On Prototypes for Winslett's Semantics of DL-Lite ABox Evolution

Evgeny Kharlamov, and Dmitriy Zheleznyakov

KRDB Research Centre, Free University of Bozen-Bolzano, Italy
last_name@inf.unibz.it

Abstract. Evolution of Knowledge Bases expressed in Description Logics (DLs) proved its importance. Most studies on evolution in DLs have focused on model-based approaches to evolution semantics and in particular on Winslett's semantics (WS). It was understood that evolution under WS even in tractable DLs, such as *DL-Lite*, suffers from inexpressibility, i.e., the result of evolution cannot be expressed in the same logics. In this work we show which combination of *DL-Lite* logical constructs is responsible for the inexpressibility and explain reasons for such a behaviour. We present novel techniques, based on what we called prototypes, to capture Winslett's evolution in $\mathsf{FO}[2]$ for *DL-Lite*$_{\mathcal{R}}$. We also discuss which fragments of *DL-Lite*$_{\mathcal{R}}$ are closed under evolution.

1 Introduction

Description Logics (DLs) provide excellent mechanisms for representing structured knowledge by means of Knowledge Bases (KBs) $\mathcal{K}$ that are composed of two components: TBox (describes intensional or general knowledge about an application domain) and ABox (describes facts about individual objects). DLs constitute the foundations for various dialects of OWL, the Semantic Web ontology language.

Traditionally DLs have been used for modeling *static* and structural aspects of application domains [1]. Recently, the scope of KBs has broadened, and they are now used also for providing support in the maintenance and *evolution* phase of information systems. This makes it necessary to study *evolution of Knowledge Bases* [2], where the goal is to incorporate a new knowledge $\mathcal{N}$ into an existing KB $\mathcal{K}$ so as to take into account changes that occur in the underlying application domain. In general, $\mathcal{N}$ is represented by a set of formulas denoting those properties that should be true after $\mathcal{K}$ has evolved, and the result of evolution, denoted $\mathcal{K} \diamond \mathcal{N}$, is also intended to be a set of formulas. In the case where $\mathcal{N}$ interacts with $\mathcal{K}$ in an undesirable way, e.g., by causing the KB or relevant parts of it to become unsatisfiable, $\mathcal{N}$ cannot simply be added to the KB. Instead, suitable changes need to be made in $\mathcal{K}$ so as to avoid this undesirable interaction, e.g., by deleting parts of $\mathcal{K}$ conflicting with $\mathcal{N}$. Different choices for changes are possible, corresponding to different approaches to *semantics* for KB evolution [3,4,5].

One approach to evolution semantics that proved its importance is *Winslett's semantics* (WS) [6], which is an *update* semantics in terms of Katsumo and Mendelzon [4], and was originally proposed for propositional theories. Under this semantics the result of evolution $\mathcal{K} \diamond \mathcal{N}$ is a *set of models* of $\mathcal{N}$ that are minimally distanced from models of $\mathcal{K}$, where the distance is based on symmetric difference between models (see Section 3 for details). Since the result of evolution $\mathcal{K} \diamond \mathcal{N}$ is a set of models, while $\mathcal{K}$ and $\mathcal{N}$ are logical theories, it is desirable to represent $\mathcal{K} \diamond \mathcal{N}$ as a logical theory using the same language as for $\mathcal{K}$ and $\mathcal{N}$. Thus, looking for representations of $\mathcal{K} \diamond \mathcal{N}$ is the main challenge in a study of evolution under WS. When $\mathcal{K}$ and $\mathcal{N}$ are propositional theories, representing $\mathcal{K} \diamond \mathcal{N}$ is well understood [5], while it becomes dramatically more complicated as soon as $\mathcal{K}$ and $\mathcal{N}$ are first-order, e.g., DL KBs [7].

In this work we study how WS can be applied to evolution of KBs under the following two assumptions. First, we assume that both $\mathcal{K}$ and $\mathcal{N}$ are written in a language of the *DL-Lite* family [8]. The focus on *DL-Lite* is not surprising since *DL-Lite* is tightly connected with conceptual data models and it is the basis of OWL 2 QL, a tractable OWL 2 profile.Second, we assume that $\mathcal{N}$ is a new ABox and the TBox of $\mathcal{K}$ should remain the same after the evolution. That is, we study a so-called *ABox evolution*. ABox evolution is important for areas, e.g., bioinformatics, where the structural knowledge TBox is well crafted and stable, while ABox facts about specific individuals may get changed, or/and new facts can be inserted in the ABox. These ABox changes should be reflected in KBs in a way that the TBox is not affected.

There are several works on WS for both *DL-Lite* and more expressive DLs. Liu, Lutz, Milicic, and Wolter studied Winslett's evolution in expressive DLs [7], for KBs with empty TBoxes. Most of DLs they considered are not closed under WS and in order to close these logics they used "@" operator. Poggi, Lembo, De Giacomo, Lenzerini, and Rosati applied WS to *DL-Lite* [9] and proposed an algorithm to compute the result of evolution. It turned out that their algorithm is wrong, i.e. it is neither sound, nor complete [10]. Actually, such an algorithm cannot exist since Calvanese, Kharlamov, Nutt, and Zheleznyakov showed that, e.g., *DL-Lite*$_{\mathcal{F}R}$ is not closed under WS of evolution [11], that is, there are $\mathcal{K}$ and $\mathcal{N}$ such that $\mathcal{K} \diamond \mathcal{N}$ is not axiomatizable in this family. Recently [12] we introduced *prototypes*, which are in a way generalization of the notion of canonical model, and proposed a way to capture some fragments of *DL-Lite* in $\mathsf{FO}[2]$, a fragment of first-order logic that uses two variables only.

Current work extends the preliminary results of [12]. Our goals here are

(i) to clarify our prototype-based techniques which was only sketched in [12],
(ii) to extend the techniques to wider *DL-Lite* fragments,
(iii) to gain a better understanding on which fragments of *DL-Lite* are closed under WS and how to approximate evolution results in *DL-Lite*.

We would also like to promote prototypes since we believe they are an useful tool to study evolution of ontologies and might be not only of *DL-Lite* ones.

In Sections 2 and 3 we define *DL-Lite*$_{\mathcal{R}}$ and ABox evolution under WS. In Section 4 we give an intuition of our approach to capture WS of evolution for *DL-Lite*$_{\mathcal{R}}$ KBs using prototypes and $\mathsf{FO}[2]$ theories. In Sections 5 and 6 we formalize the approach. Finally, we discuss properties and approximation of these theories.

