The impact of market preferences on the evolution of market price and product quality

Hongliang Liu, Enda Howley and Jim Duggan

***Abstract*—A significant challenge for firms in an open-competition marketplace is to balance the conflicting attributes of price and quality. Higher quality levels tend to lead to increased product costs, which, depending on market preferences, can trigger an increase in consumer demand. This paper presents a multi-agent model that allows for an exploration of how price and quality evolve as a result of direct market competition between firms. A new competition model, based on price and quality, is defined. Agents compete by determining their price and quality levels with a view to maximizing their profit. Our goal is to examine a range of market configurations and study how agent strategies evolve over time. We focus on those factors which contribute to each agent's survival in this evolutionary setting. We use game theoretic simulation as a basis to examine various agent strategies. A genetic algorithm is used to characterize a changing environment which evolves over time to reflect the emergence of fitter strategy attributes. Individuals can evolve their own market preferences over subsequent generations and adapt to their preferred market strategy. Agent strategies evolve rapidly to reflect the bias of their individual market. The price and quality relationship of a given market is a primary driver of the evolution of agent strategies in that market. Significantly, our results show the emergence of strategies that prefer low price and high quality sensitive markets. This is despite the penalties which are incurred by the higher costs of increased quality. These results have potentially interesting applications to real-world market dynamics, particularly as companies strive to position their products optimally on different markets.**

***Index Terms*—Price and Quality Competition, Agent Computational Economics, Agent-based Simulation, Genetic Algorithm**

I. I

Consumers from different markets exhibit wide preference differences due to natural variation in tastes and income disparities. These are mainly reflected by consumers' accepted price and quality levels. For example, consumers in rural areas may prefer lower price products while consumers in urban areas maybe willing to pay higher prices for higher quality products. These consumer preferences can indirectly establish trends in production. From the view of the firms, higher quality usually requires the use of more expensive components, and less standardized production process, and so on. As a result, higher quality levels tend to lead to increased product costs. Nevertheless, higher quality, depending on market preferences, can trigger an increase in consumer demand, and probably gain market share [1]. Therefore, there are trade-offs between quality and cost for firms. In terms of price, it is also a decision challenge. Firms can charge a higher price for their product in order to get a higher unit profit. However, higher price levels usually lead to a reduction in customer demand. Therefore, ensuring a good balance between the conflicting attributes of price and quality is a significant challenge for firms in an open-competition marketplace.

In order to better understand this problem, a number of game theoretic models have been proposed [2] [3] [4]. This existing research has focused on the strategic or rational behavior of competition between two firms. However, what will happen when there are more than two firms and their decisions are affected by the effect of bounded rationality? Another common feature of the current research is that researchers limit their analysis on one market in these models. However, in the real world, firms usually compete with each other in different marketplaces. In order to address this issue, we propose a new multi-agent competition model, based on price and quality. In this model, we consider many firm agents competing in a number of markets. Markets are defined by their own unique properties. Price and quality sensitivities are used to represent these properties, and reflect a consumers' preferred product. Variations in these values effect the demand of the products in the market. Different markets may have different price and quality sensitivities. Each market demand is determined by the average price and quality levels of firm agents in that market. Thus, each firm agent faces decision challenges including their product price and quality levels, and their preferred markets. Furthermore, the effectiveness of one agent's strategy depends on the strategies of others. In this paper, we model firm agents as individuals in a genetic algorithm which has been widely used as learning mechanism for economic agents [5] [6]. The genetic algorithm is also used to characterize a competitive market environment where the agents compete with each other for the market share. The firm agents can make price and quality decisions and evolve their own market preferences. However, they have a limited knowledge of their environment and their performance is largely determined by the actions of their peers. These features of our model are significantly different with models in the existing research.

