# An Extension-Based Argument-Ranking Semantics: Social Rankings in Abstract Argumentation

Lars Bengel<sup>1,†</sup>, Giovanni Buraglio<sup>2,†</sup>, Jan Maly<sup>3,2,†</sup> and Kenneth Skiba<sup>1,†</sup>

<sup>1</sup>Artificial Intelligence Group, University of Hagen, Hagen, Germany

<sup>2</sup>Institute of Logic and Computation, TU Wien, Wien, Austria

<sup>3</sup>Institute of Data, Process and Knowledge Management, Vienna University of Economics and Business, Wien, Austria

#### Abstract

In this paper, we introduce a new family of argument-ranking semantics which can be seen as a refinement of the classification of arguments into skeptically accepted, credulously accepted and rejected. To this end we use so-called social ranking functions which have been developed recently to rank individuals based on their performance in groups. We provide necessary and sufficient conditions for a social ranking function to give rise to an argument-ranking semantics satisfying the desired refinement property. Moreover, we analyse the properties of the argument-ranking semantics induced by the most prominent social ranking function that satisfies all of these conditions by investigating the satisfaction of principles known from the argument-ranking literature.

#### Keywords

Argumentation, Social Rankings, Extension-ranking semantics, Argument-ranking semantics

### 1. Introduction

One of the problems of computational models of argumentation is to classify the quality of arguments in the context of the larger discussion. In abstract argumentation, this is usually achieved by checking whether an argument is contained in a set of arguments, called extensions, that are acceptable according to one of several well-established semantics. While these semantics provides a natural way to rank arguments based on the larger context of the argumentation framework, it only allows us to distinguish three types of arguments: the ones that are skeptically accepted, i. e. that are contained in every extension; the ones that are credulously accepted, i. e. that are contained in at least one extension; and the ones that are not contained in any extension. For this reason, more fine-grained ways of comparing arguments have been proposed, the so called argumentranking semantics [1, 2, 3, 4, 5]. However, generally, these argument-ranking semantics are technically quite distinct from the extension-based classifications of arguments that are more commonly used.

In this paper, we propose a new way of ranking arguments which can be seen as a true refinement of the more common classification in skeptically, credulously and not accepted arguments. To this end, we combine two strands of literature that have emerged recently, namely extensionranking semantics and social ranking functions, in a novel way. Intuitively, social ranking functions allow us to rank elements based on the quality of sets they are contained in. These functions were first introduced in the economics literature [6], in order to judge the performance of individuals based on the success of groups that they were involved in, and has received significant attention from economists and computer scientists [7, 8, 9, 10]. Unfortunately, extension semantics in formal argumentation only distinguish sets of arguments that are accepted and the ones that are not accepted. This approach does not provide enough information to construct a fine-grained ranking of arguments by applying a social ranking function.

Closer to our needs, Skiba et al. [11] recently introduced so-called extension-ranking semantics that refine and extend classical argumentation semantics by providing a partial ranking over sets of arguments.

We show that, by applying the right social ranking functions to an extension-ranking semantics, we can define argument-ranking semantics that are a refinement of the traditional skeptical/credulous acceptance of arguments, both in spirit and in a strict technical sense. More precisely, we show that by applying the *lexicographic excellence operator* introduced by Bernardi et al. [9] to the extension-ranking semantics of Skiba et al. [11] we generate an argument ranking such that all skeptically accepted arguments are ranked before all credulously accepted arguments, which are, in turn, ranked before all non-accepted arguments. More generally, we show which axiomatic properties are sufficient and necessary for a social ranking operator to give rise to such a ranking (Section 4). We conclude by studying the axiomatic properties of the argument-ranking semantics induced by the lexicographic excellence operator (Section 5) and conclude that it is a well-behaved argument ranking semantics even beyond the refinement properties that motivated our work (Sections 6 and 7).

### 2. Preliminaries

In this section, we introduce the basics of abstract argumentation literature that are necessary for our work. We will start with the standard model of abstract argumentation, before introducing argument-ranking and extension-ranking semantics.

**Abstract Argumentation Frameworks** An abstract argumentation framework (AF) is a directed graph F = (A, R) where A is a (finite) set of arguments and  $R \subseteq A \times A$  is an attack relation among them [12]. An argument a is said to attack an argument b if  $(a, b) \in R$ . We say that an argument a is defended by a set  $E \subseteq A$  if every argument  $b \in A$  that attacks a is attacked by some  $c \in E$ . For  $a \in A$  we define  $a_F^- = \{b \mid (b, a) \in R\}$  and  $a_F^+ = \{b \mid (a, b) \in R\}$  as the sets of arguments attacking a and the sets of arguments attacking a set a = a.

<sup>22</sup>nd International Workshop on Nonmonotonic Reasoning, November 2-4, 2024, Hanoi, Vietnam

<sup>&</sup>lt;sup>†</sup>These authors contributed equally.

<sup>☆</sup> lars.bengel@fernuni-hagen.de (L. Bengel);

giovanni.buraglio@tuwien.ac.at (G. Buraglio); jan.maly@tuwien.ac.at (J. Maly); kenneth.skiba@fernuni-hagen.de (K. Skiba)

<sup>© 0222</sup> Copyright for this paper by its authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0).



**Figure 1:** Argumentation framework  $F_1$  from Example 1.

ments that are attacked by a in F. For a set of arguments  $E \subseteq A$  we extend these definitions to  $E_F^-$  and  $E_F^+$  via  $E_F^- = \bigcup_{a \in E} a_F^-$  and  $E_F^+ = \bigcup_{a \in E} a_F^+$ , respectively. If the AF is clear in the context, we will omit the index.

Most semantics [13] for abstract argumentation are relying on two basic concepts: *conflict-freeness* and *admissibility*.

**Definition 1.** Given F = (A, R), a set  $E \subseteq A$  is: conflict-free iff  $\forall a, b \in E$ ,  $(a, b) \notin R$ ; admissible iff it is conflict-free, and every element of E is defended by E.

For an AF F we use cf(F) and ad(F) to denote the sets of conflict-free and admissible sets, respectively. In order to define the remaining semantics proposed by Dung [12] and in addition the semi-stable semantics [14] we use the *characteristic function*.

**Definition 2.** For an AFF = (A, R) and a set of arguments  $E \subseteq A$  the characteristic function  $\mathcal{F}_F(E) : 2^A \to 2^A$  is defined via:

$$\mathcal{F}_F(E) = \{a \in A | E \text{ defends } a\}$$

An admissible set  $E \subseteq A$  is a complete extension (co) iff  $E = \mathcal{F}_F(E)$ ; a preferred extension (pr) iff it is a  $\subseteq$ -maximal complete extension; the unique grounded extension (gr) iff E is the least fixed point of  $\mathcal{F}_F$ ; a stable extension (stb) iff  $E_F^+ = A \setminus E$ ; a semi-stable extension (sst) iff it is a complete extension, where  $E \cup E_F^+$  is  $\subseteq$ -maximal.

The sets of extensions of an AF F for these five semantics are denoted as (respectively) co(F), pr(F), gr(F), stb(F)and sst(F). Based on these semantics, we can define the status of any argument, namely *skeptically accepted* (belonging to each  $\sigma$ -extension), *credulously accepted* (belonging to some  $\sigma$ -extension) and *rejected* (belonging to no  $\sigma$ -extension). Given an AF F and an extension-based semantics  $\sigma$ , we use (respectively)  $sk_{\sigma}(F)$ ,  $cred_{\sigma}(F)$  and  $rej_{\sigma}(F)$  to denote these sets of arguments.

**Example 1.** Consider the AF  $F_1 = (A, R)$  depicted as a directed graph in Figure 1, with the nodes corresponding to arguments  $A = \{a, b, c, d\}$ , and the edges corresponding to attacks  $R = \{(a, b), (b, c), (c, d), (d, c)\}$ . We see that  $F_1$  has three complete extensions  $\{a\}, \{a, c\}$  and  $\{a, d\}$  only the last two are preferred in addition. Also, we see that,  $a \in sk_{co}(F_1), c, d \in cred_{co}(F_1)$ , and  $b \in rej_{co}(F_1)$ .

An isomorphism  $\gamma$  between two AFs F = (A, R) and F' = (A', R') is a bijective function  $\gamma : F \to F'$  such that  $(a, b) \in R$  iff  $(\gamma(a), \gamma(b)) \in R'$  for all  $a, b \in A$ .

**Argument-ranking Semantics** Instead of reasoning based on the acceptance of sets of arguments, *argument-ranking semantics* (also know as *ranking-based semantics*) [2] were introduced to focus on the strength of a single argument. Note that the order returned by an argument-ranking semantics is not necessarily total, i.e. not every pair of arguments is comparable.

**Definition 3.** An argument-ranking semantics  $\rho$  is a function which maps an AF F = (A, R) to a preorder<sup>1</sup>  $\succeq_F^{\rho}$  on A.

Intuitively  $a \succeq_F^{\rho} b$  means that a is at least as strong as b in F. We define the usual abbreviations as follows;  $a \succ_F^{\rho} b$  denotes *strictly stronger*, i. e.  $a \succeq_F^{\rho} b$  and  $b \not\succeq_F^{\rho} a$ . Moreover,  $a \simeq_F^{\rho} b$  denotes *equally strong*, i. e.  $a \succeq_F^{\rho} b$  and  $b \succeq_F^{\rho} a$ .  $a \bowtie_F^{\rho} b$  denotes *incomparability* so neither  $a \succeq_F^{\rho} b$  nor  $b \succeq_F^{\rho} a$ .