2 DL-Lite$_{\mathcal{R}}$

We introduce some basic notions of DLs (see [1] for more details). We consider a logic *DL-Lite*$_{\mathcal{R}}$ of *DL-Lite* family of DLs [8,13]. *DL-Lite*$_{\mathcal{R}}$ has the following constructs for (complex) *concepts* and *roles*: *(i)* $B ::= A \mid \exists R$, *(ii)* $C ::= B \mid \neg B$, *(iii)* $R ::= P \mid P^-$, where A and P stand for an *atomic concept* and *role*, respectively, which are just names. A DL *knowledge base* (KB) $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ is compound of two sets of *assertions*: *TBox* $\mathcal{T}$, and *ABox* $\mathcal{A}$. *DL-Lite*$_\mathcal{R}$ TBox assertions are *concept inclusion assertions* of the form $B \sqsubseteq C$ and *role inclusion assertions* $R_1 \sqsubseteq R_2$, while ABox assertions are *membership assertions* of the form $A(a)$, $\neg A(a)$, and $R(a,b)$. The *active domain* of $\mathcal{K}$, denoted *adom*$(\mathcal{K})$, is the set of all constants occurring in $\mathcal{K}$. The *DL-Lite* family has nice computational properties, for example, KB satisfiability has polynomial-time complexity in the size of the TBox and logarithmic-space in the size of the ABox [14,15].

The semantics of DL-Lite KBs is given in the standard way: using first order *interpretations* $\mathcal{I}$, all over the same countable domain Δ. We assume that Δ contains the constants and $c^\mathcal{I} = c$, i.e., we adopt *standard names*. Alternatively, we view interpretations as sets of atoms: $A(a) \in \mathcal{I}$ iff $a \in A^\mathcal{I}$ and $P(a,b) \in \mathcal{I}$ iff $(a,b) \in P^\mathcal{I}$.

Definitions of $\mathcal{I}$ being a *model* of an ABox or a TBox assertion F, denoted $\mathcal{I} \models F$, and a KB $\mathcal{K}$, denoted $\mathcal{I} \models K$, are standard, as well as the notion of *satisfiability*. We use *Mod*$(\mathcal{K})$ to denote the set of all models of $\mathcal{K}$. We use entailment on KBs $\mathcal{K} \models \mathcal{K}'$ in the standard sense. An ABox $\mathcal{A}$ $\mathcal{T}$*-entails* an ABox $\mathcal{A}'$, denoted $\mathcal{A} \models_\mathcal{T} \mathcal{A}'$, if $\mathcal{T} \cup \mathcal{A} \models \mathcal{A}'$, and $\mathcal{A}$ is $\mathcal{T}$*-equivalent* to $\mathcal{A}'$, denoted $\mathcal{A} \equiv_\mathcal{T} \mathcal{A}'$, if $\mathcal{A} \models_\mathcal{T} \mathcal{A}'$ and $\mathcal{A}' \models_\mathcal{T} \mathcal{A}$.

The deductive *closure of a TBox* $\mathcal{T}$, denoted *cl*$(\mathcal{T})$, is the set of all TBox assertions F such that $\mathcal{T} \models F$. For satisfiable KBs $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, a *full closure of* $\mathcal{A}$ (wrt $\mathcal{T}$), *fcl*$_\mathcal{T}(\mathcal{A})$, is the set of all membership assertions f (both positive and negative) over *adom*$(\mathcal{K})$ such that $\mathcal{A} \models_\mathcal{T} f$. Clearly, in *DL-Lite*$_\mathcal{R}$ both *cl*$(\mathcal{T})$ and *cl*$_\mathcal{T}(\mathcal{A})$ are computable in time quadratic in, respectively, $|\mathcal{T}|$, i.e., the number of assertions of $\mathcal{T}$, and $|\mathcal{T} \cup \mathcal{A}|$. For the ease of exhibition and wlg we assume that all TBoxes and ABoxes are closed.

A *homomorphism* h from a model $\mathcal{I}$ to a model $\mathcal{J}$ is a structure-preserving mapping from Δ to Δ satisfying: *(i)* $h(a) = a$ for every constant a; *(ii)* if $\alpha \in A^\mathcal{I}$ (resp., $(\alpha, \beta) \in P^\mathcal{I}$), then $h(\alpha) \in A^\mathcal{J}$ (resp., $(h(\alpha), h(\beta)) \in P^\mathcal{J}$) for every A (resp., P). We write $\mathcal{I} \hookrightarrow \mathcal{J}$ if there is a homomorphism from $\mathcal{I}$ to $\mathcal{J}$. A canonical model $\mathcal{I}$ of $\mathcal{K}$, denoted as $\mathcal{I}^{can}_\mathcal{K}$ or just $\mathcal{I}^{can}$ when $\mathcal{K}$ is clear from the context, is a model of $\mathcal{K}$ which can be homomorphically embedded in every model of $\mathcal{K}$ [8].

3 Winslett's Semantics for Evolution of Knowledge Bases

We start with ABox evolution of single models under Winslett's semantics. Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a *DL-Lite*$_\mathcal{R}$ KB, $\mathcal{I}$ a model of $\mathcal{K}$, and $\mathcal{N}$ a new ABox satisfiable with $\mathcal{T}$. Evolution of a model $\mathcal{I}$ of $\mathcal{K}$ is based on the symmetric difference $\ominus$: $S_1 \ominus S_2 = (S_1 \setminus S_2) \cup (S_2 \setminus S_1)$, and defined as follows. The (result of) *evolution of* $\mathcal{I}$ *with* $\mathcal{N}$ under Winslett's semantics (WS) [9], denoted $\mathcal{I} \diamond \mathcal{N}$, is the set of models $\mathcal{J}$ such that:

(i) $\mathcal{J} \in$ *Mod*$(\mathcal{T} \cup \mathcal{N})$, and
(ii) there is no model $\mathcal{J}' \in$ *Mod*$(\mathcal{T} \cup \mathcal{N})$ satisfying $\mathcal{I} \ominus \mathcal{J}' \subsetneq \mathcal{I} \ominus \mathcal{J}$

Note that in Case *(i)* we have *Mod* of both $\mathcal{T}$ and $\mathcal{N}$, which means that the evolution preserves both the old TBox and the new knowledge. Case *(ii)* guarantees the principle of minimal change [5]. We extend the definition to KBs:

The result of *evolution of* $\mathcal{K}$ *with* $\mathcal{N}$ under WS, denoted $\mathcal{K} \diamond \mathcal{N}$, is the following set of models:

$$\mathcal{K} \diamond \mathcal{N} = \cup_{\mathcal{I} \in Mod(\mathcal{K})} \mathcal{I} \diamond \mathcal{N}.$$

In terms of [10], WS corresponds to $\mathcal{L}^a_\subseteq$ semantics, i.e., local model-based semantics based on atoms and set inclusion.

The input for evolution is two finite syntactic objects: a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and a new information $\mathcal{N}$, while the output $\mathcal{K} \diamond \mathcal{N}$ is a set of models, which is an infinite object for *DL-Lite*$_\mathcal{R}$. Indeed, $\mathcal{K} \diamond \mathcal{N}$ is in general infinite. One can easily come up with examples where $\mathcal{K} \diamond \mathcal{N}$ has an infinite number of infinite models. These observations imply that storing $\mathcal{K} \diamond \mathcal{N}$ is infeasible and in practice one would like to represent the evolution as a KB $\mathcal{K}'$. Moreover, one would like to stay within the same formalism and express $\mathcal{K}'$ in *DL-Lite*$_\mathcal{R}$. Formally, we say that a logic $\mathcal{L}$ is *closed* under Winslett's evolution if for every $\mathcal{K}$ and $\mathcal{N}$ in $\mathcal{L}$, the result of evolution $\mathcal{K} \diamond \mathcal{N}$ is expressible in $\mathcal{L}$, that is, there is a KB $\mathcal{K}' = (\mathcal{T}, \mathcal{A}')$ in $\mathcal{L}$ such that $\mathit{Mod}(\mathcal{K}') = \mathcal{K} \diamond \mathcal{N}$.