In this paper, we aim to examine a range of market configurations and study how firm agent strategies evolve over time. We investigate how firm agents strategically position their products over time and what are the impacts of alternative market preferences on the evolution. We have conducted a series of experiments on a range of market configurations. Our results show the impacts of market preferences on the evolution of market price and product quality. The firm agent strategies evolve rapidly to reflect the bias of their individual

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market. The price and quality relationship of a given market is a primary driver of the evolution of agent strategies in that market. Significantly, our results show the emergence of strategies that prefer markets which have low price and also high quality sensitive markets. This is despite the penalties which are incurred by the higher costs of increased quality. These results have potentially interesting applications to real-world market dynamics, particularly as companies strive to position their products optimally on many markets.

The sections of this paper are structured as follows. In Section II, we will review much of the related work relevant to price and quality competition. In Section III, we will outline our model design. Section IV will provide a detailed examination of our experimental results. Finally, in Section V we will outline our conclusions and some future work.

II. B R

The study of price and quality competition has attracted many researchers' attention. There are two main streams in the current research. One is a formal study of rational behaviors among strategically interacting agents using game theory. While the alternative approach is to use agent-based modeling and simulation to examine market economies. This is also known as agent-based computational economics (ABCE) which is the computational study of economics modeled as evolving systems of autonomous interacting agents [6].

A. Game Theory Models

Since the seminal work of Hotelling [2], a rich and diverse literature on price and quality competition has emerged. Harold Hoteling analyzes a model of spatial competition which demonstrates the relationship between location and pricing behavior of firms. In this model, Hotelling assumes that potential consumers are evenly distributed in a linear geographic location such as a straight street. Consumers have no preferences to the firms and only buy products from these that provides better value in terms of price and transportation cost. Both firms have the same constant marginal costs and compete on the store location and price. From this spatial competition model, Hotelling argues that the equilibrium strategy for each firm is to choose a location at the center of the market which is commonly referred to as "Principe of Minimum Differentiation" or "Hotelling's law". This argument means that for any location of one firm, the other firm has an incentive to move toward its opponent in order to expand the the territory under its exclusive control. In this model, a customer's location can also be interpreted as a customer's preference for quality, therefore, many papers on price and quality competition are inspired by this work. For example, Moorthy considers the quality choice in a duopoly, assuming the existence of a quadratic cost function for quality which is different with the Hotelling's location model [7]. Banker et al. examine a price and quality competition also under a duopoly setting, where consumers' demand is a linear function of price and quality levels and the cost of quality is also a quadratic form [3]. Moorthy and Banker et al. analyze the impact of quality on competitive advantages. V*örö*s designs a price and quality model using decreasing and increasing exponential demand functions for price and quality, respectively, and analyzes the influences of the quality inflating which means that the same quality performance is worth less tomorrow than today [8]. Recently, Matsubayashi et al. explore the impact of different customers' loyalty to each firm on the outcome of price and quality competition [4].

B. Agent based simulations

Agent based simulations have been successfully applied many problems such as telecommunications and market strategies [9]. In many economic applications, genetic algorithms (GAs) have been widely used to represent the learning processes of agents [10] [11]. GAs were developed by Holland in 1975 as a way of studying adaptation, optimization and learning [12]. GAs are inspired by evolutionary biology such as selection, crossover (also called recombination) and mutation. A basic GA manipulates a population of chromosomes that encode candidate solutions to a problem. Each chromosome or individual in a GA is assigned a measure of performance, called its fitness. In a game context, a chromosome can be interpreted as a strategy, and the GAs processes are models of learning. In GAs, the reproduction operator can be interpreted as learning by imitation, the crossover operator can be interpreted as learning through communication, and the mutation operator is interpreted as learning by experiment [13].

GAs have been used to examine some well known game theory models such as Prisoner's Dilemma [10], Cournot competition and Bertrand competition[14]. However, almost all the existing research has employed classical game theory to examine the price and quality competition as we have examined earlier. Only recently, Tay et al. have used a genetic algorithm to test Hunt's General Theory of competition [15]. They consider an oligopolistic market with a number of sellers who are competing on price and a product attribute which reflects a consumer's ideal preferences. The sellers' demand function is a linear function of price and and the product attribute which differs from our demand function for markets. Furthermore, we are interested in different research topics. They aim to use a GA as an alternative simulation method to test a competition theory. Our purpose of this paper is to investigate the impact of market preferences on the evolution of agents' strategies.

