On the Cognitive Efficacy of Euler Diagrams in Syllogistic Reasoning: A Relational Perspective

Koji Mineshima[1], Yuri Sato[1], Ryo Takemura[2], and Mitsuhiro Okada[1]

[1] Department of Philosophy, Keio University
2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan.
{minesima,sato,mitsu }@abelard.flet.keio.ac.jp
[2] College of Commerce, Nihon University
5-2-1 Kinuta, Setagaya-ku, Tokyo 157-8570, Japan.
takemura.ryo@nihon-u.ac.jp

Abstract. Although logic diagrams are widely used as methods for introducing students to elementary logical reasoning, it is still open to debate in cognitive psychology whether diagrams can aid untrained people to successfully conduct deductive reasoning. In our previous work, some empirical evidence was provided for the effectiveness of a certain type of logic diagrams in the process of solving categorical syllogisms. However, the question of why certain diagrams but not others have such inferential efficacy in performing syllogism reasoning has not been fully answered. Based on a proof-theoretical analysis of categorical syllogisms and diagrammatic reasoning, we supplement our previous study of cognitive efficacy of diagrams and argue that the relational information underlying quantified sentences plays a crucial role in understanding the efficacy of diagrams in syllogistic reasoning. The distinctive features of our conception of diagrammatic reasoning are made clear by comparing it with the model-theoretic conception of ordinary reasoning developed in the mental model theory.

1 Introduction

In logic teaching, Venn and Euler diagrams have been widely used as tools for introducing students to elementary logical reasoning, including set-theoretical and syllogistic reasoning.[3] However, in the literature of cognitive psychology of reasoning, it is still open to debate whether external diagrams can aid logically untrained people to conduct deductive reasoning in a successful way (see Scaife & Rogers [30] for an overview of the work on external representations in cognitive science). Indeed, it is often claimed that diagrams can only serve as an auxiliary source of information in deductive problem solving. Thus, Larkin and Simon [14], in a seminal work on the efficacy of diagrammatic representations in problem solving in general, argued that reasoning is largely independent of ways of representing information, and hence, that diagrams are less beneficial in reasoning than in such tasks as searching and recognition. Additionally, previous studies reported empirical evidence for negative effects of traditional Euler diagrams on the performance of syllogistic reasoning (Calvillo, Deleeuw, & Revlin [4];

[3] In fact, Leonhard Euler [7] introduced his diagrams to teach Aristotelian syllogistic logic to a German princess.

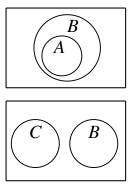
3rd International Workshop on Euler Diagrams, July 2, 2012, Canterbury, UK.

Rizzo & Palmonari [25]). Furthermore, although various systems of logic diagrams have been proposed and studied using the methods of mathematical logic (e.g. Shin [31]; Hammer [11]; for a survey, see Stapleton [34]), little attention has been paid to the question of how effective such diagrammatic systems are in people's actual reasoning.[4]

To improve this situation, we have studied how logic diagrams can support actual deductive reasoning, focusing on the case of syllogistic reasoning supported by Euler and Venn diagrams that are externally given to reasoners (Sato, Mineshima & Takemura [26, 27]).[5] Typical examples of reasoning tasks that we examined are shown in Figs.1 and 2.

All *A* are *B*.

No *C* are *B*.

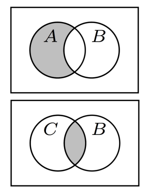


(Therefore, No *C* are *A*)

Fig.1 An example of a syllogistic reasoning task with Euler diagrams

All *A* are *B*.

No *C* are *B*.



(Therefore, No *C* are *A*)

Fig.2 An example of a syllogistic reasoning task with Venn diagrams

Euler diagrams represent set relationships in terms of inclusion and exclusion relations between circles (see the diagrams in Fig.1). By contrast, Venn diagrams have a fixed configuration of circles and represent set relationships by stipulating that shaded regions denote the empty set (see the diagrams in Fig.2). In the experiments of Sato et al. [26], subjects were divided into three groups, called the Euler group, Venn group, and Linguistic group. The Euler group and Venn group were first provided with instructions on the meanings of diagrams. A pretest was conducted to check whether the subjects understood the instructions correctly. The Euler group was then asked to solve syllogistic reasoning tasks in which subjects were presented with two sentential premises together with two corresponding Euler diagrams, as in Fig. 1, and asked to choose a valid conclusion. Similarly, the Venn group was asked to solve tasks like the one in Fig. 2. The Linguistic group was presented only with sentential premises and required to choose a valid conclusion without any aid from diagrams. The results showed that (1) the performance of the Euler and Venn groups was significantly better than that of the

[4] A notable exception is important work on *hyperproof* by Stenning, Cox, and Oberlander [36], where the effects of teaching elementary logic classes using Hyperproof methods, i.e., multimodal graphical and sentential methods, and a standard syntactic teaching method are compared.

[5] Although traditional syllogisms are less expressive than standard first-order logic, they are one of the most basic form of natural language inferences and still important for investigating human reasoning. Indeed, syllogistic logics, considered as alternative logical systems to standard first-order logic, have recently attracted increasing attention from logical and linguistic points of view, for example, in the study of decidable fragments of first-order logic; see Moss [21] and references given there.

