Strategy Formulation for Quality Control in Process Industries

Bikash Bhadury,

Professor, Department of Industrial Engineering & Management,
Indian Institute of Technology, Kharagpur, India.
E-mail: bikash@vgsom.iitkgp.ernet.in

ABSTRACT: *Control charting is synonymous with statistical process control (SPC). However, the traditional control chart assumption of data independence is often violated in process industries. This problem has been analyzed and necessary modifications to the widely used SPC tools have been suggested. The performances of the modified estimators and proposed control schemes have also been studied, through simulation. Moreover, since the majority of the actual production systems are multi-stage in nature, various strategies of integrated use of SPC and engineering process control (EPC) tools have been studied through simulation at different correlations, for both the single and two-stage production processes.*

KEYWORDS: SPC TOOLS, MEASURES OF COMMON CAUSE SIGMA, ALPHA RISKS, IN- CONTROL ARL, EWMA AND COMPOSITE SCHEMES, SPC AND EPC INTEGRATION STRATEGIES

1. INTRODUCTION

Quality Control, according to ISO 8402, includes "the operational techniques and activities that are used to fulfil the requirements for quality", and the control chart is an important statistical process control (SPC) tool for quality control in industrial organizations. The widely used Shewhart Control Chart model assumes that the process output (Y) fluctuates around a fixed mean (μ) with a constant variance (σ^2) as represented as follows:

$Y_t = \mu + {}_t$, where ${}_t$ are the random errors ~ iidn $(0, \sigma^2)$ ---------------- (1)

This paradigm has been under severe scrutiny, especially with regard to process data obtained from process industries. Herein the underlying process model is different, and the mean wanders instead of being fixed. The model can be represented as follows:

$Y_t = \mu_t + {}_t$ -- (2)

Thus, the traditional control chart assumption of data independence is often violated in process industries, and the traditional control charts constructed with the data as obtained give misleading signals abut the statistical state of the process. The control chart fails to give a signal when the process is out-of-control, in some situations, and issues frequent signals when the process is actually in control. Thus there is a great need to design and develop SPC tools which can be used for monitoring of processes in process industries. There is also a need to study the combined use of SPC and automatic process control (engineering process control, or EPC) tools in multi-stage process environments. The objectives of the research reported in this paper were as follows:

1. Studying the effect of control chart parameters on the performance of control charts in data correlated situations and suggesting suitable modifications to these parameters.
2. Studying the performance of alternative control schemes on an auto-correlated process with commonly occurring process departures, such as step, trend and impulse shifts.
3. Studying the performance of the existing exponentially weighted moving average (EWMA) statistic and suggesting steps to overcome the basic problem of inertia.
4. Studying the performance of composite control schemes in presence of data correlation.
5. Developing a scheme for integration of SPC and EPC tools in a multi-stage process environment.

2. MODIFICATIONS TO COMMONLY USED SPC TOOLS.

As noted earlier, when the process data are dependent, the process mean wanders instead of being fixed and the process is represented by equation (2). This process can be modeled, as an auto-regressive process of order one, namely AR(1), and represented as follows:

$Y_t = \mu + \phi\,(Y_{t-1} - \mu) + {}_{t,}$ --- (3)

Where ϕ is the auto-correlation coefficient and in the stable region its value varies between –1 to +1. Further, without loss of generality, it can be assumed that $\mu = 0$ and then equation (3) reduces to:

$Y_t = \phi.\ Y_{t-1} + {}_{t,}$ --- (4)

In case of individual control charting, the control chart parameters used are as follows:

(i) as measure of central tendency of process data – process mean (average of the data), and the median of the observations, and

(ii) as measure of dispersion – standard deviation statistic, which, in turn, can be further classified as follows:
  (a) standard deviation statistics based on short-term variation – standard deviation statistic based on the absolute moving range (σ_{MR}), median moving range (σ_{MMR}), and root mean square successive difference (σ_{RMSSD}), and
  (b) standard deviation statistics based on long term variation – standard deviation statistic based on pooled observations (σ_s), and mean absolute deviation (σ_{MAD}).

2.1 Robustness of Measures of Common Cause Sigma

The unbiased estimate of process standard deviation obtained from the absolute value of moving ranges is as follows:

$$\sigma_{MR} = \frac{\overline{MR}}{d_2}, \text{ where } \overline{MR} = \frac{\sum_{i=2}^{n} \left| MR_i \right|}{n-1} \quad \text{---------------------------- (5)}$$

and d_2 = bias correction factor based on the width of moving range
(here n=2 and d_2 =1.128)

This statistic is unbiased only if the data are iid normal, and its efficiency is further decreased in auto-correlated situations. Accordingly, it is suggested that the following modification to the standard deviation statistic based on moving ranges be used in data correlated situations:

$$\sigma_{PMR} = \frac{\overline{MR}}{d_2^*}, \text{ where } d_2^* = d_2 \sqrt{1-\phi} \quad \text{--------------------------------------(6)}$$

Note: The subscript PMR indicates that it is a proposed moving range statistic.

