Mathematical and numerical modeling of nonlinear deformation processes

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Abstract

To solve the problems of thermal elastic-plastic deformation of structural elements, a method of using the deformation theory of plasticity, generalized to the possibility of taking into account unloading with the development of plastic deformations during unloading, or repeated loading with the development of repeated plastic deformations, is proposed. Dependencies between the intensities of excess (differences between current values and the corresponding values recorded at the moment of unloading) stresses and excess deformations are built on the basis of Mazing's principle. An algorithm for solving problems based on the method of successive loads has been developed. The method of additional deformations was used to linearize the indicated stress-strain dependences..

Keywords

deformation theory, thermal plasticity, unloading, mathematical modeling

1.Introduction

In many technological processes, structural elements are subjected to significant force and temperature loads, as a result of which irreversible plastic deformations occur in some areas of the structure. After complete removal of the load, residual stresses and deformations occur in such structures, which can have a significant impact on the operational properties of such structures. Therefore, the problem of quantitative assessment of residual stress fields and deformations that occur in some heat treatment processes, during welding, restoration of operational properties by surfacing, is very relevant. Currently, approximate methods based on the use of unloading theorems, computational and experimental methods [1, 2, 3], as well as methods based on the theory of plastic flow [4, 5] are used to solve similar problems. The latest mathematical models are quite complex and do not always satisfy the required accuracy when tracking the load surface in the process of plastic deformation. Therefore, the paper proposes a mathematical model based on the deformation theory of plasticity, which is generalized for the case of taking into account unloading.

ITTAP'2024: 4th International Workshop on Information Technologies: Theoretical and Applied Problems, October 23–25, 2024, Ternopil, Ukraine, Opole, Poland

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To solve such problems, you can use commercial packages of computer programs such as ANSYS, SYSWELD, etc. Renting commercial packages for the appropriate term is quite expensive, and therefore, in practice, the creation of original problem-oriented mathematical support for solving problems in the field of welding and related technologies is widely used in practice. This way allows synthesizing working programs from ready-made modeling blocks and information bases, and it is much cheaper than renting a commercial package.

2.Methodology

To simulate the processes of elastic-plastic deformation we have suggested to use the theory of small thermal elastic-plastic deformations generalized for the case of unloading taken into account [6].

Stress-strain relations of small elastic-plastic deformations can be written as [6]

$$\tilde{s}_{ij} = \frac{2G(T)}{\tilde{\psi}} \tilde{e}_{ij}, \tag{1}$$

$$\tilde{\psi} = 3G(T) \frac{\tilde{\varepsilon}_i}{\tilde{\sigma}_i}, \tag{2}$$

where

$$\tilde{s}_{ij} = \frac{G(T)}{G(T_1)} s_{ij}^{(1)} - s_{ij}, \qquad \tilde{e}_{ij} = e_{ij}^{(1)} - e_{ij}, \tag{3}$$

here $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0, \quad e_{ij} = \varepsilon_{ij} - \delta_{ij}\varepsilon_0$ are the components of stresses and deformations deviators. Average stress and deformation are as follows $\sigma_0 = \frac{1}{3}\sigma_{ii}, \quad \varepsilon_0 = \frac{1}{3}\varepsilon_{ii}$ connected by the dependence $\varepsilon_0 = \frac{\sigma_0}{K} + \varepsilon^T$, where $K = \frac{2G(1+v)}{(1-2v)}$ – volume compression modulus , $G = \frac{E}{2(1+v)}$ – shear modulus, $\varepsilon^T = \alpha_t T^{¿} = \alpha_t (T - T_0)$ – average temperature deformation, α_t- coefficient of thermal linear expansion of the material, $s_{ij}^{(1)}, \quad e_{ij}^{(1)}$ – components of stresses and deformations deviators which were reached in the specified point of the environment at the moment of unloading start. The last values are equal to zero, if any unloading wasn’t observed in the specified point. T_1 is the value of temperature which was fixed in the specified point at the moment of unloading start. Values $\tilde{\sigma}_i$ і $\tilde{\varepsilon}_i$ are the stresses intensities $\tilde{s}_{ij}$ and $\tilde{e}_{ij}$, which are calculated by formulae

$$\tilde{\sigma}_i = \sqrt{\frac{3}{2} \tilde{s}_{ij} \tilde{s}_{ij}}, \tag{4}$$

$$\tilde{\varepsilon}_i = \sqrt{\frac{2}{3}\tilde{e}_{ij}\tilde{e}_{ij}} \quad (5)$$

Apparently, if any unloading isn't observed in the specified point yet, the values $\tilde{\sigma}_i$ and $\tilde{\varepsilon}_i$ are transformed into ordinary intensities of the stresses σ_i and ε_i.

