Multicriteria Linear Assignment Problem in Healthcare Improvement

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Abstract
A two-criteria linear assignment problem is applied to the distribution of medical university students for hospital internships. The optimization goal is to improve the overall level of service by maximizing total grades for the internship. The main criteria selected are the combined academic performance score and the motivational score of students to occupy specific job positions. The stated problem is reduced to the task of predicting grades for the student internship and is solved on the basis of historical data. It also solves a series of ordinary linear assignment problems with prohibitions on some assignments. A computer experiment was designed, and test calculations were conducted. The results showed the effectiveness of using the chosen technique to solve the original problem.

Keywords
multicriteria linear assignment problem, multi-objective optimization, medical service, linear convolution method, Hungarian method, polynomial solvability

Introduction

The n-dimensional linear assignment problem (LAP) is formulated as follows: there are n job positions and n applicants for these positions. It is required to find the applicants' assignments to the positions so that each position is occupied and each applicant receives a position while minimizing the total cost of the overall assignment. LAP has numerous applications in various areas [1]. A broader application is the assignment problem, where each position and each applicant are evaluated according to several criteria, and the final assignment aims to optimize all these criteria. This transforms the problem into a multi-objective linear assignment problem (MLAP). Multicriteria optimization methods are applied to solving MLAP, most of which reduce it to a single-objective LAP. The main challenge is choosing a way to reduce MLAP to LAP, particularly forming the final cost matrix of the assignments. Methods of mathematical statistics can be applied to solve this task by finding and utilizing predicted values of these costs instead of the unknown real costs of the assignments.

This paper examines the practical problem of assigning medical university students to internships in hospitals. Students are assessed by their academic performance grades, while the available job positions are assessed by the ratings given to them by the students. The main goal is to place students in positions that maximize the overall grade for the internship. The main challenge is that students are assigned to the jobs at the beginning of the internship, while the scores are received at the end. Therefore, it is necessary to predict the internship grades, reduce the problem to a two-criteria assignment problem aiming to maximize both the total academic score and the total motivational score, and

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finally reduce this two-criteria assignment problem to a single-criteria one using the predicted values of grades for practice as the effectiveness metric of the assignments. To solve the problem, methods of multicriteria optimization, linear integer optimization, and statistics, particularly regression analysis, are used.

1. Main Part

The classical linear assignment problem (LAP) [2] is the problem of optimizing a linear function on a set of permutation matrices:

$$z = C \bullet X \to \min; \tag{1}$$

$$X \in \Pi_n. \tag{2}$$

Here, Π_n is a set of permutation matrices of order n [3, 4, 5, 6], where n is the number of applicants for the positions and the positions themselves, $C \in R_+^{n\times n}$ is the matrix of individual assignments costs.

$$\Pi_n = \left\{ X \in B^{n\times n} \,:\, X \cdot e = X^T \cdot e = e \right\},$$

$$B = \{0, 1\}, e = (1, ..., 1)^T.$$

LAP is widely used in many theoretical and practical fields, such as scheduling theory, logistics, graph theory, and computational logic, both as a standalone problem and as a subproblem in more complex problems [1, 7, 8].

As a rule, LAP is formulated as a minimization problem but it is also found in the literature as a maximization problem. In this case, LAP has the form

$$z' = U \bullet X \to \max. \tag{3}$$

Under the constraints (2), where the matrix $U \in \mathbb{R}^{n\times n}$ is the utility matrix and represents the profit from the assignments, it is clear that LAP (2), (3) can be reduced to the standard LAP (1), (2) using a linear transformation. For example, let the elements of the matrix U express the utility of individual assignments on a scale from 0 to 100 score.

$$U \in [0, 100]^{n\times n}. \tag{4}$$

Then the transition from LAP (2),(3) to the classical LAP (1), (2) can be done as follows

$$U \to C = \left[c_{ij}\right]_{ij} = \left[100 - u_{ij}\right]_{ij} \in [0, 100]^{n\times n}. \tag{5}$$

As can be seen, the elements of the matrix C express the cost of individual assignments on a scale from 0 to 100 points.

