Mathematical and computer modelling to assess accuracy and adequacy in torque measurement

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Abstract

In this paper, a mathematical and computer modelling approach is considered to assess the accuracy and adequacy of torque measurement by strain gauge dynamometer. Computer modelling allows predicting the performance of a strain gauge dynamometer and simulating it under various mechanical and physical data. Strain gauging is based on the phenomenon of the strain effect, which is a change in the active resistance of the conductor of the primary transducer (strain gauge) under the influence of mechanical stresses and strains. The use of strain gauges in scientific and technical research allows monitoring deformations and stresses under static and dynamic loads. The main task of this work is to develop and control the torque on the motor shaft, and to assess the adequacy of the mathematical model and the degree of accuracy of the results obtained using the developed model of the experimental data or test problem

Keywords

strain gauge, shaft torque, engine, mathematical model, computer modelling

1. Introduction

Tensometry, as a set of methods and tools for determining the stress-strain states of objects and structures, is widely used not only to measure the degree of deformation, but also to determine the weight in the control of belt conveyors, the weight of vehicles (cars, railroad cars), to substantiate the reliability and safety of nuclear power structures, etc [1].

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Research in the field of torque measurement using strain gauges has been actively developing for several decades. One of the pioneers in this field are scientists who studied the accuracy and reliability of strain gauge measurements.

Special attention should be paid to the works of American and European researchers, in particular, R. Williams and K. Martin, who studied the influence of temperature and vibration on the accuracy of measurements [2].

In addition, the German scientist Hans-Peter Schmidt is known for his work on increasing the sensitivity of tension dynamometers through the optimization of design solutions. In the context of the adequacy of mathematical models, a significant contribution was made by Japanese scientists such as Y. Takehara, who developed effective methods for correcting errors caused by non-stationary operating conditions. Their research laid the foundation for modern approaches to modeling and accuracy analysis of measuring systems using strain gauges [3].

The more we know about the object of study and the better and more accurate its mathematical model we can build, the more accurately we can predict its behavior and manage it more effectively. Mathematical modeling of the behavior of processes of various origins and their optimal control are of key importance for conducting systematic research in many areas of activity [4].

The main objective of this work is to develop a torque control unit that can be built into existing standard equipment during its modernization or repair.

Testing of various types of engines and power generation equipment to improve design, increase operational reliability, reduce thermal, mechanical, electrical and other losses, and increase efficiency requires the creation of more advanced measuring equipment to determine the torques transmitted by the rotating shaft.

The use of torque measurement equipment, which is caused by the need to determine the power of power equipment, as well as to conduct operational and resource tests of mobile power units (engines of tractors, agricultural machinery and other vehicles) to determine the load resistance forces in mechanisms with a rotating output shaft and similar tasks and problems [5].

2. Overview of torque measurement methods and tools

The applications for torque measurement devices are quite diverse, as are the requirements for them. Therefore, there are many torque measurement systems available.

The classification of devices for measuring torque is based on various features: purpose, design features, operating principle, operating conditions, accuracy of the information obtained, etc [6].

The most convenient for use is the classification of torque measurement by the principle of operation, according to which all instruments and devices for measuring torque can be divided into:

Mechanical: mechanical brakes, electric brakes, aerodynamic brakes with differential mechanism, inductor brakes, drive motors.

Hydraulic: chamber, volumetric, disk with oil film.

Optical: photoelectric, photoelastic, stroboscopic, optoelectric, optomechanical, holographic.

Electric, which are divided into:

1 with a sensor on a rotating shaft, capacitive, string, isotope;

2 with the sensor not on the rotating shaft - nonius, time phases;

3 strain gauges, which are divided into two types:

- with a sensor on the rotating shaft - non-contact, ohmic;
- with the sensor not on the rotating shaft - magnetoelastic, using eddy currents.

Most of them are used to measure torque in stationary and laboratory conditions.

Their advantages include simplicity of design and ease of use. However, they have a number of disadvantages, including the inability to use them in the presence of vibrations, shaft runout, large fluctuations in humidity and ambient temperature.