In the formula (1) $\tilde{\psi}$ is the parameter of plasticity determined by formula (2). Moreover, they consider that there is a unique dependence between the intensities $\tilde{\varepsilon}_i$ and $\tilde{\sigma}_i$, which is not influenced by the kind of stressed state and can be found on the basis of experimental data for the simplest homogeneous stressed states.

At the stage of initial deformation from stress-free and deformation-free state in the points where some active loading is taking place the intensity of total deformation is equal to the sum of intensities of elastic and plastic deformations components $\varepsilon_i = \varepsilon_i^p + \varepsilon_i^e$.

Stress-strain relations (1) in this stage look like

$$s_{ij} = \frac{2G(T)}{\psi} e_{ij}, \quad (6)$$

$$\psi = 3G(T)\frac{\varepsilon_i}{\sigma_i}. \quad (7)$$

The relationship between the intensities of stresses and deformations in this stage for the most structural materials can be written as

$$\sigma_i = \Phi'(\varepsilon_i, T) = \begin{cases} \sigma_s(T)\cdot\dfrac{\varepsilon_i}{\varepsilon_{is}}, & \varepsilon_i \le \varepsilon_{is} = \dfrac{\sigma_s(T)}{3G(T)}, \\ \sigma_s(T)\cdot\left(\dfrac{\varepsilon_i}{\varepsilon_{is}}\right)^{\gamma}, & \varepsilon_i > \varepsilon_{is}, \end{cases} \quad (8)$$

where $\sigma_s(T)$ – material plasticity limit which depends on the temperature. The identical relationship $\tilde{\sigma}_i = \Phi(\tilde{\varepsilon}_i, T)$ can be obtained on the basis of Mazing principle [7], if it is generalized on isothermal processes of deformation. After such generalization we have found [8]

$$\tilde{\sigma}_i = \begin{cases} 2\sigma_s(T)\dfrac{\tilde{\varepsilon}_i}{\tilde{\varepsilon}_{is}}, & \tilde{\varepsilon}_i \le \tilde{\varepsilon}_{is} = 2\varepsilon_{is}, \\ 2\sigma_s(T)\left(\dfrac{\tilde{\varepsilon}_i}{\tilde{\varepsilon}_{is}}\right)^{\gamma}, & \tilde{\varepsilon}_i > \tilde{\varepsilon}_{is}. \end{cases} \tag{9}$$

Stress-strain relations (2) can be presented as the solved ones relative to the deformation tensor component

$$\varepsilon_{ij}=\frac{-\tilde{\psi}}{2G}\left[\frac{G}{G_1}\sigma_{ij}^{(1)}-\sigma_{ij}-\frac{(1+\nu)\tilde{\psi}-(1-2\nu)}{(1+\nu)\tilde{\psi}}\delta_{ij}\left(\frac{G}{G_m}\sigma_0^{(1)}-\sigma_0\right)\right]-\delta_{ij}\left(\varepsilon^{T(1)}-\varepsilon^{T}\right)+\varepsilon_{ij}^{(1)}, \tag{10}$$

or relative to the stress tensor component

$$\sigma_{ij}=\frac{G}{G_1}\sigma_{ij}^{(1)}-\frac{2G}{\tilde{\psi}}\left[\varepsilon_{ij}^{(1)}-\varepsilon_{ij}+\frac{\tilde{\psi}(1+\nu)}{1-2\nu}\delta_{ij}\left[\left(\varepsilon_0^{(1)}-\varepsilon_0\right)-\left(\varepsilon^{T(1)}-\varepsilon^{T}\right)\right]-\delta_{ij}\left(\varepsilon_0^{(1)}-\varepsilon_0\right)\right]. \tag{11}$$