In what follows, when we refer to LAP, we mean the classical problem (1), (2), to which, in particular, the Hungarian method is applicable [9]. The attractiveness of LAP from a computational point of view is determined by its equivalence to the following continuous linear relaxation (1).

$$X \in P_n = conv\Pi_n, \tag{6}$$

where P_n is a polyhedron of stochastic matrices, which is an integral [3]. This determines the comparative simplicity of solving LAP using linear programming methods compared to other combinatorial problems. Moreover, LAP is polynomially solvable. namely, there is a Hungarian algorithm for solving it with computational complexity $O(n^3)$ [9].

1.1. LAP generalizations

LAP generalizes in several ways. The first is the complication of the objective function, transitioning from LAP to the nonlinear assignment problem (NAP) [2, 10, 11], where (1) is replaced by

$$z = f(X) \to \min, f(X) \text{ is nonlinear.} \tag{7}$$

In particular, this is about quadratic (QAP) [11], cubic, and biquadratic assignment problems.

The next generalization of LAP concerns the number of objective functions. Thus, the problem

$$z^{(k)} = C^{(k)} \bullet X \to \min, k = \overline{1, K}, K > 1 \tag{8}$$

under the constraints (2) is called the multi-objective linear assignment problem (MLAP) [12, 13, 14]. Here, $C^{(k)} \in R_{+}^{n\times n}, k = \overline{1, K}$ are the cost matrices for each criterion.

Finally, the third generalization concerns the presence of additional constraints in addition to the main constraints (2). So the problem (1), (2),

$$A^{(l)} \bullet X \leq b^{(l)}, l = \overline{1, L}, L \geq 1 \tag{9}$$

is called the constrained linear assignment problem (CLAP) [10].

By combining the conditions (7) – (9) with (1), (2), other generalizations of LAP are formed. For example, problem (2), (8), (9) will be a constrained multicriteria linear assignment problem (CMLAP).

From a computational point of view, solving these generalizations is much more challenging than LAP. Problems in the NAP and CLAP classes are known to be NP-hard [2, 10, 11]. As for MLAP, we encounter the typical challenges of solving multi-objective optimization problems here. When using standard approaches, such as the weighting method (the linear convolution method) [15, 16, 17] and the priority method [15, 16, 17], additionally addresses the problem of expert assessment and determination of weights of the objective functions representable by a vector

$$W = \left(w^{(1)}, ..., w^{(K)}\right) \in R_{+}^{K}, w^{(1)} + ... + w^{(K)} = 1, \tag{10}$$

after which, using the linear convolution method, MLAP is converted into LAP with a cost matrix

$$C = \sum_{k=1}^{K} w^{(k)} C^{(K)}. \tag{11}$$

If the priority method is used, then a sequence of CLAPs is solved with one of the criteria (8) and iterative addition of constraints on non-deterioration of higher priority criteria, which are optimized in previous iterations.

Likewise, some CLAPs are reduced to NAPs and sometimes even to LAPs. Thus, the traditional way of solving constrained optimization problems, the penalty method [18], consists of incorporating all or part of additional constraints into the objective function using penalty terms. . In contrast, linear optimization problems turn into nonlinear ones without constraints, i.e., CLAP becomes NAP. If additional constraints express prohibitions on certain pair-wise assignments, then instead of using the penalty method, one can transfer from CLAP to an equivalent LAP of the following form:

$$z = C' \bullet X \to \min \tag{12}$$

$$C' = \left[c'_{ij}\right]_{i,j}, c'_{ij} = \begin{cases} c_{ij}, (i, j) \notin Z \\ \infty, (i, j) \in Z \end{cases}. \tag{13}$$

Here, Z is the set of pairs (i, j) of prohibited assignments, where i is the student number and j is the job position number. The reducibility of this CLAP to LAP justifies its polynomial solvability, for example, by the Hungarian method [9].