Based on the tests, it was found that only vibration-frequency, time, phase, ohmic, and non-contact torque measurement methods are suitable for long-term and stable use in the field. Vibration frequency sensors are manufactured with shafts made of special high-quality steels [7]. Due to the variety of shaft sizes of tractors and agricultural machines, these sensors are not widely used for torque measurement.

Time and phase methods of torque measurement are used on shafts with a large base (shaft length) or a large torsion angle.

The disadvantages of these methods include the dependence of the measurement error on the shaft speed and its runout. In this regard, time and phase methods of measuring torque can only be used exclusively under local operating conditions. The simplest and most reliable way to record torque is to directly measure the strain on the surface of the shaft under test using strain gauges. Such measurements of torque are called ohmic, and in most cases, wire sensors are used.

As a rule, a bridge scheme with two to four load cells is used to measure the torque value, which are glued at an angle of 45° to the axis.

The use of a bridge circuit increases sensitivity, improves linearity of the characteristics, reduces the sensitivity of the transducer to bending deformations, as well as to stresses arising from compression or tension of the shaft, and reduces the influence of temperature on the measurement process [8].

The advantages of strain gauges are their small size and simple mechanical design, while the disadvantages are the readings taken from a rotating shaft to a fixed measuring device and the fact that the bridge circuit does not fully compensate for the effects of bending deformations.

3. Search for analogs

The prototype dynamometer for torque measurement is GD1L3/04 "Strain gauge dynamometer for torque measurement". The invention relates to measuring technology and can be used to measure the torque on the shafts of various machines.

The purpose of the invention is to increase the measurement accuracy of the strain gauge and simplify its mounting on shafts.

The strain gauge shown in Figure 1 consists of a housing 1 with a drive part (sprocket, pulley), in which ball bearings 2 are installed, which are seated with inner rings on the outer part of the hub 3 and rest against the shoulder through a set of adjustable shims 4. The elastic element 5 is made in the form of a hollow cylinder with flanges, with strain gauges 6 glued to its outer surface, centered relative to the common axis of the tensiodynamometer by fitting a special bore machined into the compressive surface of the rear flange onto the protruding outer ring of the bearing, and connected by the front flange to the hub 3, and from the rear - to the drive part on the body 1 [9].

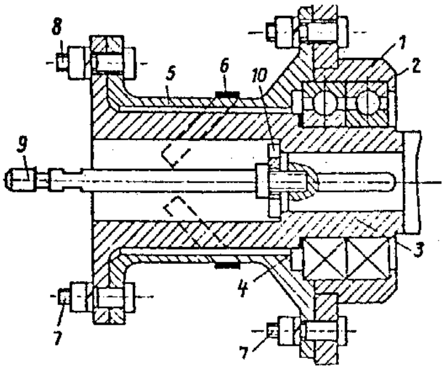

Figure 1: Prototype of the torque measuring unit

Placing the housing 1 with the drive part on the bearings 2 relieves the elastic element 5 from the radial forces perceived by the drive part and also makes it possible to transmit torque to the elastic element 5 from the hub 3 with little resistance. To ensure the rigidity of the connection of the elastic element 5 with the hub and the body 1 with the drive part, each flange connection has two reamer bolts 7, which are placed diametrically opposite, the remaining bolts 8 are of the usual design.

The connecting rod 9, in addition to its direct purpose - connecting the machine shaft with the rotor of the current collector and the output conductors of the strain gauge circuit laid on its surface, also serves for axial fixation of the strain gauge on the machine shaft with the help of a washer 10. The strain gauge works as follows. The torque from the machine drive is transmitted by means of a V-belt or chain to the drive part (pulley, sprocket) on the body 1. From it to the elastic element 5, and then to the hub 3, and then through the keyway of the shaft on which the strain gauge is mounted. The deformation of the elastic element is perceived by strain gauges 6, the electric current of the unbalance of the strain gauge circuit is transmitted through the conductors laid along the rod 9 to the measuring equipment through the current collector.