A simulation study was then carried out to estimate the α-risks and the average number of false alarms of the Shewhart individual control chart, using the six estimates of common cause sigma, namely σ_{MR}, σ_{PMR}, σ_{MMR}, σ_{RMSSD}, σ_s and σ_{MAD}. The comparative bias of the two estimates using absolute moving ranges, namely σ_{MR}, and σ_{MMR}, and the two estimates based on long term variation, namely σ_s and σ_{MAD}, were also tested through simulation. The important conclusions drawn from these studies are as follows:

1. σ_{MR} and σ_{RMSSD} should not be used for data with any amount of correlation.
2. Whereas σ_{MMR} is a robust estimate in most of the negative correlation period, it is sensitive to correlation for the rest of the AR (1) process.
3. Bias study revealed that σ_{MR} is sensitive and σ_{PMR} robust to data correlation.
4. σ_s and σ_{MAD} are robust to data correlation in most of the AR (1) process.
5. Bias study revealed that σ_s is a better statistic than σ_{MAD} in most of the stable period of AR (1) process. However, σ_{MAD} is better than σ_s in the near non-stationary region of AR(1), that is, with $\phi > 0.90$.

Based on these results, it was decided that for subsequent investigative studies, σ_{PMR}, σ_s and σ_{MAD} would be used for control charting on AR (1) data and σ_{MR} for forecast residuals based on time-series residuals model.

2.2 Use of EWMA Control Scheme

The use of exponentially weighted moving average (EWMA) in quality control was first reported by Roberts (1959), and in his report, the author compared the performance of the EWMA control chart with its counterparts, namely, Shewhart and the simple moving average control charts, and inferred that the performance of the EWMA chart is superior, especially at low and medium shifts in mean, considering the average run length as the performance measure. The application base of EWMA was widened after the publication of the expository article of Hunter (1986), wherein the author advocated the use of EWMA in automatic process controls and showed its resemblance to dynamic controls (through integral (I), proportional-integral (PI) and proportional integral and differential (PID) controllers) which are widely used in continuous process industries. Montgomery and Mastengelo (1991) have advocated the use of EWMA in autocorrelated situations. They have also suggested supplementary methods, such as tracking signals, which enhance the efficiency of process monitoring, and applied the EWMA control chart to the forecast residuals, in a manner similar to the two chart system, namely the common cause and special cause charts, as had been proposed by Alwan and Roberts (1988). The major drawback of the EWMA control chart is that it cannot respond to the process departures immediately as can be done by the Shewhart individual control chart. This reduces the detection capability of the chart (its inertia in responding to various process departures which reduces its detection capability immediately after the occurrence of the shift). Further, the performance of the EWMA control chart in data correlated situations has yet not been studied.

Let Y_t be the observation obtained from the process at time t, then the exponentially weighted moving average is defined as follows:

$$Z_t = \; . Y_t + (1 - \;). Z_{t-1}, \text{ --(7)}$$

Where $0 < \lambda \leq 1$ is the weighting constant. Also the initial value of Z_t is taken as the process target, so that $Z_0 = \mu_0$. Alternatively, the estimate of the process average, that is $\overline{y}$ or $\overline{\overline{y}}$, can also be used as Z_0. Thus the EWMA statistic Z_t is the weighted average of all the preceding observations. The curtailed EWMA (CEWMA) statistic reported in Margavo et al (1995) is as folows:

$$Z_{CK} = \sum_{i=t-k+1}^{n} \frac{(1-\lambda)^{t-i}}{1-(1-\lambda)^{k}} . Y_t, \text{ given } t > k \text{ -----------------------------(8)}$$

Where k is the period of curtailment or the number of past observations considered in estimating the CEWMA statistic. Equation (8) can also be written in terms of the constant multiples of the difference between the current and past EWMA as follows:

$$Z_{CK} = \frac{1}{1-(1-\lambda)^{k}} (Z_t - (1-\lambda)^{k} . Z_{t-k}), \text{ -----------------------------(9)}$$

where Z_{t-k} is the regular EWMA statistic k time periods earlier. The proposed statistic is represented in terms of two regular EWMA statistics and because of this, it requires less memory space as it needs to save only two values, namely Z_t and Z_{t-k}, as opposed to Margavo et al (1995), which needs to save k past observations. The period of curtailment, k, has considerable bearing on the performance of the CEWMA control scheme. For designing the control limits on the CEWMA control chart, the asymptotic process mean and variance of the CEWMA statistic are required. These were also derived.