Traditionally the development of argument-ranking semantics is guided by a principle-based approach [15]. Each principle embodies a different property for argument rankings. We recall some of the most fundamental principles [4] as well as newer ones, which are closer to the extensionbased reasoning process [16]. Before we start, we need additional notations. Let F = (A, R) be an AF with arguments  $a, b \in A$ . A *path* of length  $l_P = n$  between two arguments a, b is a sequence of arguments  $P(a, b) = (a_0, a_1, ..., a_n)$ with  $(a_i, a_{i+1}) \in R$  for all *i* with  $a_0 = a$  and  $a_n = b$ . The connected components cc(F) of an AF F are the maximal subgraphs F' = (A', R'), where for every pair of arguments  $a, b \in A'$  there exists an undirected path  $P_u(a, b) =$  $(a = a_0, a_1, ..., a_{n-1}, a_n = b)$  s.t. for every *i* there is either  $(a_i, a_{i+1}) \in R$  or  $(a_{i+1}, a_i) \in R$ . For an extensionbased semantics  $\sigma$ , an argument *a* weakly  $\sigma$ -supports *b* if  $b \in cred_{\sigma}(F)$  and for all  $E \in \sigma(F)$ , with  $b \in E$  then  $a \in E$  and a strongly  $\sigma$ -supports b if  $b \in cred_{\sigma}(F)$  and for all  $E \in \sigma(F)$ , with  $b \in E$  then there is  $E' \in \sigma(F)$  with  $E' \subseteq E, a \in E' \text{ and } b \notin E'.$ 

**Definition 4.** An argument-ranking semantics  $\rho$  satisfies the respective principle iff for all AFs F = (A, R) and any  $a, b \in A$ :

- **Abstraction (Abs).** Names of arguments should not be relevant. For a pair of AFs F = (A, R) and F' = (A', R') and every isomorphism  $\gamma : F \to F'$ , we have  $a \succeq_F^{\rho} b$  iff  $\gamma(a) \succeq_{F'}^{\rho} \gamma(b)$ .
- **Independence (In).** Unconnected arguments should not influence a ranking. For every  $F' = (A', R') \in cc(F)$ and for all  $a, b \in A'$ :  $a \succeq_F^{\rho} b$  iff  $a \succeq_{F'}^{\rho} b$ .
- Void Precedence (VP). Unattacked arguments should be ranked better then attacked ones. If  $a_F^- = \emptyset$  and  $b_F^- \neq \emptyset$  then  $a \succ_F^{\rho} b$ .
- Self-Contradiction (SC). Self-attacking arguments should be ranked worse than any other argument. If  $(a, a) \notin R$  and  $(b, b) \in R$  then  $a \succ_F^{\rho} b$ .
- **Cardinality Precedence (CP).** Two arguments are compared based on the number of attackers. If  $|a_F^-| < |b_F^-|$  then  $a \succ_F^{\rho} b$ .
- **Quality Precedence (QP).** Two arguments are compared based on the strength of their attackers. If there is  $c \in b_F^-$  s.t. for all  $d \in a_F^-$  it holds that  $c \succ_F^{\rho} d$  then  $a \succ_F^{\rho} b$ .
- **Counter-Transitivity (CT).** Two arguments are compared based on the number and quality of their attackers. If some injective  $f : a_F^- \to b_F^-$  exists s.t.  $f(x) \succeq_F^{\rho} x$ for all  $x \in a_F^-$  then  $a \succeq_F^{\rho} b$ .

<sup>&</sup>lt;sup>1</sup>A preorder is a (binary) relation that is *reflexive* and *transitive*.

- Strict Counter-Transitivity (SCT). Strict version of CT. If some injecitve  $f:a_{F}^{-}\rightarrow b_{F}^{-}$  exists s.t.  $f(x)\succeq_{F}^{\rho}x$ for all  $x \in a_F^-$  and either  $|a_F^-| < |b_F^-|$  or there exists some  $x \in a_F^-$  with  $f(x) \succ_F^{\rho} x$ , then  $a \succ_F^{\rho} b$ .
- Defense Precedence (DP). For two arguments with the same number of attackers, a defended argument is ranked better than a non-defended argument. If  $|a_F^-| = |b_F^-|$ ,  $(a_F^-)_F^- \neq \emptyset$  and  $(b_F^-)_F^- = \emptyset$ , then  $a \succeq_F^{\rho} b.$
- Distributed Defense precedence (DDP). The best defender attacks exactly one attacker. If  $|a_F^-| = |b_F^-|$ and  $|(a_F^-)_F^-| = |(b_F^-)_F^-|$ , and if defense of a is simple - every direct defender of a directly attacks exactly one direct attacker of a - and distributed - every direct attacker of a is attacked by at most one argument and defense of b is simple but not distributed, then  $a \succ_F^{\rho} b.$
- Non-attacked Equivalence (NaE) Two unattacked arguments should be equally ranked. If  $a_F^- = b_F^- = \emptyset$ then  $a \simeq_F^{\rho} b$ .
- Attack vs. Full Defense (AvsFD). Arguments without any unattacked indirect attackers should be ranked better than arguments only attacked by one unattacked argument. If F acyclic and every path P(u, a) in F from unattacked u to a has  $l_p = 0 \mod 2$  and there exists unattacked  $v \in b_F^-$ , then  $a \succ_F^{\rho} b$ .
- $\sigma$ -Compatibility ( $\sigma$ -C). Credulously accepted arguments should be ranked better than rejected arguments. For an extension-based semantics  $\sigma$  it holds that if  $a \in$  $cred_{\sigma}(F)$  and  $b \in rej_{\sigma}(F)$ , then  $a \succ_{F}^{\rho} b$ .
- weak  $\sigma$ -Support ( $w\sigma$ -S). If an argument a is an unavoidable side-effect of accepting another argument b, then a should be at least as acceptable as b. If a weakly  $\sigma$ -supports b, then  $a \succeq_F^{\rho} b$ .
- strong  $\sigma$ -Support (s $\sigma$ -S). If an argument a is a prerequisite for accepting another argument b and b is irrelevant for accepting a, then a should be ranked better then b. If a strongly  $\sigma$ -supports b, then  $a \succ_F^{\rho} b$ .

Note that these principles are not always compatible with each other, especially SC and CP are not compatible [2].

Extension-ranking Semantics Extension-ranking semantics defined in Skiba et al. [11] are a generalisation of extension-based semantics. These semantics are used to formalise whether a set E is more plausible to be accepted than another set E'.

**Definition 5.** Let F = (A, R) be an AF. An extension ranking on F is a preorder over the powerset of arguments  $2^A$ . An extension-ranking semantics  $\tau$  is a function that maps each F to an extension ranking  $\Box_F^{\tau}$  on F.

For an AF F = (A, R), an extension-ranking semantics  $\tau$  and two sets  $E, E' \subseteq A$  we say E is at least as plausible to be accepted as E' with respect to  $\tau$  in F if  $E \sqsupset_F^{\tau} E'$ . We define the usual abbreviations as follows: E is strictly more plausible to be accepted than E' (denoted as  $E \sqsupset_F^{\tau} E'$ ) if  $E \supseteq_F^{\tau} E'$  and not  $E' \supseteq_F^{\tau} E; E$  and E' are equally as plausible to be accepted (denoted as  $E \equiv_F^{\tau} E'$ ) if  $E \supseteq_F^{\tau} E'$  and

 $E' \sqsupseteq_F^{\tau} E; E \text{ and } E' \text{ are incomparable (denoted } E \asymp_F^{\tau} E')$ if neither  $E \sqsupseteq_F^{\tau} E'$  nor  $E' \sqsupseteq_F^{\hat{\tau}} E$ .

Skiba et al. [11] defined a family of approaches to define such extension-ranking semantics. Their semantics are generalisations of the classical extension-based semantics. Using these semantics we can state that a set is "closer" to being admissible, than another set. Before we define the semantics, we recall the base relations, each of them generalises one aspect of extension-based reasoning.

**Definition 6** (Base Relations [11]). Let F = (A, R) be an AF and  $E \subseteq A$  where the function  $\mathcal{F}_F^*$  :  $\mathcal{P}(A) \rightarrow$  $\mathcal{P}(A)$  is defined as  $\mathcal{F}_F^*(E) = \bigcup_{i=1}^{\infty} \mathcal{F}_{i,F}^*(E)$  over the powerset  $\mathcal{P}(A)$  of A with  $\mathcal{F}_{1,F}^*(E) = E$  and  $\mathcal{F}_{i,F}^*(E) = \mathcal{F}_{i,F}^*(E)$  $\mathcal{F}_{i-1,F}^*(E) \cup (\mathcal{F}_F(\mathcal{F}_{i-1,F}^*(E)) \setminus E_F^-)$ . Each base relation  $\alpha \in \{CF, UD, DN, UA\}$  is defined via:

• 
$$CF_F(E) = \{(a, b) \in R | a, b \in E\};$$

- $UD_F(E) = E \setminus \mathcal{F}_F(E);$
- $DN_F(E) = \mathcal{F}_F^*(E) \setminus E;$   $UA_F(E) = \{a \in A \setminus E | \neg \exists b \in E : (b, a) \in R\};$

For every base relation, the corresponding  $\alpha$  base extension ranking  $\Box_F^{\alpha}$  for  $E, E' \in A$  is given by:

$$E \sqsupseteq_F^{\alpha} E'$$
 iff  $\alpha_F(E) \subseteq \alpha_F(E')$ 

By combining these base relations, we denote the extension-ranking semantics.

