing of the received acoustic signal on the basis of wavelet analysis using the Daubechies decomposition basis are obtained. The vector of optimal weighted Daubechies wavelet coefficients is formed in accordance with one of the known criteria of optimality of spatial and temporal processing, for example, in accordance with the criterion of maximum signal-to-noise ratio.

The obtained simulation results reflect the effectiveness of the spatio-temporal wavelet method for processing acoustic signals of UAVs in the Daubechies decomposition basis compared to the less effective Fourier basis, and as a consequence, indicate its applicability for solving problems related to the detection of UAVs by the acoustic method.

Further scientific research, continuing this topic, will be related to the construction of primary acoustic features for UAV recognition. Acoustic noise emitted by a UAV is a realization of a broadband random process, the description of which can be given by an energy wavelet spectrum. Therefore, the information attributes of acoustic recognition of UAVs can serve as estimates of spectral wavelet coefficients determined from a discrete realization containing a given number of samples. The transition to secondary information features is carried out by constructing the covariance matrix of spectral wavelet coefficients and its diagonalization. After the calculations, the set of acoustic signs of UAV recognition, which came to the input of the system, corresponds to some class, if the average value of the similarity coefficient for all pairs of vectors is greater than a certain threshold value. The conducted theoretical studies allow us to develop a module for the formation of a collection of acoustic recognition features of UAVs and a module that implements the decision-making rule for the classification of feature vectors.

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