Here the plastic deformation can be determined by formulae

$$\varepsilon_{ij}^{p}=\varepsilon_{ij}^{p(1)}-\frac{(\tilde{\psi}-1)}{\tilde{\psi}}\left[\varepsilon_{ij}^{(1)}-\varepsilon_{ij}+\delta_{ij}\left(\varepsilon_0^{(1)}-\varepsilon_0\right)\right]. \tag{12}$$

Now we use the same symbols for ordinary stress and deformation tensors component which have been introduced earlier for deviator components

$$\tilde{\sigma}_{ij}=\frac{G}{G_1}\sigma_{ij}^{(1)}-\sigma_{ij},\ \tilde{\varepsilon}_{ij}=\varepsilon_{ij}^{(1)}-\varepsilon_{ij},$$
$$\tilde{\sigma}_0=\frac{G}{G_1}\sigma_0^{(1)}-\sigma_0,\ \tilde{\varepsilon}_0=\varepsilon_0^{(1)}-\varepsilon_0,\ \tilde{\varepsilon}^{T}=\varepsilon^{T(1)}-\varepsilon^{T}. \tag{13}$$

Then the dependence (10) is written as

$$\tilde{\varepsilon}_{ij}=\frac{\tilde{\psi}}{2G}\left(\tilde{\sigma}_{ij}-\frac{(1+\nu)\tilde{\psi}-(1-2\nu)}{(1+\nu)\tilde{\psi}}\delta_{ij}\tilde{\sigma}_0\right)+\delta_{ij}\tilde{\varepsilon}^{T}. \tag{14}$$

Having introduced the symbols $\tilde{e}_{ij}^{p}=\varepsilon_{ij}^{p(1)}-\varepsilon_{ij}^{p}$, the formula (10) will look like

$$\tilde{e}_{ij}^{p}=\frac{\tilde{\psi}-1}{\tilde{\psi}}\tilde{e}_{ij}. \tag{15}$$

We can also show that

$$\tilde{e}_{ij}^{e} = \frac{1}{2G}\tilde{s}_{ij}, \qquad (16)$$

where $\tilde{e}_{ij}^{e} = e_{ij}^{e(1)} - e_{ij}^{e}$.

To linearize the specified stress-strain relations the method of additional deformation (MAD) is used. We will demonstrate this method for the case when the unloading is taking place with the development of plastic deformation.

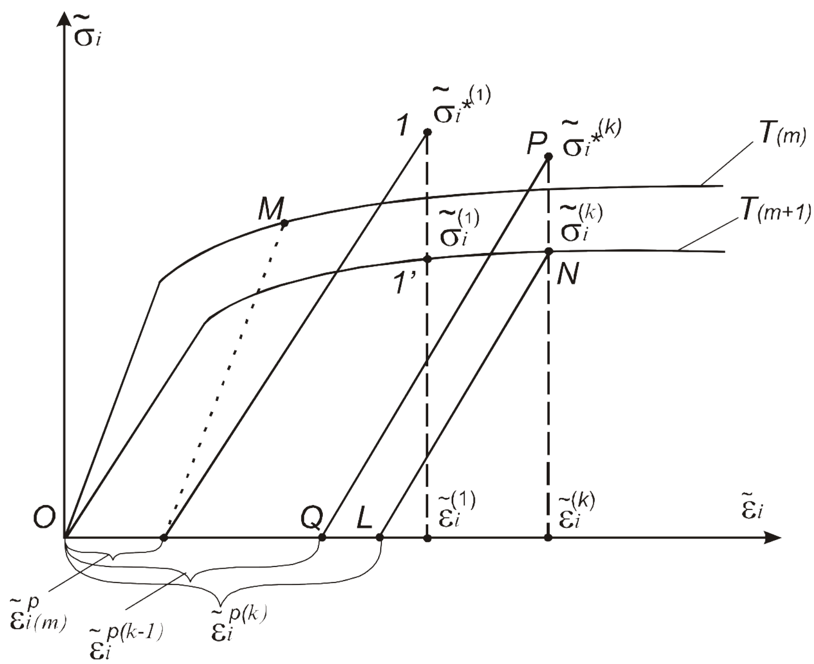

Figure 1: Picture of using the additional deformation method on the unloading stage with plastic deformation development

We assume that in the beginning of some k step we have $\tilde{\varepsilon}_{ij}^{p(k-1)}$ and point Q corresponds to these deformations (fig.1). We must admit that for visual clarity we have superposed the beginnings of axes of references $\tilde{\varepsilon}_i, \tilde{\sigma}_i$ for two different temperatures.