**Definition 7.** Let F = (A, R) be an AF and  $E, E' \subseteq A$ . We define: Admissible extension-ranking semantics r-ad via  $E \supseteq_F^{r-ad} E'$  iff  $E \supseteq_F^{CF} E'$  or  $(E \equiv_F^{CF} E')$  and  $E \supseteq_F^{LD} E'$ . Complete extension-ranking semantics r-co via  $E \supseteq_F^{r-co} E'$  iff  $E \supseteq_F^{r-ad} E'$  or  $(E \equiv_F^{r-ad} E')$  and  $E \supseteq_F^{DN} E'$ . Preferred extension-ranking semantics r-pr via  $E \supseteq_F^{r,pr} E'$  iff  $E \supseteq_F^{r,ad} E'$  or  $(E \supseteq_F^{r,ad} E'$  and  $E' \subseteq E$ ). Grounded extension-ranking semantics r-gr via  $E \sqsupseteq_F^{r-gr} E'$  iff  $E \sqsupset_F^{r-co} E'$  or  $(\breve{E} \equiv_F^{r-co} E'$  and  $E\subseteq E').$  Semi-stable extension-ranking semantics r-sstvia  $E \supseteq_F^{r-sot} E'$  iff  $E \supseteq_F^{r-co} E'$  or  $(E \cong_F^{r-co} E'$  and  $E \supseteq_F^{LA} E')$ .

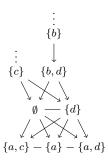
In words, one set  ${\cal E}$  is at least as plausible to be accepted as E' with respect to the admissible extension-ranking semantics, if E has less conflicts than E' or if they have the same conflicts, then we look at the undefended arguments.

Example 2. Continuing Example 1. Consider for instance the sets  $E_1 = \{b\}, E_2 = \{b, d\}$  and  $E_3 = \{d\}$ . For the complete extension-ranking semantics, we first of all have that all three sets contain no conflicts. However, both  $E_1$  and  $E_2$  contain the argument b which they do not defend against a. It follows that  $E_3 \supseteq_{F_1}^{r\text{-}co} E_1$  and  $E_3 \supseteq_{F_1}^{r\text{-}co} E_2$ . Furthermore, both  $E_1$ and  $E_2$  defend d from c, but d is not contained in  $E_1$ . Thus, we have that  $E_2 \sqsupseteq_{F_1}^{r\text{-}co} E_1$ .

The relevant excerpt of the extension-ranking for r-co can be found in Figure 2.

Extension-ranking semantics also follow a principlebased approach. Before we recall the principles defined in Skiba et al. [11], we need to introduce the notion of most plausible sets, i. e. sets for which we cannot find any other sets ranked strictly better.

**Definition 8** (Most plausible sets). Let F = (A, R) be an AF,  $E, E' \subseteq A$  two sets of arguments and  $\tau$  an extensionranking semantics. We denote by  $max_{\tau}(F)$  the maximal (or most plausible) elements of the extension ranking  $\Box_F^{\tau}$ , i.e.  $max_{\tau}(F) = \{ E \subseteq A \mid \nexists E' \subseteq A \text{ with } E' \sqsupset_F^{\tau} E \}.$ 



**Figure 2:** Excerpt of the extension-ranking  $\Box_{F_1}^{r-co}$  for the complete semantics, where  $E \to E'$  means  $E' \sqsupset_{F_1}^{r_{-co}} E$  and E - E'means  $E' \equiv_{F_1}^{r\text{-}co} E$ 

The principle  $\sigma$ -generalisation states, that the most plausible sets should coincide with the  $\sigma$ -extensions.

**Definition 9** ( $\sigma$ -Gen). Let  $\sigma$  be an extension-based semantics and  $\tau$  an extension-ranking semantics.  $\tau$  satisfies  $\sigma$ -soundness iff for all AF:  $max_{\tau}(AF) \subseteq \sigma(AF)$ .  $\sigma$ -completeness iff for all AF:  $max_{\tau}(AF) \supseteq \sigma(AF)$ .  $\sigma$ -generalisation iff  $\tau$  satisfies both  $\sigma$ -soundness and  $\sigma$ completeness.

Additional principles can be found in Skiba et al. [11].

### 3. Social Ranking

Let us now introduce the final piece of our puzzle, social rankings. Let S be a set of arbitrary objects like players of a sports team, employees of a company or arguments in an AF and  $\mathcal{P}(S)$  its powerset. A social ranking function  $\xi$ , as introduced by Moretti and Öztürk [6], maps a preorder  $\supseteq$ on  $\mathcal{P}(S)$  to a partial order on S. In the context of this work, we consider the preorder to be an extension-ranking  $\Box_F^{r-\sigma}$ for some argumentation framework F and a semantics  $\sigma$  as defined above. The most prominent social ranking function is the lexicographic excellence operator (lex-cel), which was first proposed by Bernardi et al. [9]. It ranks elements based on the best sets they appear in, proceeding lexicographically if there are ties. However, as proposed by Bernardi et al. [9], the lex-cel operator requires a total order of the sets as input, while the extension ranking semantics defined above only provide a partial ranking. To circumvent this problem, we make use a measure of the quality of a set that allows us to compare any two sets, the *rank* of a set.

**Definition 10.** Let  $X \subseteq S$  be a subset of S and  $\supseteq$  a preorder on  $\mathcal{P}(S)$ . Moreover, let  $X_1, X_2, \ldots, X_k$  be the longest sequence such that  $X_1 \sqsupset X_2 \sqsupset \cdots \sqsupset X_k \sqsupset X$ . Then, we *define the* rank of X, as  $rank_{\supseteq}(X) := k + 1$ .

Moreover, for an element  $x \in S$ , we define

$$x_{k,\square} := |\{X \in \mathcal{P}(S) \mid rank_{\square}(X) = k, x \in X\}|,$$

as the number of rank k subsets that contain x.

As we will see later, a rank-based approach to social ranking provides many desirable properties, at least in our context of ranking arguments. With the definition of a rank at hand, we can now define our rank-based version of the lex-cel social ranking function.

**Definition 11.** Let  $x, y \in S$  be two elements of S. The lexcel ranking  $\succeq^{lex-cel}$  is defined by  $x \succ^{lex-cel}_{\supseteq} y$  if there exists a k such that  $x_{i,\square} = y_{i,\square}$  for all i < k and  $x_{k,\square} > y_{k,\square}$  and  $x \sim^{lex-cel}_{\supseteq} y$  if  $x_{i,\square} = y_{i,\square}$  for all i.

Intuitively, an object x is ranked better than y by the lexicographic excellence operator if x is contained in more highly ranked sets than y.

**Example 3.** We continue Example 2 with the complete extension-ranking as depicted in Figure 2. Then, we have three sets with rank 1, namely the complete extensions. The argument a is contained in all three sets with rank 1, while c and d are only contained in one such set each. Consequently  $a \succeq^{lex-cel} c$  and  $a \succeq^{lex-cel} d$ . Now, the final admissible sets  $\emptyset$ and  $\{d\}$  are dominated by all three complete extensions under the complete extension-ranking semantics, but dominate all non-admissible sets. Therefore, they are the only sets with rank 2. It follows that  $d \succeq^{lex-cel} c$  as both are contained in the same number of sets with rank 1, but d is contained in more sets with rank 2.

Similarly to argument- and extension-ranking semantics, social rankings have been studied axiomatically. Let us first introduce an axiom that has been part of a characterization of the lex-cel function under the assumption that the ranking over sets is a total preorder [9]. As we generally do not assume the ranking over extensions to be a total preorder, the characterisation does not hold in our setting, but it is straightforward to see that the lex-cel function still satisfies this axiom.

**Definition 12** (Independence from the worst set). Let  $\supseteq$ *be a preorder on*  $\mathcal{P}$ *, let* 

$$w = \max_{X \in \mathcal{P}} (rank_{\square}(X))$$

and assume that  $\supseteq^*$  is another preorder on  $\mathcal{P}$  for which it holds

- $rank_{\square}(X) = rank_{\square^*}(X)$  for all  $X \in \mathcal{P}$  such that
- $\operatorname{rank}_{\supseteq}(X) < w.$   $\operatorname{rank}_{\supseteq^*}(X) \geq w$  for all  $X \in \mathcal{P}$  such that  $\operatorname{rank}_{\supseteq}(X) = w.$

Then for any social ranking function that satisfies Independence from the worst set, we must have that  $x \succ_{\Box} y$  implies  $x \succ_{\exists *} y.$ 

Intuitively, this axiom states that if one element is already strictly worse than another, and we further subdivide the worst sets, this strict preference remains. As we will see later, this axiom will be crucial for satisfying our desired refinement property. Next, we introduce a new, very weak axiom inspired by the classical Pareto-efficiency concept [17], that is satisfied by most reasonable rank-based social ranking functions.

**Definition 13** (Pareto-efficiency). Let  $\supseteq$  be a preorder on  $\mathcal{P}$  and let x, y be elements such that

- $\operatorname{rank}_{\square}(Z \cup \{x\}) \leq \operatorname{rank}_{\square}(Z \cup \{y\}) \text{ for all } Z \in \mathcal{P}$ with  $x, y \notin Z$ ;
- $rank_{\perp}(Z \cup \{x\}) < rank_{\perp}(Z \cup \{y\})$  for at least one  $Z \in \mathcal{P}$  with  $x, y \notin Z$ .