Example 1. Consider the following *DL-Lite* KB $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$ and $\mathcal{N}_1 = \{C(b)\}$:

$$\mathcal{T}_1 = \{A \sqsubseteq \exists R, \exists R^- \sqsubseteq \neg C\}; \quad \mathcal{A}_1 = \{A(a), C(e), C(d), R(a,b)\}.$$

Consider the following model $\mathcal{I}$ of $\mathcal{K}_1$:

$$\mathcal{I}: \quad A^\mathcal{I} = \{a, x\}, \quad C^\mathcal{I} = \{d, e\}, \quad R^\mathcal{I} = \{(a,b), (x,b)\},$$

where $x \in \Delta \setminus \mathit{adom}$. The following models belong to $\mathcal{I} \diamond \mathcal{N}_1$:

$$\begin{array}{llll} \mathcal{J}_0: & A^\mathcal{I} = \emptyset, & C^\mathcal{I} = \{d, e, b\}, & R^\mathcal{I} = \emptyset, \\ \mathcal{J}_1: & A^\mathcal{I} = \{x\}, & C^\mathcal{I} = \{e, b\}, & R^\mathcal{I} = \{(x, d)\}, \\ \mathcal{J}_2: & A^\mathcal{I} = \{x\}, & C^\mathcal{I} = \{d, b\}, & R^\mathcal{I} = \{(x, e)\}. \end{array}$$

Indeed, all the models satisfy $\mathcal{N}_1$ and $\mathcal{T}_1$. To see that they are in $\mathcal{I} \diamond \mathcal{N}_1$ observe that every model $\mathcal{J}(I) \in (\mathcal{I} \diamond \mathcal{N}_1)$ can be obtained from $\mathcal{I}$ by making modifications that guarantee that $\mathcal{J}(I) \models (\mathcal{N}_1 \cup \mathcal{T}_1)$ and that the distance between $\mathcal{I}$ and $\mathcal{J}(I)$ is minimal. What are these modifications? Since in every $\mathcal{J}(I)$ the new assertion $C(b)$ holds and $(C \sqsubseteq \neg\exists R^-) \in \mathcal{T}_1$, there should be no R-atoms with b-fillers at the second coordinate in $\mathcal{J}(\mathcal{I})$. Hence, the necessary modifications of $\mathcal{I}$ are either to drop (some of) the R-atoms $R(a,b)$ and $R(x,b)$, or to modify (some of) them, by substituting the b-fillers with another ones, while keeping the elements a and x on the first position. The model $\mathcal{J}_0$ corresponds to the case when both R-atoms are dropped, while in $\mathcal{J}_1$ and $\mathcal{J}_2$ only $R(a,b)$ is dropped and $R(x,b)$ is modified to $R(x,d)$ and $R(x,e)$, respectively. Note that the modification in $R(x,b)$ leads to a further change in the interpretation of C in both $\mathcal{J}_1$ and $\mathcal{J}_2$, namely, $C(d)$ and $C(e)$ should be dropped, respectively. ■

4 Prototypes for Winslett's Semantics

We first present a general discussion on issues with capturing WS in *DL-Lite*, then give an intuition of our approach for capturing it in $\mathsf{FO}[2]$, and finally give an example of how the approach works. In the next section we formalize the approach.

ABox Evolution of a *DL-Lite* KB $\mathcal{K}$ with an ABox $\mathcal{N}$ is the set of models $\mathcal{K} \diamond \mathcal{N}$ that may not have a canonical one [12]. This immediately yields that $\mathcal{K} \diamond \mathcal{N}$ cannot be described (aka axiomatized) in any language of the *DL-Lite* family.

Example 2. We now illustrate the lack of canonical models in $\mathcal{K}_1 \diamond \mathcal{N}_1$ from Example 1. One can verify that any model $\mathcal{J}_{can}$ that can be homomorphically embedded into $\mathcal{J}_0$, $\mathcal{J}_1$, and $\mathcal{J}_2$ is such that $A^{\mathcal{J}_{can}} = R^{\mathcal{J}_{can}} = \emptyset$, and $e, d \notin C^{\mathcal{J}_{can}}$. It is easy to check that such a model does not belong to $\mathcal{K}_1 \diamond \mathcal{N}_1$. Hence, there is no canonical model in $\mathcal{K} \diamond \mathcal{N}$ and it is inexpressible in *DL-Lite*. ■

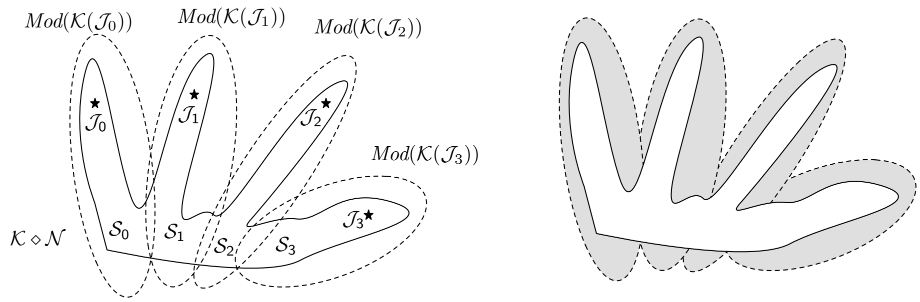

Fig. 1. Graphical representation of our approach to capture the result of evolution under WS.

A closer look at sets $\mathcal{K} \diamond \mathcal{N}$ for different $\mathcal{K}$ and $\mathcal{N}$ gave a surprising outcome: all of them satisfy the following property.

Theorem 3. *$\mathcal{K} \diamond \mathcal{N}$ can be divided (but in general not partitioned) into finitely many subsets $\mathcal{S}_0, \ldots, \mathcal{S}_n$ of models, where each $\mathcal{S}_i$ has a canonical model $\mathcal{J}_i$. Each of these canonical models is a minimal element in $\mathcal{K} \diamond \mathcal{N}$ wrt homomorphisms.*

We called these $\mathcal{J}_i$s *prototypes* [12]. Thus, capturing $\mathcal{K} \diamond \mathcal{N}$ in some logics boils down to *(i)* capturing each $\mathcal{S}_i$ with some theory $\mathcal{K}_{\mathcal{S}_i}$ and *(ii)* taking the disjunction across all $\mathcal{K}_{\mathcal{S}_i}$. This will give the desired theory $\mathcal{K}' = \mathcal{K}_{\mathcal{S}_1} \vee \cdots \vee \mathcal{K}_{\mathcal{S}_n}$ that captures $\mathcal{K} \diamond \mathcal{N}$. Unfortunately, some of $\mathcal{K}_{\mathcal{S}_i}$ are not *DL-Lite* theories (while they are $\mathsf{FO}[2]$ theories, see Section 5 for details).