The process of successive approximations by the method of additional deformation is carried out by formulae

$$\tilde{e}_{ij}^{(k)} = \tilde{e}_{ij*¿^{e(k)} + \tilde{e}_{ij}^{p(k-1)} = \frac{1}{2G}\tilde{s}_{ij}^{¿(k)} + \tilde{\varepsilon}_{ij}^{p(k-1)}¿} \qquad (17)$$

Having solved the problem under stress-strain relations conditions (17), we will find the solution $\tilde{e}_{ij}^{(k)}$, $\tilde{\sigma}_{ij}^{¿(k)}$ which point P corresponds to on fig.1. By the known values of component $\tilde{e}_{ij}^{(k)}$ we have calculated the intensity of total deformations $\tilde{\varepsilon}_i^{(k)}$. Using the surface equation $\tilde{\sigma}_i = \tilde{\Phi}(\tilde{\varepsilon}_i, T)$ for the specified temperature value T for the specified stage and value $\tilde{\varepsilon}_i^{(k)}$ we have found the intensity of stresses $\tilde{\sigma}_i^{(k)}$ (point N on the figure). It has enabled us to find by formula $\tilde{\psi}^{(k)} = 3G\frac{\tilde{\varepsilon}_i^{(k)}}{\tilde{\sigma}_i^{(k)}}$ the value of plasticity parameter $\tilde{\psi}^{(k)}$ for the specified approximation, and by the formulae $\tilde{e}_{ij}^{p(k)} = \frac{\tilde{\psi}^{(k)} - 1}{\tilde{\psi}^{(k)}}\tilde{e}_{ij}^{(k)}$ we have calculated the components of plastic residual deformation of this k approximation which can be used in formulae of the method of additional deformations (17) in the next approximation.

The formulae of the method of additional deformations in this case can be written as

$$\tilde{\varepsilon}_{ij}^{(k)} = \frac{1}{2G}\left(\tilde{\sigma}_{ij}^{¿(k)} - \frac{3\nu}{1+\nu}\delta_{ij}\tilde{\sigma}_0^{¿(k)}\right) + \delta_{ij}\tilde{\varepsilon}^T + \tilde{\varepsilon}_{ij}^{p(k-1)}, \tag{18}$$

$$\tilde{\varepsilon}_{ij}^{p(k)} = \frac{\tilde{\psi}^{(k)} - 1}{\tilde{\psi}^{(k)}}\left(\tilde{\varepsilon}_{ij}^{(k)} - \delta_{ij}\tilde{\varepsilon}_0^{(k)}\right), \tag{19}$$

$$\tilde{\psi}^{(k)} = 3G\frac{\tilde{\varepsilon}_i^{(k)}}{\tilde{\sigma}_i^{(k)}}. \tag{20}$$

The whole process of loading (heating, cooling) is divided into separate stages. Specifying the values of the deformation plasticity component for zero approximation in (18) equal to these components which were reached for the previous stage of loading (at deformation from initial undeformed state they are accepted as zero) the elastic problem with additional deformations is solved. according to the found total deformations in $k-$approximation the intensities $\tilde{\varepsilon}_i^{(k)}$ and $\tilde{\sigma}_i^{(k)}$ are calculated. Then according to the formula (20) for each point of the structure $k-$ approximation of the plasticity parameter $\tilde{\psi}^{(k)}$ is calculated and by formula (20) the components of the deformation plasticity which will be further used in formulae (18) in the next approximation to find the component $\tilde{e}_{ij}^{(k+1)}$. Iteration process lasts till its complete coincidence, after that the transition to the next stage of loading is taking place.