A social ranking function  $\xi$  satisfies Pareto-efficiency, iff  $x \succ_{\Box}^{\xi} y.$ 

Furthermore, we establish the novel *Dominating set* axiom which captures the intuition that if there exists a set containing the object x that is ranked better than every set that contains some other object y, then x must be ranked better than y by the social ranking function.

**Definition 14** (Dominating set). Let  $\supseteq$  be a preorder on  $\mathcal{P}$ and let x, y be elements such that  $\exists X \subseteq \mathcal{P}$  with  $x \in X$  such that  $\forall Y$  with  $y \in Y$  then  $X \sqsupset Y$ . A social ranking function  $\xi$  satisfies Dominating set iff  $x \succ_{\Box}^{\xi} y$ .

Crucially, Independence from the Worst Set and Paretoefficiency together imply Dominating set.

**Theorem 1.** Any social ranking function that satisfies Independence from the worst set and Pareto-efficiency also satisfies Dominating set.

*Proof.* Let  $\Box$  be a preorder on  $\mathcal{P}$  and let x, y be elements such that  $\exists X^d \subseteq \mathcal{P}$  with  $x \in X^d$  such that  $\forall X'$  with  $y \in$ X' then  $X^d \sqsupset X'$ . Furthermore, let  $w := \operatorname{rank}_{\Box}(X^d) + 1$ . We consider the preorder  $\supseteq^*$  that is defined as follows: For any two sets  $X, Y \in \mathcal{P}$  we have  $X \supseteq^* Y$  if and only if  $X \supseteq Y$  and either  $\operatorname{rank}_{\Box}(X) < w$  or  $\operatorname{rank}_{\Box}(Y) < w$ . We claim that

$$\max_{\mathbf{X} \in \mathcal{D}} (\operatorname{rank}_{\exists^*}(X)) = w$$

First, to see that  $\max_{X \in \mathcal{P}} (\operatorname{rank}_{\square^*}(X)) \leq w$  we assume for the sake of a contradiction that there is a set X with  $\operatorname{rank}_{\square^*}(X) = w^* > w$ . Then, by definition, there is a sequence  $X_1 \sqsupseteq^* X_2 \sqsupseteq^* \cdots \sqsupset^* X_{w^*} \sqsupseteq^* X$ . As every preference in  $\square^*$  is also valid in  $\square$ , the same sequence exists for  $\square$ , i.e.  $X_1 \sqsupseteq X_2 \sqsupseteq \cdots \sqsupseteq X_{w^*} \sqsupseteq X$ . However, this means  $\operatorname{rank}_{\square}(X_{w^*}) \geq w^* - 1 \geq w$  and  $\operatorname{rank}_{\square}(X) \geq$  $w^* > w$ , which contradicts  $X_{w^*} \sqsupseteq^* X$ .

To see that  $\max_{X \in \mathcal{P}} (\operatorname{rank}_{\square^*}(X)) \geq w$  we first observe that as  $\operatorname{rank}_{\square}(X^d) = w - 1$  there is a sequence  $X_1 \supseteq X_2 \supseteq \cdots \supseteq X_{w-1} \supseteq X$ . As this sequence is maximal,  $\operatorname{rank}_{\square}(X_i) < w$  for all elements  $X_i$  of the sequence. Hence the same sequence exists in  $\square^*$ . Finally, as  $X^d$  is a dominating set, we know  $X^d \supseteq \{y\}$  and as  $\operatorname{rank}_{\square}(X^d) < w$ , we also have  $X^d \supseteq^* \{y\}$ . Therefore,  $X_1 \supseteq^* X_2 \supseteq^* \cdots \supseteq^* X_{w-1} \supseteq^* X \supseteq^* \{y\}$  witnesses that  $\operatorname{rank}_{\square^*}(\{y\}) \geq w$ .

Next, we claim that  $x \succ^{\exists^*} y$  for all social ranking functions that satisfy Pareto-efficiency: By definition, rank $\exists^*(X^d) = w - 1$ . Furthermore, we have  $X^d = (X^d \setminus \{x\}) \cup \{x\} \exists^* (X^d \setminus \{x\}) \cup \{y\}$ , and thus rank $\exists^*((X^d \setminus \{x\}) \cup \{y\}) > w - 1$ . This shows that rank $\exists^*(X^d) < \operatorname{rank}_{\exists^*}((X^d \setminus \{x\}) \cup \{y\})$ . On the other hand, there can be no Z such that  $\operatorname{rank}_{\exists^*}(Z \cup \{y\}) <$ rank $\exists^*(Z \cup \{x\})$ : As  $Z \cup \{y\}$  is dominated by  $X^d$ , we know rank $\exists(Z \cup \{x\}) \ge w$  and thus  $\operatorname{rank}_{\exists}(Z \cup \{y\}) \ge w$ . Thus, the claim follows directly from  $\max_{X \in \mathcal{P}}(\operatorname{rank}_{\exists^*}(X)) = w$ Finally, if  $\succeq$  also satisfies Independence from the upper

Finally, if  $\succeq$  also satisfies Independence from the worst set, if follows that also  $x \succ \supseteq y$ , as  $\supseteq$  is just a refinement of the worst set of  $\supseteq^*$ .

# 4. Defining Argument-ranking Semantics via Social Rankings

The idea of combining extension-ranking semantics with argument-ranking semantics was briefly discussed by Skiba et al. [11], where, based on a ranking over sets of arguments, a ranking over arguments was defined. In this section, we take a more general view on this approach and define argument-ranking semantics based on an extensionranking.

#### 4.1. The Singleton Approach

The most immediate way of ranking objects based on a ranking over sets of objects is to restrict the ranking over sets of objects to the singleton sets. The behaviour of these singleton sets then gives us insight into the relationship between the objects. If  $\{a\}$  is ranked better than  $\{b\}$  then a is also ranked better than b in the restricted ranking.

**Definition 15.** Let F = (A, R) be an AF and  $\tau$  any extension-ranking semantics. For any two arguments  $a, b \in A$ , the singleton argument-ranking semantics  $ST_{\tau}$  is defined via  $a \succeq_F^{ST_{\tau}} b$  iff  $\{a\} \sqsupseteq_F^{\tau} \{b\}$ .

Bernardi et al. [9] have already discussed that a ranking based solely on singleton sets is too simplistic, as it ignores all the information provided by rankings over sets with cardinality larger than one. In the context of abstract argumentation, this is also the case.

**Example 4.** Consider the AF  $F_1$  from Example 1. We use r-ad as the underlying extension-ranking semantics, then since  $\{a\}$  and  $\{d\}$  are admissible we have  $a = {}_{F_1}^{ST_{r-ad}} d$  and both  $\{b\}$  and  $\{c\}$  are conflict-free and not defended, so

$$a =_{F_1}^{\mathcal{ST}_{r-ad}} d \succ_{F_1}^{\mathcal{ST}_{r-ad}} b =_{F_1}^{\mathcal{ST}_{r-ad}} c$$

6

The example shows that  $ST_{r-ad}$  has a limited expressiveness, since  $ST_{r-ad}$  has at most three ranks. The first rank contains arguments for which the singleton set is admissible and the lowest rank are all self-attacking arguments, in between are the non-admissible sets, but conflict-free singleton sets. Observe also that this approach does not refine the classical skeptical/credulous acceptance classification, as in Example 4 the credulously accepted argument c is ranked the same as the rejected argument b.

#### 4.2. Generalised Social Ranking Argument-ranking Semantics

In the literature, a number of different social ranking functions that are more complex than the singleton approach can be found [18, 9, 8, 7]. To understand what constitutes a good social ranking function in this context, we define a general argument-ranking semantics using social ranking solutions with respect to an extension ranking.

**Definition 16.** Let F = (A, R) be an AF and  $\xi$  a social ranking function with respect to extension ranking  $\tau$ . For any  $a, b \in A$  we call  $\xi_{\tau}$  the Social ranking argument-ranking semantics such that:

$$a \succeq_F^{\xi_{\tau}} b iff a \succeq_{\tau}^{\xi} b$$

In words, an argument a is at least as strong as argument b if the social ranking function  $\xi$  applied to the extension ranking  $\Box^{\tau}$  returns that a is at least as strong as b.

**Example 5.** In Example 3 the social ranking argument ranking lex-cel<sub>r-co</sub> was applied to the AF  $F_1$  from Example 1 where lex-cel is used and the underlying extension-ranking semantics is r-co. Thus, the resulting argument ranking is:

$$a \succ_{F_1}^{lex-cel_{r-co}} d \succ_{F_1}^{lex-cel_{r-co}} c \succ_{F_1}^{lex-cel_{r-co}} b$$

Any social ranking function can be used to rank arguments. Skiba et al. [11] have used a variation of the lex-cel social ranking function in their definitions, where an argument a is ranked better than another argument b if we can find a set E containing a which is ranked better than any set containing b.

**Definition 17** ([11]). Let F = (A, R) be an AF,  $a, b \in A$ , and  $\tau$  be an extension-ranking semantics. We define an argument-ranking semantics  $\succeq_F^{\tau}$  via  $a \succeq_F^{\tau} b$  iff there is a set E with  $a \in E$  s.t. for all sets E' with  $b \in E'$  we have  $E \supseteq_F^{\tau} E'$ .