We construct $\mathcal{K}'$ in two steps. First, we construct *DL-Lite*$_\mathcal{R}$ KBs $\mathcal{K}(\mathcal{J}_i)$ for each $\mathcal{J}_i$ such that $\mathcal{K}(\mathcal{J}_i)$ is a sound approximations of $\mathcal{S}_i$s, that is, $\mathcal{S}_i \subseteq Mod(\mathcal{K}(\mathcal{J}_i))$. Second, based on $\mathcal{K}$ and $\mathcal{N}$, we construct an $\mathsf{FO}[2]$ formula Ψ, which cancels out all the models in $Mod(\mathcal{K}(\mathcal{J}_i)) \setminus \mathcal{S}_i$, that is, $\mathcal{K}_{\mathcal{S}_0} \vee \cdots \vee \mathcal{K}_{\mathcal{S}_n} = \Psi \wedge (\mathcal{K}(\mathcal{J}_0) \vee \cdots \vee \mathcal{K}(\mathcal{J}_n))$.

To get a better intuition on our approach, consider Figure 1, where the result of evolution $\mathcal{K} \diamond \mathcal{N}$ is depicted as the figure with solid-line borders (each point within the figure is a model in $\mathcal{K} \diamond \mathcal{N}$). Assume that $\mathcal{K} \diamond \mathcal{N}$ can be divided in four subsets $\mathcal{S}_0, \ldots, \mathcal{S}_3$. To emphasize this fact, $\mathcal{K} \diamond \mathcal{N}$ looks similar to a hand with four fingers, where each finger represents an $\mathcal{S}_i$. Consider the left part of Figure 1. Each of $\mathcal{S}_i$s has a canonical model depicted as a star. Using *DL-Lite*$_\mathcal{R}$, we can provide KBs $\mathcal{K}(\mathcal{J}_0), \ldots, \mathcal{K}(\mathcal{J}_3)$ that are sound approximation of corresponding $\mathcal{S}_i$s. We depict the models $Mod(\mathcal{K}(\mathcal{J}_i))$ as ovals with dashed-line boarders. Consider the right part of Figure 1. In this figure we depict in grey the models $Mod(\mathcal{K}(\mathcal{J}_i)) \setminus \mathcal{S}_i$ that are cut off by Ψ.

Before proceeding to the next section where we formalize our approach, we introduce prototypes formally.

Definition 4. *Let $\mathcal{K}$ be a DL-Lite$_\mathcal{R}$ KB and $\mathcal{N}$ be an ABox. A* prototypal set for $\mathcal{K} \diamond \mathcal{N}$ *is a minimal subset $\mathfrak{J} = \{\mathcal{J}_0, \ldots, \mathcal{J}_n\}$ of $\mathcal{K} \diamond \mathcal{N}$ satisfying the property:*

$$\text{for every } \mathcal{J} \in \mathcal{K} \diamond \mathcal{N} \text{ there is } \mathcal{J}_i \in \mathfrak{J} \text{ such that } \mathcal{J}_i \hookrightarrow \mathcal{J}.$$

We call every $\mathcal{J}_i \in \mathfrak{J}$ a *prototype for* $\mathcal{K} \diamond \mathcal{N}$. Note that prototypes generalize canonical models in the sense that every set of models with a canonical one, say $Mod(\mathcal{K})$ for a *DL-Lite*$_\mathcal{R}$ KB $\mathcal{K}$, has a prototype, which is exactly the canonical model.

5 Computing Winslett's Semantics When No Roles Interact

We first discuss some of the reasons of WS inexpressibility in our examples and *DL-Lite*$_\mathcal{R}$.

$BZP(\mathcal{K}, \mathcal{N})$

1. $\mathcal{J}_0 := Align(\mathcal{I}^{can}, \mathcal{N}) \cup \mathcal{N}$, where $\mathcal{I}^{can}$ is the canonical model of $\mathcal{K}$.
2. For each $R(a,b) \in AA(\mathcal{K}, \mathcal{N})$, do $\mathcal{J}_0 := \mathcal{J}_0 \setminus \{R(a,b)\}$,
   if there is no $R(a,\beta) \in \mathcal{A} \setminus AA(\mathcal{K}, \mathcal{N})$ do $\mathcal{J}_0 := \mathcal{J}_0 \setminus root^{at}_{\mathcal{T}}(\exists R(a))$.
3. Return $\mathcal{J}_0$.

Fig. 2. The procedure of building zero-prototype

Dual-Affection of Roles. As we discussed in the previous section and illustrated in Example 1, sets of models $\mathcal{K} \diamond \mathcal{N}$ that result from Winslett's evolution do not have canonical models. We now give an intuition *why* in $\mathcal{K} \diamond \mathcal{N}$ canonical models are missing. Observe that in Example 1 the role R is affected by the old TBox $\mathcal{T}_1$ as follows:

(i) $\mathcal{T}_1$ *places* (i.e., enforces the *existence* of) R-atoms in the evolution result, and on *one* of coordinates of these R-atoms, there are constants from specific sets, e.g., $A \sqsubseteq \exists R$ of $\mathcal{T}_1$ enforces R-atoms with constants from A on the first coordinate, and

(ii) $\mathcal{T}_1$ *forbids* R-atoms in $\mathcal{K}_1 \diamond \mathcal{N}_1$ with specific constants on the *other* coordinate, e.g., $\exists R^- \sqsubseteq \neg C$ forbids R-atoms with C-constants on the second coordinate.

Due to this *dual-affection* (both positive and negative) of the role R in $\mathcal{T}_1$, we were able to provide an ABox $\mathcal{A}_1$ and $\mathcal{N}_1$, which together triggered the case analyses of modifications on the model $\mathcal{I}$, that is, $\mathcal{A}_1$ and $\mathcal{N}_1$ were *triggers* for R. Existence of such an affected R and triggers $\mathcal{A}_1$ and $\mathcal{N}_1$ made $\mathcal{K}_1 \diamond \mathcal{N}_1$ inexpressible in *DL-Lite*$_{\mathcal{R}}$. Therefore, we now learn how to detect dually-affected roles in TBoxes and how to understand whether these roles are triggered by an ABox and a new (ABox) information.

Formally, let $\mathcal{T}$ be a TBox, a role R is *dually-affected* in $\mathcal{T}$ if for some concepts A and B it holds that $\mathcal{T} \models A \sqsubseteq \exists R$ and $\mathcal{T} \models \exists R^- \sqsubseteq \neg B$. Let $\mathcal{N}$ be an ABox satisfiable with $\mathcal{T}$, then a dually-affected role R is *triggered* by $\mathcal{N}$ if there is a concept B such that $\mathcal{T} \models \exists R^- \sqsubseteq \neg B$ and $\mathcal{N} \models_{\mathcal{T}} B(b)$ for some constant b. The set *TR*$(\mathcal{T}, \mathcal{N})$ (or simply *TR*) is the set of all roles (dually-affected in $\mathcal{T}$) that are triggered by $\mathcal{N}$.