This paper addresses the practical problem of enhancing the level of medical service, which entails solving problems of regression analysis and a finite sequence of CMLAPs. These CMLAPs are reduced to ordinary LAPs by incorporating prohibitions on certain assignments and applying linear convolution of optimization criteria. As a result, we propose an algorithm for solving the formulated complex problem, solvable in polynomial time, depending on the problem dimension n, the number of criteria K, and the number of periods T for which the forecast of the assignment cost matrix is made.

2. Problem statement

The following practical task is considered: at the end of the academic year, students of medical universities undergo hospital internships. The objective is to assign students to suitable hospital positions to maximize the medical service they provide. A quantitative indicator of the level of service is the grades given to students in hospitals as scores for their internship. These scores, in turn, accumulate patient and staff evaluations of the student performance.

Historical data demonstrates that the level of success of an internship depends on two main indicators.

The first indicator is academic performance, expressed by the factor Kg (the academic score, grade coefficient), which depends on scores in key academic disciplines $D_1, \dots, D_M$ and the average score in all academic disciplines. A higher value of Kg indicates better learning outcomes, increasing the likelihood of the student possessing sufficient knowledge to succeed in solving assigned practical problems.

The second indicator is the student's desire to work in a specific position or hospital, referred to as the motivational factor and assessed by the numerical indicator Km (the motivation coefficient). A higher Km indicates higher motivation, suggesting that the student is more likely to succeed in the position entrusted to them.

As can be seen from the formulation, this is the standard linear assignment problem of assigning students to positions, assuming that grades for the internship are known for all students and all job positions:

$$z^* = C \bullet X^* = \min_{X \in \Pi_n} \{z = C \bullet X\}, \tag{14}$$

minimizing the total cost of the assignment z and, as a consequence, maximizing the total efficiency of the final assignment:

$$z'^* = U \bullet X^* = \max_{X \in \Pi_n} \{z = U \bullet X\}. \tag{15}$$

Here,

$$C = \left[c_{ij}\right]_{i,j} \tag{16}$$

is a matrix of costs of assigning the applicants to the positions, in particular, c_{ij} is the cost of assigning applicant i (further a student S_i), to the position j (further a job J_j). Respectively,

$$U = \left[u_{ij}\right]_{i,j} \tag{17}$$

is a matrix of efficiency (utility matrix) of assigning the students to the jobs, particularly u_{ij} represents the cost of assigning a student S_i to a job J_j.

However, the challenge arises because grades for the internship will only be given after the students have been assigned to the positions and have completed it. Therefore, the matrix U is unknown. In the mathematical model, instead of real estimates (17), only predicted values of these utilities can be used:

$$U \to \widehat{U} = \left[\widehat{u}_{ij}\right]_{i,j}, \tag{18}$$

where $\widehat{U}$ is the forecast utility matrix of assigning the students to the positions, in particular, $\widehat{u}_{ij}$ is the forecast utility of assigning the applicant i to the position j.

Thus, instead of the problem (15) we come to the following problem:

$$\widehat{z}^* = \widehat{U} \bullet \widehat{X}^* = \max_{X \in \Pi_n} \left\{ \widehat{z} = \widehat{U} \bullet X \right\}, \tag{19}$$

in which the real values of the estimates are replaced by the predicted ones, and the utility matrix U is replaced by $\widehat{U}$. Accordingly, the optimal solution to the problem (19) will be the pair $\left\langle \widehat{X}^*, \widehat{z}^* \right\rangle$, where $\widehat{X}^*$ is the predicted matrix of optimal assignments, $\widehat{z}^*$ is the predicted value of the optimal total utility of the assignments z^*.