**Example 6.** Continuing with Example 1. Using r-ad as the underlying extension-ranking semantics, we see that  $\{a, c\}$  and  $\{a, d\}$  are admissible sets, hence also among the most plausible sets. Since r-ad satisfies ad-generalisation there cannot be any set containing b ranked strictly better, than these two sets. This observation result in the ranking  $a \simeq_{F_1}^{r$ -ad}  $c \simeq_{F_1}^{r$ -ad}  $d \succ_{F_1}^{r$ -ad} b. Since  $\{a, c\}, \{a, d\} \in \sigma(F_1)$  for  $\sigma \in \{co, pr, stb, sst\}$  the ranking is the same for any r- $\sigma$ . Only for r-gr the induced ranking differs:

$$a \succ_{F_1}^{r \cdot gr} c \simeq_{F_1}^{r \cdot gr} d \succ_{F_1}^{r \cdot gr} b$$

The previous examples show that where lex-cel<sub>r-co</sub> can differentiate a, b, c, and d, the argument ranking of Definition 17 under *r*-*co* does not allow to distinguish among a, c and d. Indeed, lex-cel is more informative than the operator of Skiba et al. [11].

**Proposition 1.** Let F = (A, R) be an AF,  $a, b \in A$  and  $\tau$  an extension ranking. If  $a \succeq_{F}^{lex-cel_{\tau}} b$ , then  $a \succeq_{T}^{\tau} b$ .

 $\begin{array}{l} \textit{Proof.} \ \mbox{Let}\ F = (A,R) \mbox{ be an AF}, a,b \in A \mbox{ and } \tau \mbox{ an extension ranking. Assume} \ a \succeq_F^{\text{lex-cel}_{\tau}} \ b, \mbox{ then there is an } k \mbox{ s.t for all} \ i < k \mbox{ we have } a_{i,\tau} = b_{i,\tau} \mbox{ and } a_{k,\tau} \geq b_{k,\tau}. \end{array}$ 

If  $b_{j,\tau} \neq 0$  for  $1 \leq j \leq k$ , then there is one  $Y \subseteq A$  with  $rank_{\tau}(Y) = j$  and  $b \in Y$ . W.l.o.g. let j be the smallest number s.t.  $b_{j,\tau} \neq 0$ . Then  $Y \sqsupseteq_F^{\tau} X$  for all  $X \subseteq A$  with  $a \in X$ , therefore  $b \succeq_F^{\tau} a$ . Since,  $b_{j,\tau} \leq a_{j,\tau}$ , there has to be an  $X' \subseteq A$  with  $rank_{\tau}(X') = j$  and  $a \in X'$  s.t.  $X' \equiv_F^{\tau} Y$ , so  $a \succeq_F^{\tau} b$ .

If  $b_{j,\tau} = 0$  for all  $j \in \{1, \ldots, k\}$  and  $a_{k,\tau} > 0$ , then there is at least one  $X \subseteq A$  with  $a \in X$  and  $rank_{\tau}(X) = k$ s.t.  $X \sqsupset_F^{\tau} Y$  for all  $Y \subseteq A$  with  $b \in Y$ , and therefore  $a \succ_F^{\tau} b$ .

In particular, lex-cel<sub>r-co</sub> allows us to distinguish among skeptically and credulously accepted arguments (*a* is ranked before *c* and *d*). To capture this, we define a skeptical variation of  $\sigma$ -Compatibility. Skeptical accepted arguments are part of every  $\sigma$ -extension, therefore they should be ranked better than any other argument.

**Definition 18.** Let F = (A, R) be an AF,  $a, b \in A$ , and let  $\sigma$  be a extension-based semantics. Argument-ranking semantics  $\rho$  satisfies  $\sigma$ -skeptical-Compatibility ( $\sigma$ -sk-C) iff  $a \in sk_{\sigma}(F)$  and  $b \notin sk_{\sigma}(F)$  then  $a \succ_{F}^{\rho} b$ .

Crucially, a well-behaved argument ranking semantics should be able to rank skeptically accepted arguments before all credulously accepted ones, which should be, in turn, ranked before all non-accepted arguments. This translated to the following refinement property.

**Definition 19** ( $\sigma$ -Refinement). Argument-ranking semantics  $\rho$  satisfies  $\sigma$ -Refinement if  $\rho$  satisfies  $\sigma$ -C and  $\sigma$ -sk-C for extension-based semantics  $\sigma$  for all AFs F. Next, we investigate principles for social ranking based argument-ranking semantics from a general point of view. In particular, we are interested in understanding which combinations of axioms for extension-ranking semantics  $\tau$  and social ranking functions  $\xi$  represent necessary and sufficient conditions for the corresponding social ranking argument-ranking semantics  $\xi_{\tau}$  to satisfy fundamental principles of argument rankings, chiefly among them our desired refinement property. This translates to the following research questions:

- **RQ1** What properties of  $\xi$  and  $\tau$  are adequate to ensure that  $\xi_{\tau}$  satisfies a specific principle for argument-ranking semantics?
- **RQ2** What properties of  $\xi_{\tau}$  are adequate to ensure that  $\xi$  satisfies a specific principle for social ranking functions when combined with a certain extension-ranking semantics  $\tau$ ?

Next, we address RQ1 and RQ2 for a selected number of principles for argument ranking semantics.

#### 4.2.1. Sufficient Conditions for Social Ranking Argument-ranking semantics

We start by considering  $\sigma$ -*Compatibility*. For this we show that *Independence from the worst set* together with the quite weak condition *Pareto-efficiency*, is sufficient for satisfying  $\sigma$ -*C*.

**Theorem 2.** Let F = (A, R) be an argumentation framework,  $\tau$  an extension-ranking semantics, satisfying  $\sigma$ -generalisation for extension semantics  $\sigma$  and  $\xi$  a social ranking function that satisfies Independence from the worst set and Pareto-efficiency. Then  $\xi_{\tau}$  satisfies  $\sigma$ -C.

Proof: Consider first the extension ranking  $\exists^{\sigma}$  defined by  $X \exists_{F}^{\sigma} Y$  if and only if  $X \in \sigma(F)$  and  $Y \notin \sigma(F)$ . Furthermore, let  $x \in cred_{\sigma}(F)$  and  $y \in rej_{\sigma}(F)$ . Then, we claim that  $x \succ^{\exists^{\sigma}} y$  for any social ranking function  $\xi$  that satisfies Pareto-efficiency: As x is credulously accepted, there exists a  $X \in \sigma(F)$  with  $x \in X$  and as y is rejected, we have  $Y \notin \sigma(F)$  for all  $y \in Y$ . It follows that  $\operatorname{rank}_{\exists^{\sigma}}(X \setminus \{x\}) \cup \{x\}) = 1 < \operatorname{rank}_{\exists^{\sigma}}(X \setminus \{x\}) \cup \{x\})$  and there can be no S such that  $\operatorname{rank}_{\exists^{\sigma}}(S \cup \{y\}) < \operatorname{rank}_{\exists^{\sigma}}(S \cup \{x\})$  as, due to the fact that  $w = \max_{X \subseteq A}(\operatorname{rank}_{\exists^{\sigma}}(X)) = 2$ , this would imply  $\operatorname{rank}_{\exists^{\sigma}}(S \cup \{y\}) = 1$  and therefore  $S \cup \{y\} \in \sigma(F)$ .

Furthermore, as  $\tau$  satisfies  $\sigma$ -generalisation, we know that  $\operatorname{rank}_{\supseteq_F^{\sigma}}(X) = 1$  if and only if  $\operatorname{rank}_{\supseteq_F^{\tau}}(X) = 1$ . Therefore, it follows from Independence from the worst set that  $x \succ_F^{\xi_{\supseteq\sigma}} y$  implies  $x \succ_F^{\xi_{\tau}} y$ . Consequently, we know that  $\xi_{\tau}$  satisfies  $\sigma$ -*C*.

Next, we show that Independence from the worst set and Pareto-efficiency together also imply that every skeptically accepted argument is ranked before any argument that is not skeptically accepted.

**Theorem 3.** Let F = (A, R) be an AF,  $\tau$  an extension-ranking semantics satisfying  $\sigma$ -generalisation for an extension-based semantics  $\sigma$ , then if social ranking function  $\xi$  satisfies Pareto-efficiency and Independence from the worst set then  $\xi_{\tau}$  satisfies  $\sigma$ -sk-C.

*Proof.* Let F = (A, R) be an AF,  $\tau$  an extension-ranking semantics satisfying  $\sigma$ -generalisation for an extension-based semantics  $\sigma$ , and  $\xi$  a social ranking function satisfying Pareto-efficiency and Independence from the worst set. Since  $\sigma$ -generalisation is satisfied by  $\tau$  we can view  $\tau$  as a refinement of the extension-ranking semantics  $\tau'$  defined by  $X \sqsupseteq_F^{\tau'} Y$  iff  $X \in \sigma(F)$  and  $Y \notin \sigma(F)$  for  $X, Y \subseteq A$ .

Now consider two arguments,  $a, b \in A$ , such that  $a \in sk_{\sigma}(F)$  and  $b \notin sk_{\sigma}(F)$ . Assume there exists a  $Z \subseteq A \setminus \{a, b\}$  s.t.  $\operatorname{rank}_{\supseteq_{F}^{\tau'}}(Z \cup \{b\}) < \operatorname{rank}_{\supseteq_{F}^{\tau'}}(Z \cup \{a\})$ . Since  $\tau'$  only has two levels, this implies  $Z \cup \{b\} \in max_{\tau'}(F)$  and thus  $Z \cup \{b\} \in \sigma(F)$ . As  $a \in sk_{\sigma}(F)$ , we must have  $a \in Z \cup \{b\}$ . However, as  $a \notin Z$  we know that also  $a \notin Z \cup \{b\}$ . This is a contradiction and hence such a Z cannot exist.