In summary, there is a body of literature in economics on price and quality competition. However, these models rely on very strong assumptions such as rational behaviour of two firms and one market. The research from ABCE has not been addressed this perspective on price and quantity competition. In this paper, we propose a multi-agent model and aim to address these issues.

III. P Q C M

In this section, we propose our game theoretic model. We consider many firm agents competing with each other over a number of competitive markets. Different markets may have different preferences over price and quality which are reflected through market demands in the markets. Firms in the same

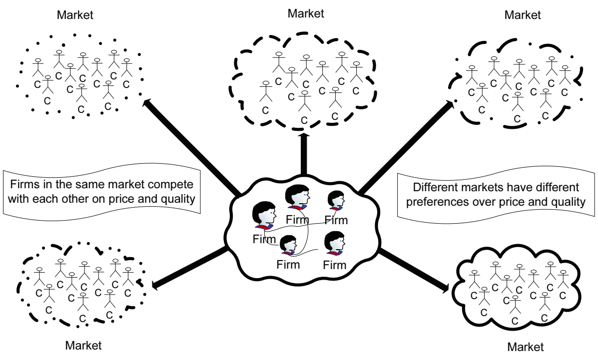


Fig. 1. Price and Quality Competition Model

marketplace compete with each other on price and quality for higher profits. As for firms, a relative lower price level or a higher quality level may attract more consumers. This depends not only on other firm agents' strategies but also on the preferences of the markets. Furthermore, the lower price or higher quality strategies also reduce unit profit level as higher quality levels incur higher unit cost levels. Therefore, in our model, each firm agent faces decision challenges including price levels, quality levels and their preferred markets as shown in Figure 1. In the following, we first present our market properties, then the firm agents and their decision-making process. Finally, our simulator design is outlined.

A. Market Properties

We consider m markets. Each market demand is dependent on the average price and quality levels (p, q) of all firm agents in the market. The market demand will increase as the price level goes down given any quality level, and on the contrary, it increases as the quality of the product improves for any price level. In order to reflect these relationships in real markets, we use Equation (1) to model market k's demand $D_k(p, q)$.

$$D_k(p, q) = \mathcal{A}_k e^{-\alpha_k p}(1 - be^{-\beta_k q}) \tag{1}$$

where $\mathcal{A}_k$ is the potential maximum demand, $b \in (0, 1]$, $\alpha_k \in [0, 1]$, and $\beta_k \in [0, 1]$ are parameters. Note that the demand function is monotonically decreasing over price p and increasing over quality q since $\partial D(p, q)/\partial p < 0$ and $\partial D(p, q)/\partial q > 0$. The combination of the parameters (α,β) corresponds to a set of consumers' price and quality sensitivities for a given market.

- (α) This represents the consumers' price sensitivity as the higher α the demand goes down faster given the same price change. The higher α means higher consumers' price sensitivity.
- (β) This represents the consumers' quality sensitivity as the higher β the demand changes faster given the same quality change. Similarly, the higher β value reflects higher consumers' quality sensitivity.

B. Firm Agents

In these m markets there are f firm agents in total. Each firm agent faces decision challenges including their product price and quality levels, and their preferred markets. Let $\eta_i = (p_{i,t}, q_{i,t}, k_{i,t})$ denote the agent i's decision strategies at time step t where $p_{i,t}, q_{i,t}, k_{i,t}$ are the price level, the quality level and the market $k_{i,t}$ ($k_{i,t} \in [1, m]$). The firm agents from the same marketplace k compete with each other for a higher market share and profit over time.