Assuming that there is historical data on the appointments of previous students to the same positions, their grades in academic disciplines and summer internships for previous years and motivational scores, this forecasting task can be formulated as follows. The goal of the forecast includes searching for the matrix $\widehat{U}$ with the subsequent solution of the problem (19) and consists of finding a forecast matrix of assignments $\widehat{X}^*$, which is close to the optimal matrix of assignments X^*. This goal can be represented as

$$\widehat{z}^* \approx z^*.$$

Another metric for comparison

$$\widetilde{z}^* = U \bullet \widehat{X}^* \approx \widehat{z}^*,$$

which we can calculate at the end of the process of acceptance and implementation of the found assignments, when scores for the internship are given, that is, $\widetilde{z}^*$ is a posteriori estimate.

3. Problem formalization

Let us introduce some notations:

- $i \in \overline{1, n}$ is a student index;
- $j \in \overline{1, n}$ is a position index;
- $t \in \overline{1, T+1}$ is an index of the time period, including $\overline{1, T}$ is an index of the historical period, $T + 1$ is an index of the current (forecast) period;
- $m \in \overline{1, M}$ is an index of the key academic discipline;
- $h \in \overline{1, H}$ is a hospital type.

As input data for making forecasts, we have:

1. for the forecast $T + 1$-th period and historical periods $1, \ldots, T$:
    a) by the student:

i. the score for key academic disciplines for this practice and the average score of the students:

$$G^t = \left(g^t_{im}\right)_{i,m,t}, ag^t = \left(ag^t_i\right)_{i,t},$$

where $g^t_{im} \in [0, 100]$ is the score of the student S_i in the key discipline D_m in the period t;

ii. student ratings of the positions (the motivational factor), starting from 0 for the worst and up to 100 for the best:

$$Km^t = \left[Km^t_{ij}\right]_{i,j,t} \tag{20}$$

where $Km^t_{ij} \in [0, 100]$ is the motivational score of the student S_i assigned to the position J_j in the period T_t;

b) by the job:

i. their link to the hospitals;

ii. the relative weight of disciplines and the average score of them (can be the same for all periods, the same for the positions, the same for the hospitals). For example, the vector of weights:

$$w = (w_1, ..., w_M, w_a) \in R^{M+1}_+, w_1 + ... + w_M + w_a = 1, \tag{21}$$

corresponds to the case when these weights are uniform across positions and periods. In particular, w_m is the relative weight of the key academic discipline D_m in the factor Kg, while w_a is the relative weight of the average score in the factor Kg;

iii. intervals of passing scores, both average and in individual disciplines (can be uniform across hospitals and periods).

Let us introduce a constraint:

$$\underline{g}^t_{jm} \le g^t_{im} \le \overline{g}^t_{jm}, \underline{ag}^t_j \le ag^t_i \le \overline{ag}^t_j, \forall i, j, m, t. \tag{22}$$

$\underline{g}^t_{jm}, \overline{g}^t_{jm}$ are, respectively, the lowest and highest passing scores in the discipline D_m for the occupation of the position J_j in the period T_t, while $\underline{ag}^t_j, \overline{ag}^t_j$ are the lowest and highest pass average score for position J_j during period T_t. These passing scores can also be uniform across the periods and positions within the same hospital. For example, the lower bound on passing academic scores, uniform across all periods, looks like this

$$\underline{g}_{jm} \le g^t_{im}, \underline{ag}_j \le ag^t_i, \forall i, j, m, t, \tag{23}$$

where $\underline{g}_{jm}$ is the minimal passing score in the discipline D_m for occupation of the position J_j, $\underline{ag}_j$ is the minimal passing average score for the position J_j.

c) by hospital:

i. hospital rating

ii. requirements for passing grades

2. for previous periods $1, ..., T$:

a) by student:

i. score for the practice and position occupied by them

$$\left(i, j^t_i, c^t_{i,j^t_i}\right)_i, t = 1, ..., T. \tag{24}$$

For example, $\left(i, j^1_i, c^1_{i,j^1_i}\right) = (2, 3, 85)$ means that in period $t = 1$, the student S_2 was assigned to the position J_3 and received a grade of 85.

ii. There are also statistics on which students were appointed to which positions and what grade they received