The in-control average run length (ARL) performance of the EWMA schemes (EWMA and CEWMA with different values of) were tested through simulation (1000 simulation runs were made for each case). It was observed that the in-control ARL of the EWMA schemes is either much greater or much smaller than the designed value because of over and under estimation of variance in the negative and positive correlation regions respectively. Brown (1963) had suggested a modification to the variance estimator of the EWMA based on the correlation in the data. This modification was applied and the modified variance statistics of the EWMA and CEWMA were tested through simulation. The results were encouraging and it was found that in the positive correlation region, all control charts performed well, whereas in the negative correlation region, EWMA schemes with higher values of $\lambda (\lambda > 0.40)$ performed well.

2.3 Use of Composite Control Schemes and Detection Capability of Various Control Schemes

To overcome the inherent deficiency of the EWMA charts and also for detection of process shifts of unknown magnitudes, the use of composite control schemes was investigated. Robust standard deviation estimates, identified earlier, were used for this investigation. The simulation studies showed that the in-control ARL performance of composite schemes is better than that of individual charts throughout the stable period of AR (1) process. Moreover, the in control ARL performance of EWMA schemes using σ_{MAD}, as the estimate of standard deviation, is better than ones using σ_s. This confirms the positive bias of σ_{MAD} identified earlier in study. It was also found that the composite schemes are not only useful in situations of unknown magnitude of shifts, but also have better in-control ARL performance. Moreover, in all the controls schemes, it was found that in-control ARLs are higher in the negative correlation region.

For investigation of the detection capability of Shewhart, EWMA and composite control schemes, three kinds of process disturbance, namely, step, trend and impulse shifts, were considered. The following three measures of run length performance (RLP) were considered:

(i) out-of-control ARL,

(ii) various quantiles, namely q_{50}, q_{25} and q_{10} (where q_{50} is the median run length (MdRL), and

(iii) cumulative distribution function (CDF), such as the cumulative probability of detection at the i^{th} observation (i = 1,2,3,...,,10).

10,000 simulation runs were made for the three shifts of various magnitudes and at different correlations in the stable period of AR(1) process. A large number of cases (108) were investigated for each of the three robust standard deviation estimates. The major recommendations obtained from these investigations are as follows:

1. σ_{PMR} is the best statistic as far as detection capability is concerned. This holds for its use with all the control charts. However, its in-control ARL is slightly lower than σ_{MAD} (σ_{MAD} is best from the in-control ARL point of view).

2. Significant difference has not been observed between detection capabilities of σ_{MAD} and σ_s. However, σ_{MAD}, is better from the in-control ARL view point.
3. Composite control schemes have higher probability of detection at i^{th} observation than the others.
4. Shewhart and Alwan-Roberts (A-R) control charts have better detection capability for large and medium step shifts in both high negative and high positive correlation regions.
5. EWMA schemes with $\lambda \leq 0.2$ have better detection capability of small step shift (upto 0.5σ) in most of the stable period of the AR(1) process.
6. EWMA schemes with $\lambda \geq 0.4$ have better detection capability for medium and large shifts between $-0.5 \leq \phi \leq 0.5$.
7. Composite schemes, using CEWMA with $\lambda = 0.03$, have better detection capability for small step shifts in 85 percent of the stable period of AR(1) process.
8. Composite schemes, using EWMA & CEWMA with $\lambda \geq 0.1$, have better detection capability for medium and large step shifts in around 80 percent of the stable period.
9. At high positive correlation, detection capabilities of all composite schemes are equal. However, EWMA with high values of are the most preferable, Boxley (1990).
10. Shewhart and A-R charts have better detection capabilities for all trend shifts in the high positive correlation region ($\phi > 0.70$), whereas in the high negative correlation region, their detection capabilities are better for medium and large trend shifts.
11. EWMA schemes with low λ values ($\lambda \leq 0.2$) have better detection capabilities for small and medium trend shifts in around 70 percent of the stable period of AR(1) process.
12. Composite schemes using EWMA and CEWMA, with $\lambda \leq 0.05$, have better detection capabilities for small trend shifts over 70 percent of the stable period of AR(1) process.