Description Logics DL-Lite$^{I}_{\mathcal{R}}$*.* We now show a restriction of *DL-Lite*$_{\mathcal{R}}$ for which we later present an algorithm to capture WS using prototypal set. *DL-Lite*$^{I}_{\mathcal{R}}$ (where I stands for (mutual) *independence* of roles) is a restriction of *DL-Lite*$_{\mathcal{R}}$ in which TBoxes $\mathcal{T}$ satisfy: for any two roles R and R', $\mathcal{T} \not\models \exists R \sqsubseteq \exists R'$ and $\mathcal{T} \not\models \exists R \sqsubseteq \neg\exists R'$. That is, we forbid direct role interaction (subsumption and disjointness) between role projections. Some interaction is still possible: role projections may contain the same concept. This restriction allows us to analyze evolution affecting roles independently for every role.

Components for Computation. We now introduce several notions and notations that we further use in the description of our algorithm. An *alignment* of a model I with N, denoted $Align(\mathcal{I}, \mathcal{N})$, is the interpretation:

$$Align(\mathcal{I}, \mathcal{N}) = \{f \mid f \in \mathcal{I} \text{ and } f \text{ is satisfiable with } \mathcal{N}\}.$$

An auxiliary set of atoms *AA* (*Auxiliary Atoms*) that, due to evolution, should be deleted from the original KB and have some extra condition on the first coordinate is:

$$AA(\mathcal{T}, \mathcal{A}, \mathcal{N}) = \{R(a,b) \in fcl_{\mathcal{T}}(\mathcal{A}) \mid \mathcal{T} \models A \sqsubseteq \exists R, \mathcal{A} \models_{\mathcal{T}} A(a), \mathcal{N} \models_{\mathcal{T}} \neg\exists R^-(b)\}.$$

For the set *TR* we define the set of *forbidden atoms* $\mathtt{FA}[\mathcal{T}, \mathcal{A}, \mathcal{N}](R_i)$ of the original ABox as:

$$\{D(c) \in fcl_{\mathcal{T}}(\mathcal{A}) \mid \exists R_i^-(c) \wedge D(c) \models_{\mathcal{T}} \bot, \mathcal{N} \not\models_{\mathcal{T}} D(c), \text{ and } \mathcal{N} \not\models_{\mathcal{T}} \neg D(c)\}.$$

$BP(\mathcal{K}, \mathcal{N}, \mathcal{J}_0)$

---

1. $\mathfrak{J} := \{\mathcal{J}_0\}$.
2. For each subset $\mathcal{D} = \{D_1(c_1), \dots, D_k(c_k)\} \subseteq \mathtt{FA}$ do
   for each $\mathcal{R} = (R_{i_1}, \dots, R_{i_k})$ such that $D_j(c_j) \in \mathtt{FA}(R_{i_j})$ for $j = 1, \dots, k$ do
   for each $\mathcal{B} = (A_{i_1}, \dots, A_{i_k})$ such that $A_j \in SC(R_j)$ do
   $\mathcal{J}[\mathcal{D}, \mathcal{R}, \mathcal{B}] := \left[\mathcal{J}_0 \setminus \bigcup_{i=1}^{k} root_{\mathcal{T}}(D_i(c_i))\right] \cup \bigcup_{i=1}^{k} \left[fcl_{\mathcal{T}}(R'_i(x_i, c_i)) \cup \{A_{R'_i}(x_i)\}\right]$,
   where all x_i's are different constants from $\Delta \setminus adom(\mathcal{K})$, fresh for $\mathcal{I}^{can}$.
   $\mathfrak{J} := \mathfrak{J} \cup \{\mathcal{J}[\mathcal{D}, \mathcal{R}, \mathcal{B}]\}$.
3. Return $\mathfrak{J}$.

Fig. 3. The procedure of building prototypes in *DL-Lite*$^{I}_{\mathcal{R}}$ based on the zero prototype $\mathcal{J}_0$

Consequently, the set of forbidden atoms for the entire KB $(\mathcal{T}, \mathcal{A})$ and $\mathcal{N}$ is

$$\mathtt{FA}(\mathcal{T}, \mathcal{A}, \mathcal{N}) = \cup_{R_i \in TR} \mathtt{FA}(\mathcal{T}, \mathcal{A}, \mathcal{N})(R_i).$$

In the following we omit the arguments $(\mathcal{T}, \mathcal{A}, \mathcal{N})$ whenever they are clear from the context. For a role R, the set $SC(R)$, where *SC* stands for *sub-concepts*, is a set of those concepts which are *immediately* under $\exists R$ in the concept hierarchy generated by $\mathcal{T}$:

$$SC(R) = \{A \mid \mathcal{T} \models A \sqsubseteq \exists R \text{ and there is no } A' \text{ s.t. } \mathcal{T} \models A \sqsubseteq A' \text{ and } \mathcal{T} \models A' \sqsubseteq \exists R\}.$$

If f is an ABox assertion, then $root^{at}_{\mathcal{T}}(f)$ is a set of all the atoms that $\mathcal{T}$-entail f. For example, $A(x) \in root^{at}_{\mathcal{T}}(\exists R(x))$ if $\mathcal{T} \models A \sqsubseteq \exists R$.

We are ready to proceed to construction of prototypes.

Constructing Zero-Prototype. The procedure $BZP(\mathcal{K}, \mathcal{N})$ (*Build Zero Prototype*) in Figure 2 constructs the main prototype $\mathcal{J}_0$ for $\mathcal{K}$ and $\mathcal{N}$ from *DL-Lite*$^{I}_{\mathcal{R}}$, which we call *zero-prototype*. Based on $\mathcal{J}_0$ we will construct all the other prototypes. To build $\mathcal{J}_0$ one has to align the canonical model of $\mathcal{K}$ with $\mathcal{N}$, and then delete from the resulting set of atoms all the auxiliary atoms $R(a, b)$ (from $AA(\mathcal{K}, \mathcal{N})$). In the case when *no* $R(a, \beta)$ for some constant β such that $R(a, \beta) \in AA(\mathcal{K}, \mathcal{N})$ is in the canonical model, we also delete atoms $root^{at}_{\mathcal{T}}(\exists R(a))$, since their presence in the model and the absence of R-atoms with a at the first coordinate would contradict the TBox.