The use of an elastic element in the strain gauge in the form of a hollow cylinder with a wall thickness selected from the ratio [10]

$$\delta < 0.1 d_c \tag{1}$$

where d_c is the diameter of the annular cross-section of the elastic element along the center line, allows to create on its surface by varying the geometric dimensions of the annular cross-section of the elastic element the maximum level of tangential stresses equivalent to the measured torque and at the same time a significant value of the torsional stiffness of the elastic element, which provides a high natural frequency of the strain gauge to increase the measurement accuracy.

In addition, the design of the strain gauge with a hollow elastic element reduces the number of additional parts and connections between the drive part and the machine shaft to the lowest possible level, which greatly simplifies its mounting on the shaft and removal without any disassembly, reducing assembly and manufacturing costs and metal consumption [11].

4. Comparative analysis and selection of a solution to the task

Strain gauges are divided into two types according to their design features:

- with the placement of strain gauges on the moving elements of the drive;
- with the placement of strain gauges on non-moving drive elements;

The forces in the latter are transmitted to the strain gauges, which are fixed stationary, from the drive, which can be rotated under the influence of torque due to the installation on bearings, auxiliary transmissions (gearing on an elastic torsion shaft, a cable through a pulley on the axis of rotation of the drive, installation of the drive on a lever, etc.) [12].

The advantages of these designs are the absence of current collectors from moving elements, which reduces measurement error and simplifies the electrical part of the device. The disadvantages are the bulkiness and complexity of the mechanical design, and the presence of additional errors due to mechanical transmissions, which in turn creates impossible conditions for the use of the device in difficult field conditions.

Strain gauges with strain gauges placed on moving elements, when optimally designed, have minimal dimensions and a maximum simple structure that compensates for errors caused by the information signal taken from rotating elements.

To solve this problem, many designs of both contact and non-contact types have been created.

So, to solve this issue, we focus on the design of a device for measuring torque with strain gauges mounted on the moving elements of the drive based on the copyright certificate GD1L3/04 "Strain gauge dynamometer for measuring torque" [13].

5. Mathematical representation and calculation of model adequacy

5.1 Estimates of strain gauge resistance measurement errors

Selecting a modeling object and building its mathematical model.

A mathematical model is used to describe certain properties of a projected object that are significant at the stage of a particular design procedure [14]. It can reflect:

- a set and interconnection of the constituent elements of an object when solving problems of linking structural elements to certain spatial positions (for example, PCB tracing) or relative moments of time (for example, the sequence of technological operations).
- geometric properties of the object in terms of spatial forms and the relative position of its elements.
- quantitative and qualitative relationships between external conditions, process parameters, and parameters of the system in which the processes under study occur.

Atthe same time, itis necessaryto indicate the area of adequacy of the adopted description (the limits of the system parameters within which the adopted description reflects the properties of the designed object with an accuracy not less than the specified one).

When modeling the behavior of a technical object in conditions close to its operating conditions, it should be borne in mind that the technical object operates under the influence of three main factors [15]:

1. energy source that causes the desired processes to occur in the object;
2. external actions from the environment; among these actions, one can distinguish useful actions that also cause the necessary (useful) processes in the technical object, and harmful actions that cause deviations in the functioning of the technical object from the planned one;
3. a receiver (load), which is another real (material) object that perceives movement, energy, or an information signal from the technical object in question; the receiver can also influence the behavior of the technical object in both a positive and negative sense.

In view of the above, a model that simulates a technical object under conditions close to its operating conditions should include the relevant components [16].

5.2 Mathematical formulation and method of solving the problem

At the initial stage of formulating a mathematical model, we turn to the conceptual statement of the problem - a list of the main issues to be solved by means of mathematical modeling, as well as a set of hypotheses regarding the properties and behavior of the modeling object [17].

A conceptual model is built as an idealized model of an object.

According to the accepted hypotheses, a set of parameters describing the state of the object and a list of laws describing the behavior of the object and the relationship between the object's parameters and the environment are determined. A mathematical problem statement (mathematical description of an object) is a set of mathematical relations that describe the behavior of a modeling object. Let's turn to the analytical representation of the dependence of the measured resistance on the supply voltage and the reference resistances in the measuring bridge.