3. INTEGRATION OF EPC AND SPC TOOLS.

In addition to integral (I), proportional and integral (PI) and proportional integral and differential (PID) controllers, the minimum mean square error (MMSE) controller has been widely discussed and considered as industry standard. EPC tools (I, PI, PID, MMSE etc.) continuously correct the controllable inputs, or process parameters, to keep the process output at or near the target. Hence the EPC tools try to reduce the short term variations as opposed to SPC tools, which try to eliminate the long term quality variations. Thus for proper control of an automatic production process, the combined use of EPC and SPC tools is necessary to overcome the deficiencies of each of these individual tools and keep the desired product quality characteristic close to the target.

The selection of a particular integration strategy depends on the manufacturing process. The following are the recommendations for general use:

i) For monitoring and control of a process or processes encountering (affected by) small to medium shifts in the process mean, the integration of MMSE, or any one of PID controllers, with the optimal SPC tools used for monitoring small shifts, that is, EWMA control schemes with smaller values of λ.

ii) For monitoring and control of a process or processes affected by larger shifts in the process mean, the integration MMSE, or any one of PID controllers, with the optimal SPC tools used for monitoring larger shifts, such as EWMA control schemes with larger values of λ or Shewhart control charts.

iii) For monitoring and control of a process or processes affected by the shifts of unknown magnitude in process mean, the integration of MMSE, or any one of PID controllers, with a composite SPC control scheme.

Strategies related to the use of specific control limits with the SPC tools in integrated schemes are as follows:

i) It is worth keeping wide control limits when the process is operating in the startup phase, since in the initial stages there is every chance of frequent signaling by control chart. This is the result of over or under corrections initiated by the controllers due to startup phase problems.

ii) It is worth keeping narrower control limits for a process operating in a state of statistical control. This is because the control actions initiated by the controller for a matured process are more stabilized in nature. Further, keeping closer limits helps in continuous quality improvement of the process.

Simulation study was performed to test the various integration strategies. The SPC tools considered were as follows:

(i) Shewhart control chart,

(ii) CEWMA and EWMA control charts with λ = 0.05, 0.10, 0.20, 0.40, 0.50 and 0.75, and

(iii) Composite schemes of Shewhart – CEWMA and Shewhart – EWMA with λ = 0.05.

The width of the control limits for the control schemes were based on the result of studies conducted on EWMA and composite schemes, noted earlier, for the in-control ARLs of 250. The statistical state of the process was assessed using the first 300 observations. Process departures of various types, such as step and trend shifts in process mean in constant multiples of the process standard deviation σ, were initiated after the 300th observation. The out-of-control signal indicated by the chart was duly recorded. As soon as the control chart signaled for an out-of-control condition, then the disturbance created on the process was brought to zero. If the signal was not obtained till the 700th observation, then the run was terminated. Performance measures such as the MSE, correlation in the output data and run lengths were recorded for each iteration. 1000 runs were simulated for each type and magnitude of shift at each of the correlations in the stable period of AR(1) process.

As noted, various integration strategies were studied through simulation at different correlations in the stable period of the AR(1) process. Initially the single stage production process was taken up for investigation. Different strategies have been tested and the optimal strategy has been determined. The same methodology was used for the two stage production system. In this case, the correlation estimate of the second stage has been derived using variance of the controlled output of the first stage and the transfer function of the second stage. The results obtained show considerable improvement through the use of a strategy of integration of EPC and SPC tools. However, there is the danger of over-correction because the MMSE controller initiates control action based on estimated parameters of the first and second stages of the process. This problem needs to be investigated further. The results of the two stage production system can be extended to the multi-stage system (m>2).

4. SCOPE FOR FUTURE RESEARCH

The possible areas of future research based on this work on SPC tools for auto-correlated data situation are as follows:

1. Robustness of control charts using proposed and identified robust variance estimates (σ_{PMR}, σ_s, σ_{MAD} and σ_{MZ}), for the model mismatch, for data with ARMA(p.q.) process.
2. The effect of under or over estimation of correlation coefficient on the RLPs of control charts using various robust standard deviation estimates.

The possible areas of future research on integration of SPC and EPC tools in multi-stage environment are as follows:

1. It is observed from the results of single stage integration over stable period of AR(1) process that the MMSE controller initiates over control in low positive correlated region of AR(1) process. This effect is aggravated in the two stage production process. This aspect needs to be further investigated to eliminate or to reduce the over correction by the MMSE controller. This is particularly important for the multi-stage process, where m>2.
2. The other, possible area of research is integration of MMSE controller with Zhang's cause selecting control scheme (1992).

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