Since  $b \notin sk_{\sigma}(F)$  we know there has to exists  $\overline{Y} \subseteq A$  s.t.  $\overline{Y} \in max_{\tau'}(F)$  and  $y \notin \overline{Y}$ . Then because  $a \in sk_{\sigma}(F)$  we know that  $(\overline{Y} \setminus \{a\}) \cup \{b\} \notin max_{\tau'}(F)$ . Consequently, Pareto-efficiency implies  $a \succ_{F}^{\xi_{\tau'}} b$ .

As  $a \succ_F^{\xi_{\tau'}} b$  holds for  $\tau'$ , and  $\tau$  is a refinement of  $\tau'$  such that  $max_{\tau'}(F) = max_{\tau}(F)$  it follows from Independence for the worst set that the same holds for  $\tau$ , i. e.  $a \succ_F^{\xi_{\tau}} b$ .  $\Box$ 

Additionally, observe that Independence from the worst set means that we might have to ignore most of the information that is available to us. The following result shows that, at least for the rank information, this is essentially unavoidable if we want to satisfy cf-C. Let us first introduce an axiom that encodes the idea that we cannot ignore overwhelming, rank based evidence.

**Definition 20** (Rank k-super majority). Let  $k \in \mathbb{N}$  be a natural number. Then we say a social ranking function  $\xi$  satisfies rank k-super majority if for all x and y such that

$$|\{Z \in \mathcal{P} \mid x, y \notin Z \land rank(Z \cup \{x\}) < rank(Z \cup \{y\})\}| > k \cdot |\{Z \in \mathcal{P} \mid x, y \notin Z \land rank(Z \cup \{y\}) < rank(Z \cup \{x\})\}|.$$

we have  $x \succeq y$ .

In words, if there are k-times as many sets Z such that the rank of  $Z \cup \{x\}$  is strictly better than the rank of  $Z \cup \{y\}$ , than the other way round, then x must be (weakly) preferred to y.

**Proposition 2.** Any social ranking function  $\xi_{r-cf}$  satisfies cf-C and violates rank k-super majority for every k.

*Proof.* Let k be an arbitrary natural number,  $\ell$  a natural number such that  $\ell \geq k$  and  $\ell \geq 3$ . Furthermore consider an argumentation framework F with the arguments  $a, b, c_1, \ldots c_\ell$  and the attacks (b, b) and  $(c_i, a)$  for all  $i \leq \ell$ . Then,  $a \in cred_{cf}(F)$ , as witnessed by the conflict free set  $\{a\}$ , but  $b \in rej_{cf}(F)$ , as it is self-attacking. It follows from the fact that  $a \succ b$ , because  $\preceq$  satisfies cf-C. However, observe that

$$\begin{split} \{Z \in \mathcal{P} \mid x, y \not\in Z \wedge \operatorname{rank}_{r \cdot \operatorname{cf}}(Z \cup \{a\})\} \\ < \operatorname{rank}_{r \cdot \operatorname{cf}}(Z \cup \{b\}) = \{\emptyset\} \end{split}$$

while

$$\begin{aligned} \{Z \in \mathcal{P} \mid x, y \notin Z \\ \wedge \operatorname{rank}_{r-\operatorname{cf}}(Z \cup \{a\}) < \operatorname{rank}_{r-\operatorname{cf}}(Z \cup \{b\})\} \\ &= \{Z \in \mathcal{P} \mid x, y \notin Z \land |Z| \ge 2\}. \end{aligned}$$

However, then, by our choice of  $\ell$  we know

$$|\{Z \in \mathcal{P} \mid x, y \notin Z \land |Z| \ge 2\}| > k = k \cdot |\{\emptyset\}|.$$

 $\square$ 

It follows that rank *k*-super majority is violated.

Next, consider the axiom *SC*. Here, we can find a property of social ranking functions that guarantees that  $\xi_{\tau}$  satisfies *SC* under the assumption that  $\tau$  satisfies the following principle:

**Definition 21** (Respects Conflicts). For AFF = (A, R) and  $E, E' \subseteq A$  extension-ranking semantics  $\tau$  satisfies respects conflicts if  $E \in cf(F)$  and  $E' \notin cf(F)$ , then  $E \sqsupset_F^{\tau} E'$ .

To show that  $\xi_{\tau}$  satisfies *SC* we also need the *Dominating set* property from Definition 14. With these two properties we can then show when SC is satisfied.

**Theorem 4.** For AFF = (A, R) if extension-ranking semantics  $\tau$  satisfies respects conflicts and social ranking function  $\xi$ satisfies Dominating set, then  $\xi_{\tau}$  satisfies SC.

*Proof.* For AF F = (A, R), let  $a, b \in A$ ,  $(b, b) \in R$  and  $(a, a) \notin R$ , then  $\{a\} \in cf(F)$  and for all E' with  $b \in E'$  it holds that  $E' \notin cf(F)$ . Because of respects conflicts we have  $\{a\} \sqsupset E'$  and therefore because of Dominating set we have  $a \succ_{F}^{\mathcal{E}_{T}} b$ .

#### 4.2.2. Necessary Conditions for Social Ranking Argument-ranking semantics

Let us try to go the other way, that is finding necessary conditions for the social ranking functions to satisfy desirable properties. First observe it is not possible to formulate any necessary conditions that also hold for any ranking that cannot be realised by any AF, i. e., we cannot find an AF that induces this ranking. This is because any property of the argument-ranking only restricts the social ranking function on realisable rankings. Therefore, we need to define the following concept in a similar vain to Dunne et al. [19].

**Definition 22.** Let X be a set and let  $\supseteq$  be a preorder on  $\mathcal{P}(X)$ . Then, we say that  $\supseteq$  is  $\tau$ -realisable for a extension-ranking semantics  $\tau$  if there is an AF F with A = X such that  $\supseteq_F^{\tau} = \supseteq$ .

For example, for a set  $\{a, b\}$  any preorder containing  $\{a, b\} \supseteq \{a\}$  is not r-cf-realisable. The conflicts in  $\{a, b\}$  must be a strict super-set of the conflicts in  $\{a\}$ . On the other hand, the preorder containing exactly the relations  $\{a\} \supseteq \{a, b\}$  and  $\{b\} \supseteq \{a, b\}$  is realised by the AF  $(\{a, b\}, \{(a, b)\})$ .

**Theorem 5.** Let  $\xi$  be a social ranking function such that  $\xi_{r-cf}$  satisfies cf-C. Then,  $\xi$  satisfies Dominating set for all r-cf-realisable preorders  $\supseteq$ .

*Proof.* Let  $\supseteq$  be a *cf*-realisable preorder and let *F* be an *AF* that realises it. Assume further that there are  $x, y \in A$  such that there exists a *X* with  $x \in X$  for which we have  $X \supseteq Y$  for all *Y* such that  $y \in Y$ .

As X contains x, its set of conflicts must be a strict superset of the conflicts in  $\{x\}$ . It follows that  $\{x\} \supseteq X \supseteq Y$ and hence by transitivity also  $\{x\} \supseteq Y$  for all Y such that  $y \in Y$ . In particular, it follows that  $\{x\} \supseteq \{y\}$ . By definition, this means  $CF_F(\{x\}) \subset CF_F(\{y\})$ , which can only hold if y is self-attacking and x is not. However, then x is credulously accepted in the under conflict-free semantics while y is not. Consequently, it follows from cf-C that  $x \succ y$ . Hence, dominating set is satisfied. It follows that dominating set is a necessary and sufficient condition for a social ranking function to satisfy cf-C when combined with r-cf.

A similar result can be found for admissible semantics.

**Theorem 6.** Let  $\xi$  be a social ranking s.t.  $\xi_{r-ad}$  satisfies ad-C. Then  $\xi$  satisfies Dominating set for all r-ad-realisable preorders  $\supseteq$ .

*Proof.* Let  $\supseteq$  be a *r*-*ad*-realisable preorder and AF F = (A, R) induces  $\supseteq$ . Assume  $x, y \in A$  such that there exists  $X \subseteq A$  with  $x \in X$  for which we have  $X \supseteq Y$  for all Y such that  $y \in Y$ .

Assume that the set X is not admissible. That means one of the following two cases must apply

(1) 
$$CF_F(X) \neq \emptyset$$
 or, (2)  $UD_F(X) \neq \emptyset$ 

to (1): Then, there is some attack  $(a,b) \in CF_F(X)$  for  $a,b \in X$ . From  $X \Box Y$  it follows that  $CF_F(X) \subseteq CF_F(Y)$  and thus  $(a,b) \in CF_F(Y)$ . Now, if y = a or y = b it follows that  $y \in X$  which directly contradicts our assumption because of  $X \equiv Y'$  for Y' = X with  $y \in Y'$ . However, if  $y \neq a$  and  $y \neq b$  we can construct  $Y' = Y \setminus \{a,b\}$ . Clearly, that means we either have  $CF_F(Y') = \emptyset$  which means  $Y \Box X$  or we have  $CF_F(Y') \neq \emptyset$  which implies  $X \asymp Y'$ . Because of  $y \in Y'$  both cases contradict the initial assumption, hence we must have that  $CF_F(X) = \emptyset$ , i. e. the set X is conflict-free.

to (2): Then, there exists an argument  $a \in UD_F(X)$ which is not defended by X. Consider now the set  $Y' = \{y\}$  for which we either have that  $UD_F(Y') = \emptyset$  or  $UD_F(Y') = \{y\}$ . If  $UD_F(Y') = \emptyset$ , it follows directly that  $Y' \supseteq X$ , contradicting our initial assumption. On the other hand, for  $UD_F(Y') = \{y\}$  we distinguish between two cases:

(2.1) 
$$y = x$$
, (2.2)  $y \neq x$ 

Clearly, if x = y we contradict our initial assumption because  $X \equiv Y''$  for Y'' = X. Consider now the case  $y \neq x$ . That means, we have that  $UD_F(X) \asymp UD_F(Y')$  and thus  $X \asymp Y'$ . Therefore, it follows that we must have  $UD_F(X) = \emptyset$ , i. e. X defends all its elements.