We must admit that initially on every iteration in each point of the structure the above-mentioned deformation has been assumed that occurred in it during the previous stage of loading, i.e. initial elastic or plastic deformation, elastic unloading or unloading with the development of further plastic deformation. After coincidence of the iteration process the examination is conducted in every point of the structure to find out if such deformation was taking place in fact. If in some points the deformation behavior does not correspond the accepted one on the basis of information from the previous stage of loading, then the stage is fully recalculated with the previous replacement of the deformation behavior to the opposite one in such points.

3.The results

Due to the above-mentioned technique a number of practical problems have been solved, namely the welding of thin-walled structural parts, building-up welding aimed at strengthening or restoring the operational characteristics.

In this way, for instance, the problem of welding procedure simulation of two cylindrical shells by circular joint providing the welding is taking place along the whole length of the welding seam simultaneously [9]. The obtained results have completely correlated with the similar results found in the paper [5] using more complicated theory of plastic flow. The results of modelling have made possible to find the fields of residual stresses, deformations and displacements, study the kinetics of stress-and-strain state of the welding process, study the diagram of deformation in different points of the structure. As an example, we will show the distribution of residual welding stresses, elastic deformations, and also residual deflection of a shell.

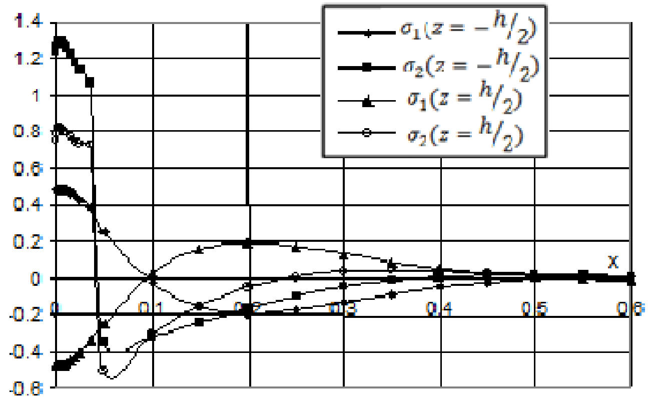

Figure 2: Distribution of residual welding stresses in cylindrical shell.

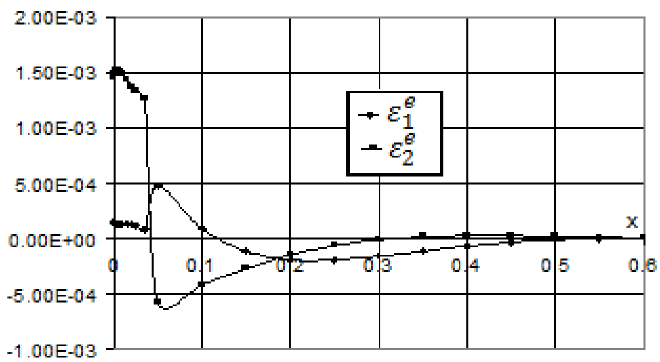

Figure 3: Distribution of elastic residual deformations.

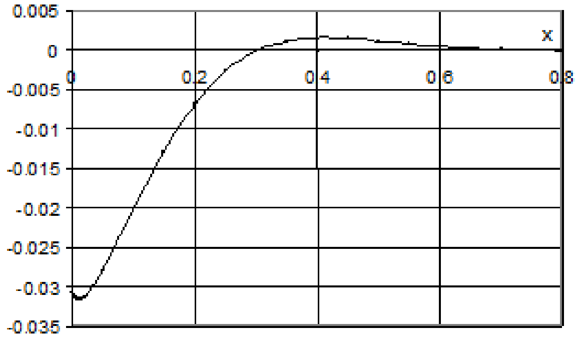

Figure 4: Residual deflection.

In the figures, $x=\frac{X}{L}-¿$ is a dimensionless coordinate along the length of the shell, the stresses are related to the yield point of the material at some initial temperature.

4.Conclusions

It is shown that to solve the complex problems of thermal elastic-plastic deformation of structural elements, it is possible to use the theory of the deformation theory of thermal plasticity deformation, which is much simpler than the flow theory, generalized to consider the possibility of unloading with the development of plastic deformations, or repeated loading with the development of repeated plastic deformations.

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