The firm agent i's market share ($s_{i,k,t}$) in market k at time step t depends not only on its own price and quality levels but also on the other agents' strategies. We propose a new mechanism as follows.

$$s_{i,t} = \frac{D_k(p_{i,t}, q_{i,t})}{\sum_{j=1}^{w} D_k(p_{j,t}, q_{j,t})} \tag{2}$$

where w is the number of firm agents in the market k at time t. This mechanism is different with the mechanisms used in the existing price and quality competition models [7] [16] [3] [4]. In the existing models, researchers only consider two firms competing with each other and one firm's demand is a linear function of both firms strategies.

The firm agents from the same market compete in determining their price and quality levels to maximise their profits. The profit ($\pi_{i,t}$) for agent i in market k at time step t is given as follows:

$$\pi_{i,t} = (p_{i,t} - C(q_i))s_{i,t}D_k(\bar{p}, \bar{q}) \tag{3}$$

where $\bar{p} = \sum_{i=1}^{w} p_{i,t}$, $\bar{q} = \sum_{i=1}^{w} q_{i,t}$, $D_k(\bar{p}, \bar{q})$ is the demand of market k, $s_{i,t}D_k(\bar{p}, \bar{q})$ is the firm agent i's demand, and $C(q_i)$ the agent i's quality cost.

Higher quality levels are usually accompanied by higher costs in most businesses. In our model, we use a quadratic cost function: $C(q) = \epsilon q^2$. The ϵ is a positive parameter. This type of cost function reflects the nonlinear impact of quality levels on costs and is often used in the marketing literature [7] [3] [4].

1) Decision-making process: From the discussion above, we note that the firm agents face decision challenges on their product price and quality levels, and their preferred markets $(p_{i,t}, q_{i,t}, k_{i,t})$. In this paper, the GA is not only used to characterize a competitive market environment, where the firm agents interact and compete with each other over time, but also model firm agents' decision-making process. In our GA, each firm agent is represented through an agent chromosome. This chromosome holds a number of genes which represents how that particular agent behaves.

$$Chromosome = (G_P, G_Q, G_M) \tag{4}$$

The G_P gene represents the agent's price decision strategy. The G_Q gene represents the agent's quality decision strategy. Finally, the G_M gene represents the preferred market's ID and is used to determine which market the agent participants in.

Furthermore, we use the profit function as the fitness function in our GA (See Equation 3. We do not distinguish between profit and fitness and will alternatively use both words in the following context.

TABLE I

P .

Variable	Range/value	Description	Variable	Range/value	Description
T	200	Simulation length	β	[0, 1]	market preference over quality
f	60	Firm agent number		0.05	Selection rate (GA)
m	5	Market number		0.8	Crossover rate (GA)
$\mathcal{A}_k$	6000	Potential maximum demand		0.05	Mutation rate (GA)
b	0.9	Weight parameter	G_P	[0, 5]	Price gene
ϵ	1.0	Quality cost parameter	G_Q	[0, 1]	Quality gene
α	[0, 1]	market preference over price	G_M	{1, 2, 3, 4, 5}	Market ID gene

In our GA, we use an elitism mechanism to implement our selection operator. We select the best agents directly into the following generation which is controlled by the selection rate. This means, in each generation, a small number of agents do not change their strategies as their current strategies perform well. The rest of individuals or firm agents, have a certain probability to learn new strategies through our crossover operator and mutation operator. A single point crossover operator is implemented. For our mutation operator, the degree of change of each strategy gene is $0.1 * (max - min)$ where max and min is a gene's range.

C. Simulator Design

In order to examine the impact of market preferences on the evolution of market price and product quality, the GA is used to facilitate evolution and a competitive dynamic market environment. Our competitive market consists a number of markets and many firm agents interacting with each other. We assume that the firm agents can freely participant in any market, however, one firm agent can only participant in one market at each period. The firm agents in the same market compete with each other. In other words, firm agents compete locally in a market of their peers, where they have no knowledge about their peers, or the individual market preferences.