Constructing Other Prototypes. The procedure $BP(\mathcal{K}, \mathcal{N}, \mathcal{J}_0)$ (*Build Prototypes*) of constructing $\mathfrak{J}$ for the case of *DL-Lite*$^{I}_{\mathcal{R}}$, takes $\mathcal{J}_0$ and manipulates with it by first dropping atoms from $\mathtt{FA}$ and then adding atoms in order to compensate the dropped ones so that the result is an evolved *model* under WS. It can be found in Figure 3

We conclude the discussion on the algorithms with a theorem:

Theorem 5. *Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a DL-Lite$^{I}_{\mathcal{R}}$ KB, and $\mathcal{N}$ a DL-Lite$_{\mathcal{R}}$ ABox consistent with $\mathcal{T}$. Then the set $BP(\mathcal{K}, \mathcal{N}, BZP(\mathcal{K}, \mathcal{N}))$ is a prototypal set for $\mathcal{K} \diamond \mathcal{N}$.*

Continuing with Example 1, it is easy to check that the prototypal set for $\mathcal{K}_1$ and $\mathcal{N}_1$ is $\{\mathcal{J}_0, \mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3\}$, where $\mathcal{J}_0$, $\mathcal{J}_1$, and $\mathcal{J}_2$ are described in the example and

$$\mathcal{J}_3: \quad A^{\mathcal{I}} = \{x, y\}, \quad C^{\mathcal{I}} = \{b\}, \quad R^{\mathcal{I}} = \{(x, d), (y, e)\}.$$

We proceed to correctness of BP in capturing evolution in *DL-Lite*$^{I}_{\mathcal{R}}$, where we use the following set $\mathtt{FC}[\mathcal{T}, \mathcal{A}, \mathcal{N}](R_i) = \{c \mid D(c) \in \mathtt{FA}[\mathcal{T}, \mathcal{A}, \mathcal{N}](R_i)\}$, that collects all the constants that participate in the forbidden atoms.

Theorem 6. *Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a DL-Lite$^{\mathcal{I}}_{\mathcal{R}}$ KB, $\mathcal{N}$ a DL-Lite$_{\mathcal{R}}$ ABox consistent with $\mathcal{T}$, and $BP(\mathcal{K}, \mathcal{N}, BZP(\mathcal{K}, \mathcal{N})) = \{\mathcal{J}_0, \ldots, \mathcal{J}_n\}$ is a prototypal set for $\mathcal{K} \diamond \mathcal{N}$. Then*

$$\mathcal{K} \diamond \mathcal{N} = Mod(\mathcal{T}) \cap Mod(\mathcal{A}_0 \vee \ldots \vee \mathcal{A}_n) \cap Mod(\Phi \wedge \Psi),$$

where A_i is a DL-Lite$_{\mathcal{R}}$ ABox such that $\mathcal{J}_i$ is a canonical model for $(\mathcal{T}, \mathcal{A}_i)$, and

$$\Phi = \bigwedge_{R_i \in TR} \bigwedge_{c_j \in \texttt{FC}[R_i]} \forall x. \left[\left(R_i(x, c_j) \rightarrow (root^{at}_{\mathcal{T}}(\exists R_i(x)) \neq \emptyset) \right) \wedge \right.$$

$$\left. \forall y. \left(R_i(x, c_j) \wedge R_i(x, y) \rightarrow y = c_j \right) \right],$$

$$\Psi = \bigwedge_{R(a,b) \in \mathcal{S}_{at}} \exists R(a) \rightarrow root^{at}_{\mathcal{T}}(\exists R(a)) \cap fcl_{\mathcal{T}}(\mathcal{A}).$$

The $\mathcal{A}_i$ mentioned in Theorem 6, can be constructed in the similar way that the corresponding prototypes $\mathcal{J}_i$, taking the original ABox $\mathcal{A}$ instead of $\mathcal{I}^{can}$. Note that an ABox may include a negative literals, like $\neg B(c)$. Those should be treated in the same way that the positive literal (atoms) are. We will denote such an ABox as $\mathcal{A}[\mathcal{J}_i]$.

Theorem 7. *A prototype $\mathcal{J}_i$ is a canonical model of the KB $(\mathcal{T}, \mathcal{A}[\mathcal{J}_i])$.*

Continuing with Example 1, the ABoxes $\mathcal{A}[\mathcal{J}_0]$ and $\mathcal{A}[\mathcal{J}_1]$ are as follows:

$$\mathcal{A}[\mathcal{J}_0] = \{C(d), C(e), C(b)\}; \quad \mathcal{A}[\mathcal{J}_1] = \{A(x), C(e), C(b), R(x, d)\}.$$

$\mathcal{A}[\mathcal{J}_2]$ and $\mathcal{A}[\mathcal{J}_3]$ can be built in the similar way. Note that only $\mathcal{A}[\mathcal{J}_0]$ is in *DL-Lite*$_{\mathcal{R}}$, while writing $\mathcal{A}[\mathcal{J}_1], \ldots, \mathcal{A}[\mathcal{J}_3]$ requires variables in ABoxes. Variables, also known as *soft constants*, are not allowed in *DL-Lite*$_{\mathcal{R}}$ ABoxes, while present in *DL-Lite*$_{\mathcal{RS}}$ ABoxes. Soft constants x are constants not constrained by the Unique Name Assumption: it is not necessary that $x^{\mathcal{I}} = x$. Since *DL-Lite*$_{\mathcal{RS}}$ is tractable and FO rewritable [13], expressing $\mathcal{A}[\mathcal{J}_1]$ in *DL-Lite*$_{\mathcal{RS}}$ instead of *DL-Lite*$_{\mathcal{R}}$ does not affect tractability.

6 Computing Winslett's Semantics with Roles Interaction

The algorithm BP for constructing prototypal set works only when roles do not interact. The following example illustrates that it does not work in a general case.

Example 8. Consider a KB $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ and a new ABox $\mathcal{N}_2 = \{C(b)\}$:
TBox $\mathcal{T}_2$: $\exists R^- \sqsubseteq \neg \exists P^-$, $\exists R^- \sqsubseteq \neg C$, $A \sqsubseteq \exists R$, $B \sqsubseteq \exists P$;
ABox $\mathcal{A}_2$: $R(a, b)$, $A(a)$, $R(f, g)$, $A(f)$, $P(c, d)$, $B(c)$, $C(e)$.
One can check that the following model $\mathcal{J}'$ is in $\mathcal{K}_2 \diamond \mathcal{N}_2$:

$$A^{\mathcal{J}'} = \{y\}, \quad B^{\mathcal{J}'} = \{z\}, \quad C^{\mathcal{J}'} = \{b, e\}, \quad R^{\mathcal{J}'} = \{(y, d)\}, \quad P^{\mathcal{J}'} = \{(z, g)\}.$$

At the same time, BP over $\mathcal{K}_2$ and $\mathcal{N}_2$ returns the following four prototypes only:

	$A^{\mathcal{J}_i}$	$B^{\mathcal{J}_i}$	$C^{\mathcal{J}_i}$	$R^{\mathcal{J}_i}$	$P^{\mathcal{J}_i}$
$i = 0$	$\{f\}$	$\{c\}$	$\{b, e\}$	$\{(f, g)\}$	$\{(c, d)\}$
$i = 1$	$\{f, x\}$	$\{c\}$	$\{b\}$	$\{(f, g), (x, e)\}$	$\{(c, d)\}$
$i = 2$	$\{f, y\}$	$\emptyset$	$\{b, e\}$	$\{(f, g), (y, d)\}$	$\emptyset$
$i = 3$	$\{f, x, y\}$	$\emptyset$	$\{b\}$	$\{(f, g), (x, e), (y, d)\}$	$\emptyset$

where x and y are fresh constants. It is easy to see that none of $\mathcal{J}_i$s is homomorphically embeddable in $\mathcal{J}'$. Thus, BP does not capture $\mathcal{J}'$ and it is incomplete. ■

$BP_{rec}(\mathcal{K},\mathcal{N})$

1. Compute $\mathfrak{J} := BP(\mathcal{K},\mathcal{N}, BZP(\mathcal{K},\mathcal{N}))$.
2. Repeat
   $\mathfrak{J}' := \mathfrak{J}$;
   for each $\mathcal{J} \in \mathfrak{J}'$ do $\mathfrak{J} := \mathfrak{J} \cup BP(\mathcal{K},\mathcal{N} \cup S, \mathcal{J})$,
   where $S = \{f \in \mathcal{J} \mid$ a fresh constant from $\Delta \setminus$ *adom*$(\mathcal{K})$ appears in $f\}$;
   until $\mathfrak{J} = \mathfrak{J}'$.
3. Return $\mathfrak{J}$.