When constructing our mathematical model, we limit our consideration to the cases (21) and (23). The academic performance coefficient looks like this:

$$Kg_i^t = \sum_{m=1}^{M} w_m g_{im}^t + w_a a g_i^t \in [0, 100], \forall i,\ t \in \overline{1, T+1}. \quad (25)$$

The two main factors of academic achievement and motivation are combined into a single metric:

$$K = f(Kg, Km) \quad (26)$$

in the form of a weighted sum of the indicators Kg and Km, which relative weights depend on the period t, i.e.

$$K_{ij}^t = \alpha^t Kg_i^t + \beta^t Km_{ij}^t \in [0, 100], \forall i, j, t, \quad (27)$$

where $\alpha^t, \beta^t \geq 0, \alpha^t + \beta^t = 1, \forall t$.

As a result, we obtain a set of matrices of total scores:

$$K^t = \left[K_{ij}^t\right]_{ij}; K_{ij}^t = \alpha^t Kg_i^t + \beta^t Km_{ij}^t \in [0, 100], \forall i, t. \quad (28)$$

As stated earlier, the weighted sum coefficients are supposed to be known for historical periods $t \in \overline{1, T}$ and unknown for the current period $T + 1$. Therefore, under the assumption the vector

$$\alpha = \left(\alpha^t\right)_{t=\overline{1,T}} \quad (29)$$

is given, while α^{T+1} is unknown, we formulate the auxiliary forecasting problem of assessing α^{T+1} (the forecast value is denoted as $\widehat{\alpha}^{T+1}$) as follows: we assume that α^t stochastically linearly depends on the period t. Therefore, we introduce a simple linear regression [19, 20]:

$$\alpha^t = a \cdot t + b + v, \quad (30)$$

where a, b are regression parameters and v is the error. After estimating its parameters by the least squares method [19], we can construct the desired forecast:

$$\hat{\alpha}^{T+1} = a \cdot (T + 1) + b. \quad (31)$$

Now, we can find the forecast matrix of total scores adapting (28) as follows:

$$\hat{K}^{T+1} = \left[\hat{K}_{ij}^{T+1}\right]_{ij}; \hat{K}_{ij}^{T+1} = \hat{\alpha}^{T+1} Kg_i^{T+1} + \hat{\beta}^{T+1} Km_{ij}^{T+1} \in [0, 100], \forall i, j. \quad (32)$$

In order to incorporate the constraints (23) into the utility matrix, we perform the transition from the matrices $\left\{K^t\right\}_t$ to

$$K^t \to K'^t = \left[K_{ij}'^t\right]_{ij}; K_{ij}'^t = \begin{cases} K_{ij}'^t, if\ (23)\ \text{holds for all}\ m \\ -\infty, \text{otherwise} \end{cases} \quad (i, j, t)$$

Similarly,

$$\hat{K}^{T+1} \to \hat{K}'^{T+1} = \left[\hat{K}_{ij}'^{T+1}\right]_{ij}; \hat{K}_{ij}'^{T+1} = \begin{cases} \hat{K}_{ij}'^{T+1}, if\ (23)\ \text{holds for all}\ m \\ -\infty, \text{otherwise} \end{cases} \quad (i, j)$$

Now we solve a series of ordinary LAPs of type (19) with the efficiency matrices:

$$z^{t*} = K^t \bullet X^{t*} = \max_{X \in \Pi_n} \left\{ z^t = K^t \bullet X \right\}$$

getting a set of auxiliary optimal solutions $\langle X^{t*}, z^{t*} \rangle, t = \overline{1,T}$.