Initially these firm agent genes are generated using a uniform distribution for the first generation. Over subsequent generations new agent chromosomes are generated using our genetic algorithm. For each generation, we firstly calculate each market's demand, and then each firm agents market share, and profit (fitness) according to Equation 1, 2 and 3. Finally, the selection operator, crossover operator and mutation operator are applied. Through these operators, a number of the least fit individuals are removed and replaced with other new strategies which may perform better or worse than those replaced.

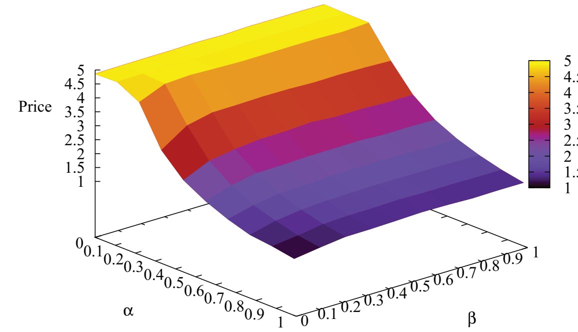
IV. E R

In this section, we will present a series of experimental results from our simulations. Table I shows the parameter settings for the markets, firm agents and our GA. By varying the different parameters in our model we investigate the impact of market preferences on the evolution of market price and product quality. We examine two different market configurations: homogeneous and heterogeneous market settings. In the homogeneous model, all markets have the same price and quality sensitivities while in the heterogeneous model, the markets have different price and quality sensitivities. In the following sections, we will firstly examine the results from homogeneous markets and then the results from heterogeneous markets.

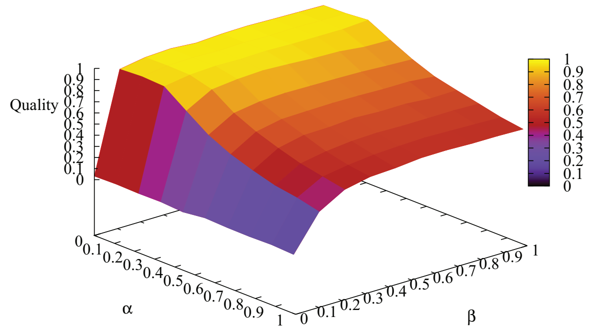
A. Competition in homogeneous markets

All 5 markets have the same setting in homogeneous markets. Each market has two parameters α and β which reflect the market preferences. A high α reflects that a market is highly sensitive to price, while a high β reflects that a market places a premium on quality. The results in Figure 2 are from 50 runs of our simulator for each combination of α and β. Figures 2(a), 2(b), 2(c), and 2(d) depict the average price, quality, profit and demand quantity for the whole agent population at generation 200, respectively.

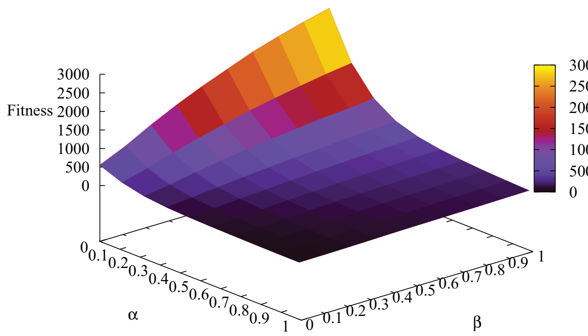
There are a number of features involving these experiments. Firstly, we observe that the agents' average price evolves to a lower level as the α value increases. In other words, the agents lower their price levels as the market becomes more sensitive to the price levels. Secondly, the agents' average quality level evolves to a higher level as the β value increases. This reflects that the agents increase the product quality levels as the markets pay more attention to quality. Therefore, we can conclude that agent strategies evolve to reflect the bias of their market. These emergent phenomena stem from firm agents' competition provided by our GA. As the markets are more sensitive to price or quality, the firm agents with lower price and higher quality products have a competitive advantage. These firms are considered the most fit agents in our GA. The lower price and higher quality genes are then promoted in the following generations. Finally, the market price and quality evolve to a lower level and a higher level respectively. Furthermore, these strategies subsequently affect the average profit as shown in Figure 2(c). Specifically, the average profit decreases as the markets are more sensitive to price. Higher market price sensitivities lead to intense competition, resulting in a decrease in profits. Conversely, we observe that the average profit increases as the markets are more sensitive to quality despite the higher costs of increased quality for firms. This is because that higher quality levels of products in the markets result in a higher market demand as shown in Figure 2(d), and subsequently an increased profit. Therefore, higher quality has a positive impact on agents' profit in our model. This feature of our model is consistent with the existing research results [1].