Fig. 4. The procedure of building prototypes in *DL-Lite*$_\mathcal{R}$

6.1 Recursive BP Algorithm.

For general *DL-Lite*$_\mathcal{R}$ KBs, BP algorithm does return prototypes but not all of them. The reason is: when, while constructing prototypes with BP, we delete a forbidden atom (an atom from FA), it may trigger another dually-affected role and such triggering may require further modifications, which are not accounted by BP. In order to compute all prototypes we should run BP recursively: considering the prototypes obtained at the previous step as zero ones. We present a recursive algorithm BP_{rec} for building prototypes for general *DL-Lite*$_\mathcal{R}$ KBs in Figure 4. The following theorem shows the correctness of the algorithm.

Theorem 9. *Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a DL-Lite$_\mathcal{R}$ KB and $\mathcal{N}$ a DL-Lite$_\mathcal{R}$ ABox consistent with $\mathcal{T}$. Then the algorithm $BP_{rec}(\mathcal{K},\mathcal{N})$ terminates and returns the finite set which is a prototypal set for $\mathcal{K} \diamond \mathcal{N}$.*

We illustrate BP_{rec} on the following example.

Example 10. Consider KB $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ and a new ABox $\mathcal{N}_2$ from Example 8. Let us compute $BP_{rec}(\mathcal{K}_2,\mathcal{N}_2)$. First we run $BP(\mathcal{K},\mathcal{N},\mathcal{J}_0)$ and it returns four prototypes: $\mathcal{J}_0$, $\mathcal{J}_1$, $\mathcal{J}_2$, and $\mathcal{J}_3$ (see Example 8). Now we apply the BP procedure to $\mathcal{J}_1$, $\mathcal{J}_2$, and $\mathcal{J}_3$. It is easy to see that $BP(\mathcal{K},\mathcal{N} \cup \{A(x), R(x,e)\}, \mathcal{J}_1) = \emptyset$, since no role atom except for $R(a,b)$ was affected. Consider $BP(\mathcal{K},\mathcal{N} \cup \{A(y), R(y,d)\}, \mathcal{J}_2)$: it consists of the only prototype $\mathcal{J}_4$:

$$A^{\mathcal{J}_4} = \{y\}, \quad B^{\mathcal{J}_4} = \{z\}, \quad C^{\mathcal{J}_4} = \{b, e\}, \quad R^{\mathcal{J}_4} = \{(y,d)\}, \quad P^{\mathcal{J}_4} = \{(z,g)\}.$$

The uniqueness of the prototype follows from the fact that the role atom that was affected in $\mathcal{J}_2$ is $P(c,d)$ and $\texttt{FA}[\mathcal{T},\mathcal{A},\mathcal{N} \cup \{A(y), R(y,d)\}](P) = \{\exists R^-(g)\}$. Finally, running $BP(\mathcal{T},\mathcal{N} \cup \{A(y), R(y,d), B(z), P(z,g)\}, \mathcal{J}_4)$ we obtain a prototype $\mathcal{J}_5$:

$$A^{\mathcal{J}_5} = \{y, v\},\ B^{\mathcal{J}_5} = \{z\},\ C^{\mathcal{J}_5} = \{b\},\ R^{\mathcal{J}_5} = \{(y,d),(v,e)\},\ P^{\mathcal{J}_5} = \{(z,g)\}.$$

Note that $BP(\mathcal{T},\mathcal{N} \cup \{A(y), R(y,d), B(z), P(z,g), A(v), R(v,e)\}, \mathcal{J}_5) = \emptyset$. Analogously, $\mathcal{J}_6$ can be obtained by running $BP(\mathcal{K},\mathcal{N} \cup \{A(x), A(y), R(x,e), R(y,d)\}, \mathcal{J}_3)$:

$$A^{\mathcal{J}_6} = \{x, y\},\ B^{\mathcal{J}_6} = \{z\},\ C^{\mathcal{J}_6} = \{b\},\ R^{\mathcal{J}_6} = \{(x,e),(y,d)\},\ P^{\mathcal{J}_6} = \{(z,g)\}.$$

Thus, the prototypal set $\mathfrak{J}$ for $\mathcal{K} \diamond \mathcal{N}$ is $\{\mathcal{J}_i\}_{i=0}^{6}$. ■

We conclude with the theorem that BP_{rec} gives a sound approximation for WS.

Theorem 11. *Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a DL-Lite$_\mathcal{R}$ KB, $\mathcal{N}$ a DL-Lite$_\mathcal{R}$ ABox consistent with $\mathcal{T}$, and $BP_{rec}(\mathcal{K}, \mathcal{N}) = \{\mathcal{J}_0, \dots, \mathcal{J}_n\}$ is a prototypal set for $\mathcal{K} \diamond \mathcal{N}$. Then*

$$\mathcal{K} \diamond \mathcal{N} \subseteq \mathit{Mod}(\mathcal{T}) \cap \mathit{Mod}(\mathcal{A}_0 \vee \dots \vee \mathcal{A}_n) \cap \mathit{Mod}(\Phi \wedge \Psi),$$

where A_i is a DL-Lite$_\mathcal{R}$ ABox such that $\mathcal{J}_i$ is a canonical model for $(\mathcal{T}, \mathcal{A}_i)$ and Φ and Ψ are as they defined in Theorem 6.

6.2 Closure Under Evolution and Approximation

Next theorem allows us to approximate results of evolution under WS, since $\mathsf{FO}[2]$ is decidable.

Theorem 12. *$\mathcal{K} \diamond \mathcal{N}$ under WS for KBs in DL-Lite$_\mathcal{R}$ can be captured in* $\mathsf{FO}[2]$.

As a future work we are going to study ways to approximate the resulted $\mathsf{FO}[2]$ theories in *DL-Lite*.