In this case, the working formulas are the ratios that define the effect of a change in resistance in the bridge arm on the unbalance voltage:

$$d_r = 4\cdot U\cdot r\ \ /U_0/\left(1-2\cdot U/U_0\right) \tag{2}$$

where U0 is the supply voltage, r0 is the constant resistance, U is the unbalance voltage and the effect of temperature on the change in resistances in the bridge arms:

$$R = r_0\cdot(1+k_1\cdot(t-T0)+k_2\cdot(t-T_0)^2)\,. \tag{3}$$

5.3 Implementation of the model in the form of a program

To model these dependencies, we used the MATLAB environment, in particular, work with symbolic variables [18].

```
clear all
r0=1000;
U0=100;
% effect of changes in resistance in the bridge arm on the unbalance voltage
%(inverse characteristic)
syms U
dr=4*U*r0/U0/(1-2*U/U0)
subplot(3,1,1)
ezplot(dr,[0,5]),grid
% effect of temperature change on the change in resistance in the bridge arm
%(inverse characteristic)
T0=20;
t=[T0:10:T0+10*T0];
k1=.001;
k2=.000005;
R=r0*(1+k1*(t-T0)+k2*(t-T0).^2);
subplot(3,1,2)
plot(R-r0,t,'o'),grid
n=length(t);
Q(1:n,1)=R(1:n)-r0;
Q(1:n,2)=t(1:n);
T=1.5e-006*dr^3-.0015*dr^2+.84*dr+22;
% dependence of the measured resistance on the unbalance voltage
% (calibration characteristic)
subplot(3,1,3)
ezplot(T,[0,5]),grid
% dependence of the measured resistance on the unbalance voltage and
resistance
% in the arms of the balanced bridge (full calibration characteristic)
syms U r0
dr=4*U*r0/U0/(1-2*U/U0);
T=1.5e-006*dr^3-.0015*dr^2+.84*dr+22;
figure
ezsurf(T,[0,5,100,1000])
box
% sensitivity of the measuring unit to changes in the measured voltage and
resistance in
% arms of the balanced bridge
Su=diff(T,U)
Sr=diff(T,r0)
figure
subplot(2,1,1)
ezsurf(Su,[0,5,100,1000])
box
subplot(2,1,2)
ezsurf(Sr,[0,5,100,1000])
box
% measurement error
dU=1;
dr0=5;
dT=abs(Su)*dU+abs(Sr)*dr0
figure
ezsurf(dT,[0,5,100,1000])
```

5.4 Modeling results

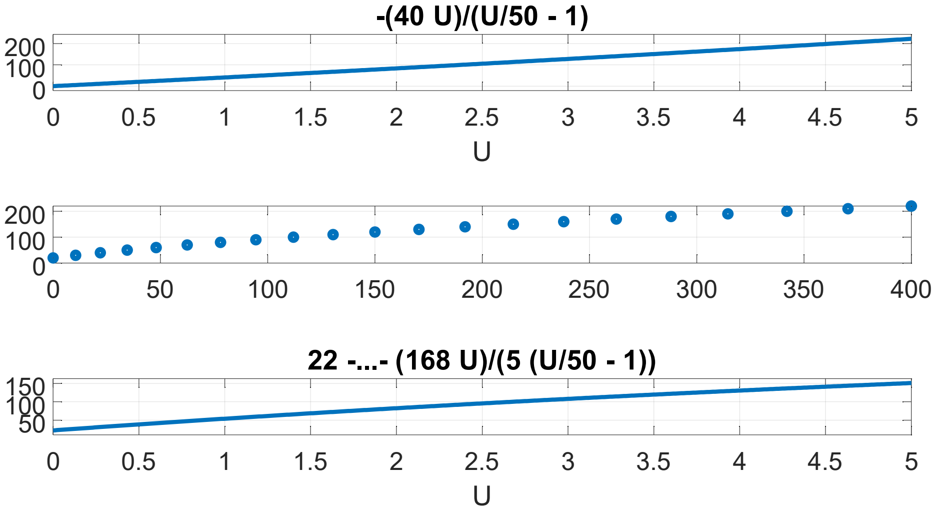

Figure 2: Inverse characteristic for assessing the effect of resistance change in the bridge arm on the unbalance voltage (top graph); Inverse characteristic for assessing the effect of temperature change on resistance change in the bridge arm (middle graph); Dependence of measured resistance on unbalance voltage (bottom graph)