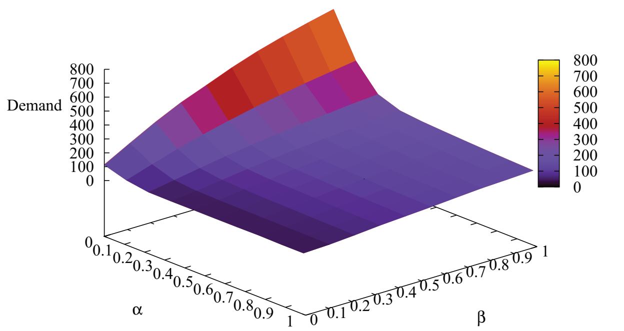
(a) Average price



(b) Average quality



(c) Average profit



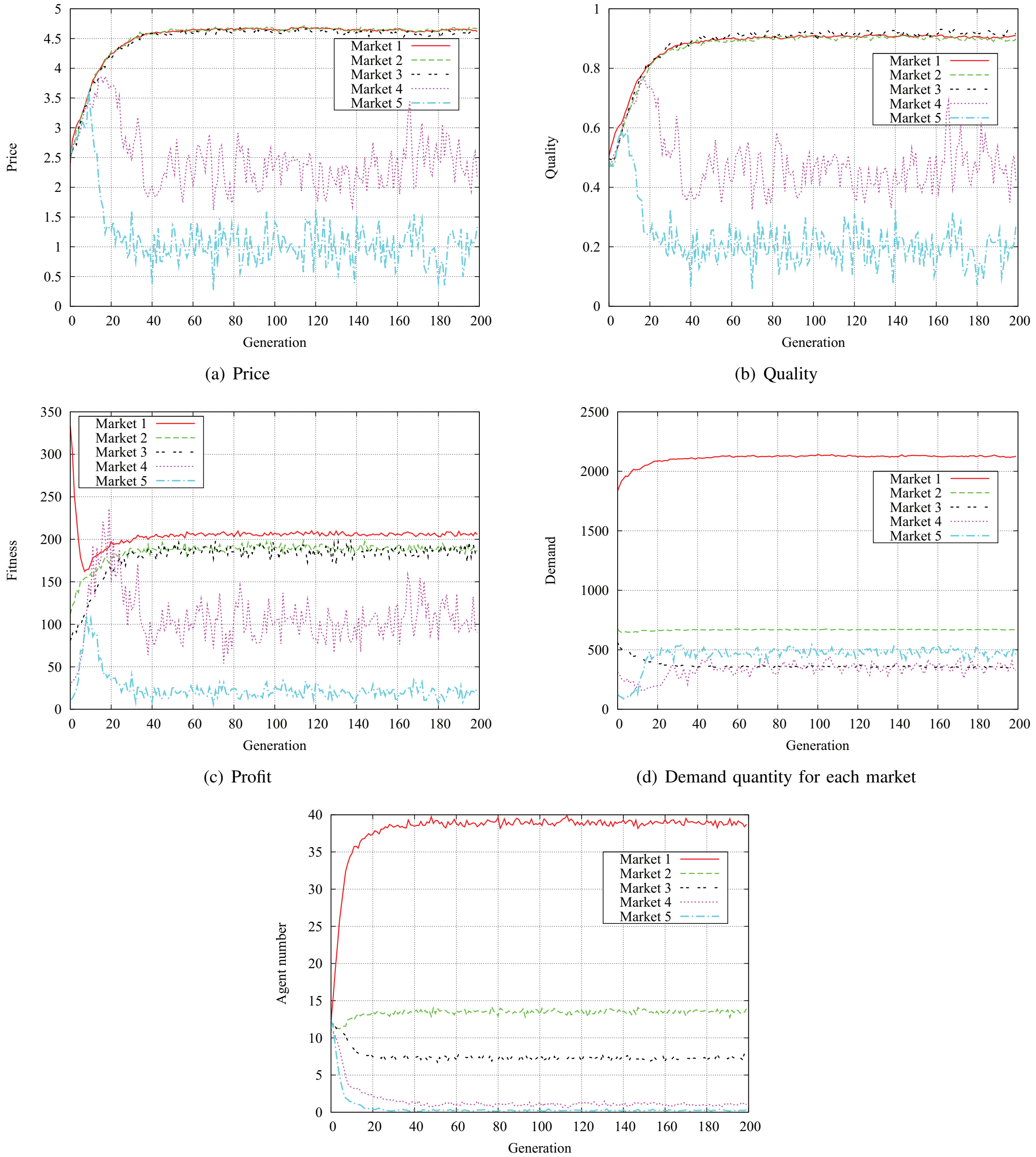
(d) Average demand

Fig. 2. Agent behaviors for values of α and β in homogeneous markets

B. *Competition in heterogeneous markets*

In this section, we examine a scenario where agents compete on price and quality in heterogeneous markets. In heterogeneous markets, each market has different price and quality sensitivities. Our purpose is to investigate how agents' strategies evolve over time in heterogeneous markets. The 5 different markets are set as (Market 1: $\alpha = 0.1$ and $\beta = 0.8$), (Market 2: $\alpha = 0.1$ and $\beta = 0.1$), (Market 3: $\alpha = 0.4$ and $\beta = 0.4$), (Market 4: $\alpha = 0.8$ and $\beta = 0.8$) and (Market 5: $\alpha = 0.8$ and $\beta = 0.1$). Each market represents different degrees of price and quality sensitivities. Our markets have the following features. Markets 1, 2 and 3 have lower price sensitivities while Markets 4 and 5 have higher price sensitivities. Markets 2, 3 and 5 have lower quality sensitivities, while Markets 1 and 4 have higher quality sensitivities.

Figure 3 shows the average data from 50 runs. Figures 3(a), 3(b), 3(c), 3(d) and 3(e) depict how the firm agents' average price, quality, profit, each market demand and the firm agent numbers evolve over time. From these figures, we notice that the markets' preferences on price are significant factors on the evolution of market price levels. As Figure 3(a) shows, the price levels evolve to higher levels in Markets 1, 2 and 3 (lower price sensitivities), while in Markets 4 and 5 (higher price sensitivities), the agent price levels evolve to lower levels. This also stems from firm agents' competition provided by our GA which we have discussed in the homogeneous markets. More interestingly, the effect of quality preferences in heterogeneous markets is different with that in homogeneous markets. For example, the quality levels in Market 4 do not evolve to a higher level although Market 4 is a higher quality sensitive market as shown in Figure 3(b). Conversely, in Market 2, the quality levels evolve to a higher level although this market has very low quality sensitivities. This derives from the features of these markets. Market 4 is very sensitive to price and subsequently, firm agents from this market have to reduce their product price levels. This drives their profits down and consequently, they have lower incentive to produce higher quality products although consumers in this market prefer higher quality products. For Market 2, we can apply similar analysis. Finally, we can observe the emergence strategies of the firm agents that many firm agents enter into Market 1 which is a lower price sensitive and higher quality sensitive market as Figure 3(e) shows. Although higher quality levels lead to a production cost, it results in a higher market demand. In Market 1, agents have to produce higher quality products which will incur higher quality cost, but also could stimulate consumer demand. In fact, due to the relationship of price and quality preferences, Market 1 becomes the biggest one among the 5 markets (see Figure 3(d)). Furthermore, we find that many agents rush into Market 1 which increases the degree