Finally, we discuss cases when the result of Winslett's evolution is expressible in *DL-Lite*$_\mathcal{R}$. The following formulas appearing in Theorem 6 are not expressible in *DL-Lite*$_\mathcal{R}$: *(i)* the disjunction of the ABoxes $\mathcal{A}_0 \vee \dots \vee \mathcal{A}_n$ and *(ii)* formula $\Phi \wedge \Psi$. The disjunction of ABoxes becomes expressible when it is of the length one, i.e., there is the only prototype: $\mathcal{J}_0$. The last statement yields that $\mathtt{FC} = \emptyset$ and therefore Φ is always true. The formula Ψ becomes trivially true when $\mathit{AA} = \emptyset$, i.e., for every atom $R(a, b) \in \mathit{fcl}_\mathcal{T}(\mathcal{A})$ either $\mathcal{N} \not\models_\mathcal{T} \neg\exists R^-(b)$ or $\mathit{root}^{at}_\mathcal{T}(\exists R_i(a_i)) \cap \mathit{fcl}_\mathcal{T}(\mathcal{A}) = \emptyset$. As one can see, the condition of expressibility of the result in *DL-Lite*$_\mathcal{R}$ (emptiness of $\mathtt{FA}$ and *AA*), depends on a TBox, an ABox, and a new information. Hence, if we do a chain of evolution, at some step the result may be not expressible in *DL-Lite*$_\mathcal{R}$. Since TBox stays unchangeable, to guarantee the expressibility we need to find TBoxes $\mathcal{T}$ such that $(\mathcal{T}, \mathcal{A}) \diamond \mathcal{N}$ is expressible in *DL-Lite*$_\mathcal{R}$ for every $\mathcal{A}$ and $\mathcal{N}$. A condition that guarantees the emptiness of $\mathtt{FA}$ and *AA* is: for every role $R \in \Sigma(K \cup \mathcal{N})$ at least one of the following items holds: *(1)* there is no concept C such that $\mathcal{T} \models \exists R^- \sqsubseteq \neg C$, or *(2)* there is no concept A such that $\mathcal{T} \models A \sqsubseteq \exists R$. The former conditions gives that $\mathit{TR} = \emptyset$ since $\mathcal{N} \not\models_\mathcal{T} \neg\exists R^-(b)$, which leads to $\mathtt{FA} = \mathit{AA} = \emptyset$. The latter one yields that $\mathit{SC}(R) = \emptyset$, therefore *TR* again is empty.

As a practical summary of this section, given a KB $\mathcal{K}$ and a new ABox $\mathcal{N}$, one can check (in polynomial time) whether any dually-affected role is "triggered" by $\mathcal{N}$. If it is *not* the case, one can compute (in polynomial time) an evolved KB $\mathcal{K}'$ that exactly captures $\mathcal{K} \diamond \mathcal{N}$. Otherwise, it is the case that $\mathcal{K} \diamond \mathcal{N}$ is inexpressible in *DL-Lite*$_\mathcal{R}$. Thus, one can compute an $\mathsf{FO}[2]$ theory that captures $\mathcal{K} \diamond \mathcal{N}$ and then approximate it in *DL-Lite*$_\mathcal{R}$, by, for example, dropping all the not *DL-Lite*$_\mathcal{R}$ formulas. We will not focus on approximation in this paper.

7 Conclusion

We studied how to capture ABox evolution for *DL-Lite*$_\mathcal{R}$ under WS. In general the result of evolution requires constructs that are not present in *DL-Lite*$_\mathcal{R}$, and even not in *DL-Lite*, such as disjunction. Moreover, in general the result of evolution, which is a set of models, does not even have a canonical model, which should always exist for any *DL-Lite* theory. It turned out that the inexpressibility is caused by a condition on the TBox level, which we called dual-interaction: by pairs of assertions of the form $A \sqsubseteq \exists R$ and $\exists R^- \sqsubseteq \neg B$. In order to capture evolution results in the presence of dual-interactions, we introduced prototypes. Our approach is based on the observation that evolution results can be divided into a finite number of subsets and each of them has a canonical model, i.e., a prototype. These subsets can be captured by theories guided by prototypes and the disjunction of these theories, compensated with two formulas, captures evolution results and is in $\mathsf{FO}[2]$. We proved that this technique works for *DL-Lite*$_\mathcal{R}$. We are currently working on efficient approximation of the obtained $\mathsf{FO}[2]$ theory in *DL-Lite* and on extending results to capture evolution for other *DL-Lite* languages.

Acknowledgements We are thankful to Diego Calvanese and Werner Nutt for insightful discussions. The authors are supported by EU projects ACSI (FP7-ICT-257593), Ontorule (FP7-ICT-231875). The first author was supported by the ERC FP7 grant Webdam (agreement n. 226513). Part of this work was carried out while the first author was visiting Department of Computer Science of Oxford University.

References

1. Baader, F., Calvanese, D., McGuinness, D., Nardi, D., Patel-Schneider, P.F., eds.: The Description Logic Handbook. Cambridge University Press (2003)
2. Flouris, G., Manakanatas, D., Kondylakis, H., Plexousakis, D., Antoniou, G.: Ontology change: Classification and survey. Knowledge Engineering Review **23**(2) (2008) 117–152
3. Abiteboul, S., Grahne, G.: Update semantics for incomplete databases. (VLDB-85)
4. Katsuno, H., Mendelzon, A.: On the difference between updating a knowledge base and revising it. (KR-91)
5. Eiter, T., Gottlob, G.: On the complexity of propositional knowledge base revision, updates and counterfactuals. Artificial Intelligence **57** (1992) 227–270
6. Winslett, M.: Updating Logical Databases. Cambridge University Press (1990)
7. Liu, H., Lutz, C., Milicic, M., Wolter, F.: Updating description logic ABoxes. (KR-06)
8. Calvanese, D., Giacomo, G.D., Lembo, D., Lenzerini, M., Rosati, R.: Tractable reasoning and efficient query answering in description logics: The *DL-Lite* family. J. of Automated Reasoning **39**(3) (2007) 385–429
9. Giacomo, G.D., Lenzerini, M., Poggi, A., Rosati, R.: On instance-level update and erasure in description logic ontologies. J. of Logic and Computation, Special Issue on Ontology Dynamics **19**(5) (2009) 745–770
10. Calvanese, D., Kharlamov, E., Nutt, W., Zheleznyakov, D.: Evolution of DL-Lite knowledge bases. In: International Semantic Web Conference (1). (2010) 112–128
11. Calvanese, D., Kharlamov, E., Nutt, W., Zheleznyakov, D.: Evolution of DL-Lite knowledge bases (extended version). Technical Report KRDB11-3, KRDB Research Centre (2011)
12. Kharlamov, E., Zheleznyakov, D.: Understanding inexpressibility of model-based ABox evolution in DL-Lite. (AMW-11)
13. Artale, A., Calvanese, D., Kontchakov, R., Zakharyaschev, M.: The DL-Lite family and relations. J. Artif. Intell. Res. (JAIR) **36** (2009) 1–69
14. Artale, A., Calvanese, D., Kontchakov, R., Zakharyaschev, M.: The *DL-Lite* family and relations. J. of Artificial Intelligence Research **36** (2009) 1–69
15. Poggi, A., Lembo, D., C., D., Giacomo, G.D., Lenzerini, M., Rosati, R.: Linking data to ontologies. J. on Data Semantics (2008) 133–173