That means X is admissible and thus it follows directly that  $x \in cred_{ad}(F)$ .

From  $UD_F(X) = \emptyset$  and  $X \sqsupset_F^{UD} Y$  for all Y it follows that  $UD_F(Y) \neq \emptyset$ . Since  $\sqsupset$  satisfies *ad-generalisation* it follows that  $Y \notin ad(F)$  for all Y and thus also  $y \in rej_{ad}(F)$ . Consequently, it follows from *ad-C* that  $x \succ y$ . Hence, Dominating set is satisfied.  $\Box$ 

## 5. Investigating Principles for lex-cel<sub>τ</sub>

In the previous section, we looked at social ranking solutions from a general perspective and were able to characterise  $\sigma$ -C and present sufficient conditions for SC and  $\sigma$ -sk-C, however a number of principles are still to be investigated. In this section, we take a closer look at lex-cel<sub> $\tau$ </sub> and analyse which principles it satisfies.

As the results from the previous section suggest we should start with checking if lex-cel satisfies *Pareto-efficiency*.

Theorem 7. lex-cel satisfies Pareto-efficiency.

*Proof.* First, consider sets  $Z_1, \ldots, Z_n \in \mathcal{P}$  for which condition (2) of Pareto-efficiency holds. Among these, take those  $Z_1, \ldots, Z_m$  (with  $m \leq n$ ) for which  $\operatorname{rank}_{\exists}(Z_j \cup \{x\}) = k$ (with  $1 \leq j \leq m$ ) is minimal. At this level in the ranking, we have that  $\operatorname{rank}_{\exists}(Z \cup \{x\}) = \operatorname{rank}_{\exists}(Z \cup \{y\})$  for each  $Z \neq Z_j$ . Hence, for every  $Z \cup \{x\}$  there is exactly one corresponding set  $Z \cup \{y\}$ , except for each  $Z_j \cup \{x\}$  (because  $\operatorname{rank}_{\exists}(Z_j \cup \{y\}) > k$ ). Thus, for each  $Z \in \mathcal{P}$  with  $x, y \notin Z$ :

$$\begin{split} |\{Z \cup \{x\} \in \mathcal{P} \mid \mathrm{rank}_{\beth}(Z \cup \{x\}) = k\}| > \\ |\{Z \cup \{y\} \in \mathcal{P} \mid \mathrm{rank}_{\beth}(Z \cup \{y\}) = k\}|. \end{split}$$

At level k, there are more sets containing x than those containing y, i. e.  $x_{k, \square} > y_{k, \square}$  by Definition 10. To prove  $x \succ_{\square}^{\text{lex-cel}} y$  it remains to show that  $x_{i, \square} = y_{i, \square}$  for all i < k. By construction, for all i < k and  $Z \in \mathcal{P} \setminus \{x, y\}$ , we know that  $\operatorname{rank}_{\square}(Z \cup \{x\}) = \operatorname{rank}_{\square}(Z \cup \{y\})$ . Hence, for each set containing x there is exactly one set containing y. By Definition 10, we obtain  $x_{i, \square} = y_{i, \square}$ , as desired.

Bernardi et al. [9] have shown that lex-cel satisfies *Independent from worst set* for total orders and it is straightforward to see that this also holds for our setting. By Theorem 1 this means lex-cel also satisfies *Dominating set*, which implies that lex-cel<sub> $\tau$ </sub> satisfies  $\sigma$ -C and  $\sigma$ -sk-C if  $\tau$  satisfies  $\sigma$ -generalisation. Since both  $\sigma$ -C and  $\sigma$ -sk-C are satisfied the resulting argument ranking has a quite interesting pattern. The argument ranking can be split into three groups, first the skeptically accepted arguments wrt.  $\sigma$  then the credulously accepted wrt.  $\sigma$  and finally the rejected arguments wrt.  $\sigma$ . Inside all these groups the arguments can still be differentiated, so the resulting ranking is a generalisation of the acceptance problems for abstract argumentation.

The following result summarises the compliance of lexcel<sub> $\tau$ </sub> with the argument-ranking principles.

**Theorem 8.** *lex-cel*<sub> $\tau$ </sub> *satisfies the respective principles as stated in Table 1 for*  $\tau \in \{r\text{-}ad, r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}.$ 

We want to discuss the following counterexample showing that *VP* is violated by  $ex-cel_{\tau}$  in particular.

**Example 7.** We examine the following  $AF F = (\{a, b, c, d, \}, \{(a, b), (a, d), (b, c), (c, b)\})$ . Consider, for instance, the r-ad extension-ranking for F:

 $0: \{a,c\}, \{a\}, \{c\}, \emptyset \qquad 1: \{c,d\}$ 

The lex-cel<sub>r-ad</sub> argument-ranking is then:

$$c \succ_{E}^{lex-cel_{r-ad}} a \succ_{E}^{lex-cel_{r-ad}} d \succ_{E}^{lex-cel_{r-ad}} b$$

However, a is unattacked, while c is attacked and therefore VP is violated. Since all other sets that contain a are not conflict-free, that means that  $\{c, d\}$  is always ranked better than these sets. Therefore c is ranked better than a wrt. lex-cel<sub> $\tau$ </sub> for all other  $\tau \in \{r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}$ 

At first glance it might seem unintuitive that VP is violated. Both arguments a and c are skeptically accepted wrt. complete semantics, so there is no reason to reject either argument. However, argument a is involved in more conflicts than c and thus c is compatible with more arguments than a. Therefore we reason that c should be ranked better than a. In general, if we think back to the motivation of social ranking functions then employees who can work together

	Abs	ln	VP	SC	СР	QP	СТ	SCT	DP	DDP	NaE	AvsFD	w $\sigma$ -S	s $\sigma$ -S	$\sigma$ -C	$\sigma ext{-sk-C}$
$ ex-ce _{\tau}$	$\checkmark$	$\checkmark$	Х	$\checkmark$	Х	Х	Х	Х	Х	Х	х	$\checkmark$	Х	$\checkmark$	$\checkmark$	$\checkmark$
Cat	$\checkmark$	$\checkmark$	$\checkmark$	Х	Х	Х	$\checkmark$	$\checkmark$	$\checkmark$	Х	$\checkmark$	Х	Х	Х	Х	Х
ser	$\checkmark$	$\checkmark$	Х	Х	Х	Х	Х	Х	Х	Х	$\checkmark$	$\checkmark$	ad	ad	ad	Х

Table 1

Principles satisfied by lex-cel<sub> $\tau$ </sub> for  $\tau \in \{r\text{-}ad, r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}$  and other ranking semantics from the literature. Existing results for Cat and ser are taken from Bonzon et al. [4] and Blümel and Thimm [16].

with more employees are considered better, so this ranking of a and c is in line with the idea behind social ranking functions.

The remaining proofs and counterexamples can be found in the supplementary material<sup>2</sup>.

### 6. Related Work

A number of social ranking functions are discussed in the literature. In the following, let A be an arbitrary set of objects and  $\supseteq$  is a preorder on the powerset  $\mathcal{P}(A)$ .

A prominent social ranking function is the *Ceteris Paribus Majority Solution* (CP), which was defined by Haret et al. [8] as follows.

**Definition 23.** For the preorder  $\square$  and for any  $x, y \in A$ , we have that  $x \succeq^{CP} \supseteq y$  if and only if

$$|\{S \in \mathcal{P}(A \setminus \{x, y\})| S \cup \{x\} \sqsupset S \cup \{y\}\}| \ge |\{S \in \mathcal{P}(A \setminus \{x, y\})| S \cup \{y\} \sqsupset S \cup \{x\}\}|$$

Another relevant social ranking function is the *Ordinal* Banzhaf Index Solution (BI) of Khani et al. [7]. For that, we denote with  $U_i = \{S \in \mathcal{P} \mid i \notin S\}$  the set of subsets that do not contain *i* and with  $U_{ij} = \{S \in \mathcal{P} \mid i, j \notin S\}$  the set of subsets that contain neither *i* nor *j*.

First, we define the notion of *ordinal marginal contribution* as follows.

**Definition 24.** Let  $\supseteq$  be a preorder on  $\mathcal{P}(A)$ . The ordinal marginal contribution  $m_i^S(\supseteq)$  of element *i* wrt. the set *S* with  $i \notin S$ , for the preorder  $\supseteq$  is defined as:

$$m_i^S(\beth) = \begin{cases} 1 & if S \cup \{i\} \sqsupset S, \\ -1 & if S \sqsupset S \cup \{i\}, \\ 0 & otherwise. \end{cases}$$
(1)

We denote with  $u_i^{+, \supseteq} (u_i^{-, \supseteq})$  the set of subsets  $S \in U_i$ such that  $m_i^S(\supseteq) = 1$  ( $m_i^S(\supseteq) = -1$ ) respectively. Furthermore, we refer to the difference  $s_i^{\supseteq} = u_i^{+, \supseteq} - u_i^{-, \square}$  as the ordinal Banzhaf score of i wrt.  $\supseteq$ .

Finally, we define the social ranking solution based on the ordinal Banzhaf score as follows.