(a) Price

(b) Quality

(c) Profit

(d) Demand quantity for each market

(e) The firm agent numbers in each market

Fig. 3. Heterogeneous markets ((Market 1: $\alpha = 0.1$, $\beta = 0.8$), (Market 2: $\alpha = 0.1$, $\beta = 0.1$), (Market 3: $\alpha = 0.4$, $\beta = 0.4$), (Market 4: $\alpha = 0.8$, $\beta = 0.8$) and (Market 5: $\alpha = 0.8$, $\beta = 0.1$))

of competition. Subsequently, this drives the average profit down at the beginning as Figure 3(c) shows. However, the average profit in Market 1 goes up a little due to their learning on Market 1's preferences. Furthermore, we observe that the distribution of agents in the markets is related to the average agent profits of the markets. This reflects the agents's rational choices on market position.

Furthermore, we compare the agent numbers in each market from homogeneous markets and heterogeneous markets. Table II shows the agent's distribution in both market settings. This data is recorded from 50 runs of our simulator over 200 generations. From this table, we can find that the number of agents is almost evenly distributed in homogeneous markets since there are no differences in markets. The distribution of agents in heterogeneous markets reflects the bias of agents' preferences which has been analysed above.

V. C S F W

The research outlined in this paper have investigated the evolution of the price and quality competition under a range

TABLE II

A

Market Name	Homogeneous markets		Heterogeneous markets	
	Agent Number	Standard Deviation	Agent Number	Standard Deviation
Market 1	11.9	1.1	36.3	3.0
Market 2	12.3	1.2	14.1	2.7
Market 3	11.8	0.9	6.9	1.8
Market 4	11.6	1.1	2.1	1.5
Market 5	12.4	0.9	0.6	1.1

of market configurations. This research holds particular significance for those interested in price and quality competition. The contributions of this paper include the following several aspects.

We have proposed a new multi-agent price and quality competition model. This model differs from existing models in a number of ways. Firstly, we consider many firms competing with each other in a number of markets simultaneously while existing models only consider one or two firms competing in one market such as Hotelling's Model [2], and Banker et al's model [3]. Furthermore, different markets may have different properties, such as the demand size, price and quality sensitivities. Secondly, we design a new mechanism to determine each firm agent's demand quantity. This mechanism indirectly reflects each firm agent's market share is not only determined by their own strategies but also affected by other agents' decisions. This model could be easily extended to include many extra features, such as advertisement effects of firms.

We have investigated the impact of market preferences on the evolution of price and quality under a range of market configurations. We find that the price and quality relationship in a given market is a primary driver of the evolution of firm agent strategies in that market. Firm agents' price strategies evolve rapidly to reflect the preferences of the markets in both homogeneous and heterogeneous markets. We can also observe the similar features for the agents' quality strategies in the homogeneous markets. However, in heterogeneous markets, the agents' average quality levels evolve to higher quality level even in the lower quality sensitive markets despite the penalties incurred by higher cost of increased quality due to the positive impact of quality. Furthermore, we notice an emergent phenomenon in the heterogeneous markets that the firm agents prefer low price and high quality sensitive markets. Based on these results, we can conclude that market preferences have significant effects on the agents' rational decisions. These results have potentially interesting applications to real-world market dynamics and help make strategic decisions on market competition and market entry.

However, there are still a number of factors that influence this study. Firstly, the assumption that all the firms have the same quality cost function is not realistic in the real world. Secondly, the firm agents in our model have no capacity constraints. In future, we would like to improve our model and explore its applications to the real-world market dynamics.

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