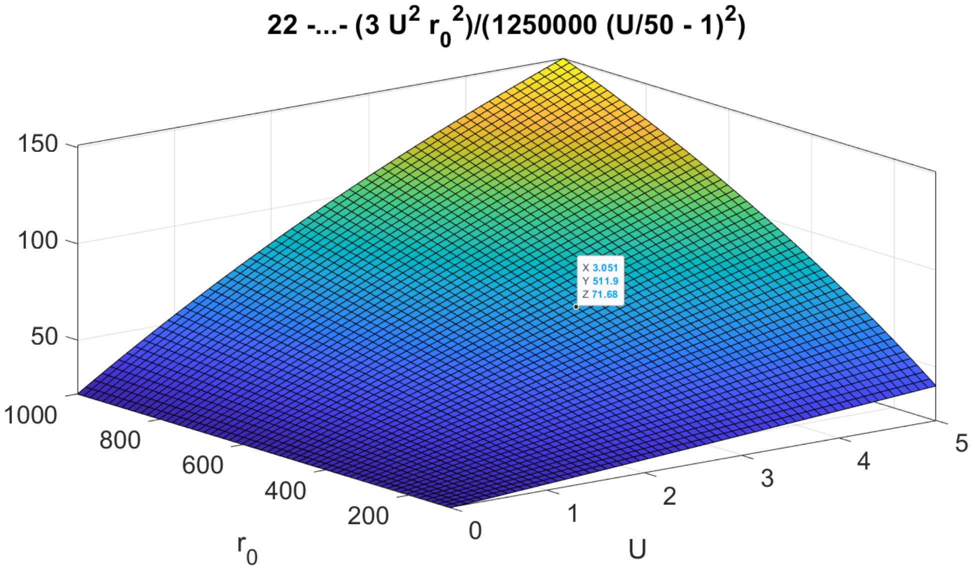

Figure 3: Dependence of the measured resistance on the unbalance voltage and resistance in the arms of the balanced bridge (full calibration characteristic)

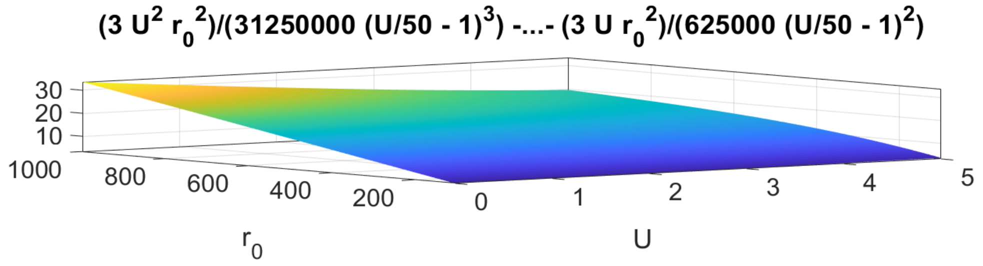

Figure 4: Sensitivity of the measuring unit to changes in the measured voltage and resistance in the arms of a balanced bridge

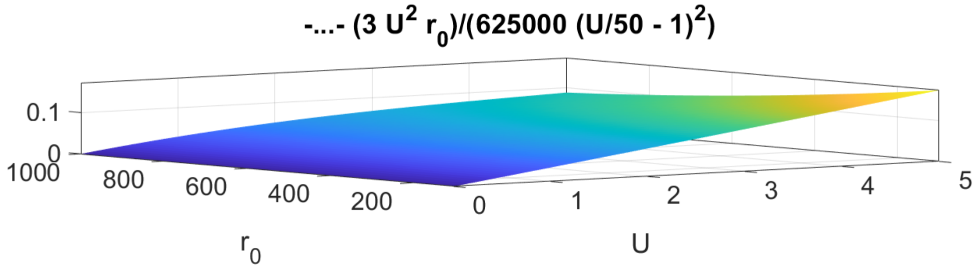

Figure 5: Measurement error as a function of unbalance voltage and balancing resistance in the bridge arm