In the same way, the following LAP is solved for the current period $T + 1$:

$$z^{T+1,*} = \hat{K}^{T+1} \bullet X^{T+1,*} = \max_{X \in \Pi_n} \left\{ \hat{z}^t = \hat{K}^{T+1} \bullet X \right\},$$

yielding the solution

$$\left\langle X^{T+1,*}, z^{T+1,*} \right\rangle, t = \overline{1,T}. \tag{33}$$

The solution (33) is the main result of optimization as it yields the student assignment based on the forecast score for their internship in the form of the matrix $X^{T+1,*}$, while the corresponding entries of the matrix $\hat{K}^{T+1}$ provide expected internship score of the students given that the assignment given by $X^{T+1,*}$ is implemented. Note that, in this case, the value of $z^{T+1,*}$ yields the expected total score for the internship.

Remark 1. *The above scheme of obtaining the assignment solution* (33) *relies on the known parameters* (29)*. However, the information can be hidden from the decision-maker. In this situation, another set of data given by assumption can be utilized: the historical scores* (24) *for the internship. We propose to assess the vector* (29) *solving a set of additional 2-factor linear regressions with a constraint:*

$$C^t = \alpha^t Kg^t + \beta^t Km^t, \alpha^t + \beta^t = 1, \alpha^t, \beta^t \geq, t = \overline{1,T},$$

where n observation is utilized for every t, while the values of j_i^t column of Km^t and C^t are participate, $i = \overline{1,n}$. Then the above algorithm becomes applicable with the only difference that now the vector α contains predicted parameters rather than real.

4. Experiment design

1. Enter n, M, T, M', where M' is total number of disciplines;
2. To generate the academic score matrices $G^t, ag^t, \forall t$, for each discipline, we set the average scores $E(D_m)$ and the standard deviation $\sigma(D_m)$, $m = \overline{1,M'}$. The generation is conducted according to the normal distribution:

   $$g_{im}^t \sim N\left(E(D_m), \sigma(D_m)\right), \forall i, m, t. \tag{34}$$

   The first m columns of the matrix form the matrix G^t. Next, the matrix-column ag^t is filled with arithmetic averages over the columns of the matrix:

   $$ag_i^t = \frac{1}{M'} \sum_{m=1}^{M'} g_{im}^t.$$

3. Using the relative weights (21), one can now find the values of the factor Kg
4. We will generate matrices of motivational scores $\left\{Km^t\right\}_t$ according to a normal distribution with a given parameters $E\left(Km_j\right), \sigma\left(Km_j\right)$:

   $$Km_{ij}^t \sim N\left(E\left(Km_j\right), \sigma\left(Km_j\right)\right), \forall i, t$$

5. To convolute the academic and motivational points into the combined score K_{ij}^t, we use the weights α^t and $\beta^t = 1 - \alpha^t$ known for historical periods.
6. We will set the low bound for the passing score for different positions in key academic disciplines and the average academic score. Thus, we do not set the upper bound. We will start from the previously specified mathematical expectation and standard deviation of these values (see (34)), allowing at most specified deviation from the mathematical expectation depending on the standard deviation.
   We use the formula (22), where the lower bounds $\underline{g}_{jm}, \underline{ag}_j, \forall j, m$ are calculated by the formula
   $$\underline{g}_{jm} = \max\{0, E(D_m) - \gamma_j \sigma(D_m)\}, \underline{ag}_j = \min\{100, E\left(\overline{D}\right) - \gamma_j \sigma\left(\overline{D}\right), \forall j, m\},$$
   assuming that the parameters $\{\gamma_j\}_j$ are given.
7. we assess the parameters of the regression (30) and use them for the forecast value $\hat{\alpha}^{T+1}$ (see (31)).

A test experiment was conducted for the following parameters $n = 14, M = 4, T = 5, M' = 10$ and the outlined experimental design scheme.

5. Conclusions

This paper addresses the practical task of improving medical care services through the internships of medical university students in hospitals. Based on historical data, the main assumption is that student performance is statistically linearly proportional to their academic performance scores and the attractiveness of the positions to the students. A mathematical model of the problem was constructed using multi-objective optimization methods, linear integer optimization, and regression analysis. A computational experiment was designed, and a test calculation for a small-scale problem was conducted to demonstrate the applicability of the proposed approach in solving the original task.

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