**Definition 25.** For the preorder  $\supseteq$  and for any  $x, y \in A$ , we define that  $x \succeq^{BI_{\supseteq}} y$  if and only if

$$s_i^{\Box} \ge s_i^{\Box}$$

However, the corresponding Social ranking argumentranking semantics  $BI_{\tau}$  and  $CP_{\tau}$  do not generalise credulous acceptance, because these two argument-ranking semantics do not satisfy the principle SC, as shown by the following examples.

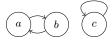
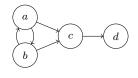


Figure 3: The AF  $F_2$  from Example 8.



Figure 4: The AF  $F_3$  from Example 9.



**Figure 5:** The AF  $F_4$  from Example 10.

**Example 8.** The argument ranking  $\succeq^{CP_{\tau}}$  violates SC for  $\tau \in \{r\text{-}ad, r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}$ . Consider the AF  $F_2$  in Figure 3. Then we have that  $c \succeq_{F_2}^{CP_{\tau}}$  a, which contradicts SC.

**Example 9.** The argument ranking  $\succeq^{Bl_{\tau}}$  violates SC for  $\tau \in \{r\text{-}ad, r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}$ . Consider the AF  $F_3$  in Figure 4. Then we have that  $a \succeq^{Bl_{\tau}}_{F_3}$  b, which contradicts SC.

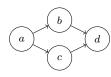
So, self-contradicting arguments are not necessarily the worst ranked arguments. Thus, these two social ranking functions are not suitable to rank arguments in the context of abstract argumentation and therefore we do not discuss them further.

A number of other argument-ranking semantics were introduced in the literature (for an overview see Bonzon et al. [4]). However, the only known argument-ranking semantics satisfying *ad*-Compatibility is the *serialisability-based argument-ranking semantics* (ser) by Blümel and Thimm [16]. The *serialisability-based argument ranking semantics* ranks arguments according to the number of conflicts that need to be resolved to include these arguments in an admissible set. However, this semantics violates *co*-sk-C.

**Example 10.** Let  $F_4$  be the AF as depicted in Figure 5. Then argument  $d \in sk_{co}(F_4)$ . So, according to co-sk-C it should hold that  $d \succ_{F_4} a$ , however this is not the case for ser, i. e.  $a \succ_{F_4}^{ser} d$ . Thus co-sk-C is violated.

Similarly, we have that the categorizer ranking semantics (Cat) violates  $\sigma$ -sk-C.

<sup>&</sup>lt;sup>2</sup>https://fernuni-hagen.sciebo.de/s/eTCZyHVIOzRtIsE



**Figure 6:** The AF  $F_5$  from Example 11.

**Example 11.** Let  $F_5$  be the AF as depicted in Figure 6. We have that  $sk_{co}(F_5) = \{a, d\}$ . However, we have for instance  $b \simeq_{F_5}^{Cat} d$ . Thus,  $\sigma$ -sk-C is violated by Cat for  $\sigma \in \{co, gr, pr, sst, st\}$ .

So lex-cel<sub> $\tau$ </sub> is the only known argument-ranking semantics that satisfies  $\sigma$ -C and  $\sigma$ -sk-C and thus satisfies  $\sigma$ -Refinement for extension-based semantics  $\sigma$ . Thus, lex-cel<sub> $\tau$ </sub> is part of none of the equivalence classes of argument-ranking semantics defined by Amgoud and Beuselinck [20].

### 7. Conclusion

In this paper we have combined well-known approaches from abstract argumentation and social ranking functions to define a new family of argument-ranking semantics. The resulting semantics are generalisations of the acceptance classifications for abstract argumentation. Thus, the skeptically accepted arguments are ranked before credulously accepted arguments and those are ranked before rejected arguments, and within each of these groupings the arguments are also ranked. While the extension ranking methods used are off the shelf approaches and already discussed in the literature, we needed to slightly generalise the existing social ranking functions in order for them to work with partial rankings. Here, our rank-based approach proved to be well suited for our specific setting. Whether this approach to social ranking also is appealing more generally is a very natural and intriguing question, that, unfortunately, is out of the scope of this paper and has to be left to future work.

The converse problem to social ranking functions are *lifting operators*, i. e. given a ranking over objects, we want to construct a ranking over sets of objects. These operators have been discussed for argumentation in the past by Yun et al. [21] and Maly and Wallner [22]. However, both theses papers do not present a complete picture of lifting operators for abstract argumentation, since they either consider only a subset of sets of arguments (Yun et al. [21]) or only discuss lifting operators for  $ASPIC^+$  (Maly and Wallner [22]). Skiba [23] discussed some shortcomings of lifting operators for argumentation frameworks and discussed the need to define lifting operators specifically tailored to abstract argumentation to fully discuss the relationship of argument-ranking semantics, extension-ranking semantics and lifting operators.

Acknowledgements. The research reported here was supported by the Deutsche Forschungsgemeinschaft under grants 375588274 and 506604007, by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 101034440) and by the Austrian Science Fund (FWF) under grant J4581.

### References

- C. Cayrol, M. Lagasquie-Schiex, Graduality in argumentation, J. Artif. Intell. Res. 23 (2005) 245–297.
- [2] L. Amgoud, J. Ben-Naim, Ranking-based semantics for argumentation frameworks, in: Scalable Uncertainty Management - 7th International Conference, SUM 2013, Springer, 2013, pp. 134–147.
- [3] L. Amgoud, J. Ben-Naim, D. Doder, S. Vesic, Ranking arguments with compensation-based semantics, in: Principles of Knowledge Representation and Reasoning: Proceedings of the Fifteenth International Conference, KR 2016, AAAI Press, 2016, pp. 12–21.
- [4] E. Bonzon, J. Delobelle, S. Konieczny, N. Maudet, A comparative study of ranking-based semantics for abstract argumentation, in: Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence 2016, AAAI Press, 2016, pp. 914–920.
- [5] J. Heyninck, B. Raddaoui, C. Straßer, Ranking-based argumentation semantics applied to logical argumentation, in: Proceedings of the Thirty-Second International Joint Conference on Artificial Intelligence, IJCAI 2023, ijcai.org, 2023, pp. 3268–3276.
- [6] S. Moretti, M. Öztürk, Some axiomatic and algorithmic perspectives on the social ranking problem, in: Algorithmic Decision Theory - 5th International Conference, ADT 2017, Springer, 2017, pp. 166–181.
- [7] H. Khani, S. Moretti, M. Öztürk, An ordinal banzhaf index for social ranking, in: Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, ijcai.org, 2019, pp. 378–384.
- [8] A. Haret, H. Khani, S. Moretti, M. Öztürk, Ceteris paribus majority for social ranking, in: Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018, ijcai.org, 2018, pp. 303–309.
- [9] G. Bernardi, R. Lucchetti, S. Moretti, Ranking objects from a preference relation over their subsets, Social Choice and Welfare 52 (2019) 589–606.
- [10] T. Suzuki, M. Horita, Consistent social ranking solutions, Social Choice and Welfare (2024).
- [11] K. Skiba, T. Rienstra, M. Thimm, J. Heyninck, G. Kern-Isberner, Ranking extensions in abstract argumentation, in: Proceedings of the Thirtieth International Joint Conference on Artificial Intelligence, IJCAI 2021, ijcai.org, 2021, pp. 2047–2053.
- [12] P. M. Dung, On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games, Artificial Intelligence (1995).
- [13] P. Baroni, M. Caminada, M. Giacomin, Abstract argumentation frameworks and their semantics, in: Handbook of Formal Argumentation, 2018, pp. 157–234.
- [14] M. W. A. Caminada, W. A. Carnielli, P. E. Dunne, Semistable semantics, J. Log. Comput. 22 (2012) 1207–1254.
- [15] L. van der Torre, S. Vesic, The principle-based approach to abstract argumentation semantics, FLAP 4 (2017).
- [16] L. Blümel, M. Thimm, A ranking semantics for abstract argumentation based on serialisability, in: Computational Models of Argument - Proceedings of COMMA 2022, IOS Press, 2022, pp. 104–115.
- [17] H. Moulin, Fair Division and Collective Welfare, MIT Press, 2004.
- [18] E. Algaba, S. Moretti, E. Rémila, P. Solal, Lexicographic

solutions for coalitional rankings, Social Choice and Welfare 57 (2021) 817–849.

- [19] P. E. Dunne, W. Dvorák, T. Linsbichler, S. Woltran, Characteristics of multiple viewpoints in abstract argumentation, Artif. Intell. 228 (2015) 153–178.
- [20] L. Amgoud, V. Beuselinck, An equivalence class of gradual semantics, in: Symbolic and Quantitative Approaches to Reasoning with Uncertainty - 17th European Conference, ECSQARU 2023, Springer, 2023, pp. 95–108.
- [21] B. Yun, S. Vesic, M. Croitoru, P. Bisquert, Viewpoints using ranking-based argumentation semantics, in: Computational Models of Argument - Proceedings of COMMA 2018, IOS Press, 2018, pp. 381–392.
- [22] J. Maly, J. P. Wallner, Ranking sets of defeasible elements in preferential approaches to structured argumentation: Postulates, relations, and characterizations, in: Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021, AAAI Press, 2021, pp. 6435–6443.
- [23] K. Skiba, Bridging the gap between ranking-based semantics and extension-ranking semantics, in: Proceedings of the 9th Workshop on Formal and Cognitive Reasoning co-located with the 46th German Conference on Artificial Intelligence (KI 2023), CEUR-WS.org, 2023, pp. 32–43.