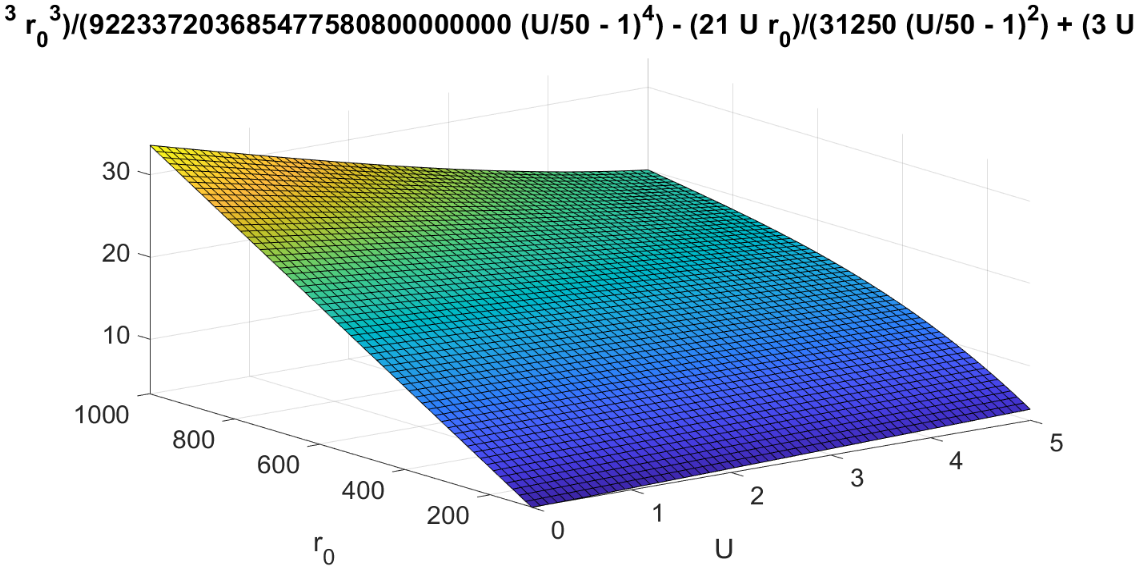

Figure 6: Sensitivity of the measuring unit to changes in the measured voltage and resistance in % of the balanced bridge arms

5.5 Checking the adequacy of the model

The adequacy of the model is understood as the degree to which the results obtained using the developed model correspond to the data of the experiment or test problem.

The purpose of the adequacy test is to:

- ensure that the accepted set of hypotheses is valid at the stage of the conceptual and mathematical model;
- establish that the accuracy of the results obtained corresponds to the accuracy specified in the terms of reference.

In models for performing estimation calculations, an error of 10...15% is considered satisfactory. In models used in control and monitoring systems, an error of 1...2% or less is acceptable.

The reasons for the inadequacy of the mathematical model may be as follows:

- the values of the specified model parameters do not correspond to the permissible range of these parameters;
- the accepted system of hypotheses is correct, but the constants and parameters in the defining relations are not set accurately enough;
- the system of hypotheses used is incorrect.

If the results are inadequate, the model should be adjusted, considering the reasons in the above sequence.

Errors in measuring the strain gauge resistance are a component of the torque measurement error. The obtained estimates allow us to track the influence of an external factor (such as temperature) and the accuracy of the system parameters (such as reference resistances in the arms of the measuring bridge) on the final result - the value of the measured torque.

Conclusion

Errors in measuring the strain gauge resistance are a component of the torque measurement error. Computer modelling allows to predict the performance of the strain gauge and simulate it under various mechanical and physical conditions. The obtained estimates allow us to track the influence of an external factor (such as temperature) and the accuracy of the system parameters (such as reference resistances in the arms of the measuring bridge) on the final result - the value of the measured torque.

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