Linguistic group, and (2) the performance of the Euler group was significantly better than that of the Venn group.

Sato et al. [26, 27] argue that the differences in performance between the three groups can be explained on the basis of the distinction between two kinds of efficacy, namely, interpretational and inferential efficacy. By interpretational efficacy we mean the effects of diagrams on determining the correct interpretation of a sentence. For example, sentence *All B are A* tends to be interpreted as equivalent to *All A are B*.[6] Diagrams can contribute to avoiding deductive reasoning errors due to such unintended interpretations of linguistic materials. For example, those who are presented with diagrams representing *All B are A* as shown in Figs. 1 and 2 can immediately see that the semantic information delivered is not equivalent to *All A are B* by virtue of their form. Diagrams can also can play a crucial role in reasoning processes. We refer to the efficacy of diagrams in reasoning processes themselves as "inferential efficacy". More specifically, when diagrams of a certain form are externally given, the process of solving deductive reasoning tasks could be replaced with the syntactic manipulation of diagrams.

It should be noted that in the experimental set-up of Sato et al. [26], subjects in the Euler and Venn groups were given instructions on the meaning of diagrams, while subjects in the Linguistic group were not. Then one might argue that the difference in training could have had a major effect on differences in performance between the Euler and Venn groups, on the one hand, and the Linguistic group, on the other. However, such an objection can be avoided if a comparison is made between the Euler group and the Venn group. The latter was also given substantial instructions and practice trials, yet the result showed that the performance of the Euler group was significantly better than that of the Venn group.

The hypothesis explored in Sato et al. [26] was that the difference in performance between the three groups can be explained by assuming that Venn diagrams only have interpretational efficacy, while Euler diagrams have both interpretational and inferential efficacy.[7] That is, Euler diagrams not only contribute to the correct interpretations of categorical sentences but also play a substantial role in the inferential processes of solving syllogisms. To substantiate this claim, Sato et al. [27] outlined a cognitive model of syllogistic inferences that are externally supported by diagrams, assuming that both (categorical) sentences and diagrams conventionally express semantic information, and furthermore, that diagrams are syntactic objects to be manipulated in reasoning processes. In Sato et al. [27], however, the semantic and syntactic (proof-theoretical) analyses of categorical sentences and diagrams were left unspecified. Consequently, the question of why certain logic diagrams, in particular, Euler diagrams, have inferential efficacy in syllogism solving has not been fully answered. The rest of the present paper is devoted to addressing this question. Building on the proof-theoretical study of categorical syllogisms and Euler diagrams in Mineshima, Okada and Takemura [19, 20], we

[6] This is known as "illicit conversion error" in the literature; see, e.g. Newstead & Griggs [22].

[7] Gurr [10] emphasizes that in addition to the process of combining information, the process of extracting information from a diagram plays a role in diagrammatic reasoning. Sato, Mineshima, and Takemura [28] examines differences in the cognitive process of extracting information between Euler and Venn diagrams in some details.

will analyze both syllogistic and diagrammatic inferences from a unified perspective, which we call a *relational* perspective. Thus, the aim of this paper is to make a connection between the logical study of syllogisms and diagrams in Mineshima et al. [19, 20] and the cognitive study of diagrams in Sato et al. [26, 27], and thereby to provide a model of reasoning in which the experimental results of Sato et al. [26, 27] can be explained in a natural way. The key assumption is that both syllogistic and diagrammatic inferences are decomposed as inferences with two primitive relations, i.e., *inclusion* and *exclusion*. We claim that the efficacy of Euler diagrams in syllogistic reasoning derives from the fact that they are effective ways of representing and reasoning about relational structures that are implicit in categorical (quantified) sentences.

The formal study of logic diagrams in Mineshima, Okada and Takemura [20] also sheds light on the question of how diagrams can contribute to judging that a given inference is *invalid* in actual reasoning. It has been noticed in cognitive psychology of reasoning that falsification tasks, including tasks that require a reasoner to judge that there is no valid conclusion drawable from a given set of premises, are often difficult for untrained people when inference materials are only presented in linguistic (sentential) form. Interestingly, the experimental results in Sato et al. [26] showed that Euler diagrams were particularly effective in supporting such falsification tasks of syllogistic reasoning. We argue that the efficacy of Euler diagrams in falsification tasks is partly explained by assuming that when such diagrams are externally given, the information that there is no valid conclusion drawable from the premise diagrams can be obtained in a direct way, specifically, by combining premise diagrams and extracting the relevant relational information. This way of understanding diagrammatic reasoning can be made clear by comparing it with model-based inferences such as those studied in the mental model theory (e.g., Johnson-Laird & Byrne [13]), where the process of constructing a particular model plays a crucial role in checking the validity and invalidity of an inference. By taking a closer look at the difference between the two conceptions of inferences, we will point out that in reasoning with Euler diagrams, constraints on unification processes of diagrams play an important role; furthermore, both processes of proving and refuting a conclusion can be realized as a uniform process of syntactic manipulation of diagrams.

This paper is structured as follows. In Section 2, we provide a preliminary background on a relational analysis of categorical syllogisms, originally provided in Mineshima, Okada and Takemura [20]. In Section 3, we turn to the relational analysis of Euler diagrams. In Section 4, our model of diagrammatic reasoning is compared with that of the mental model theory. Finally, in Section 5, we give a summary of the discussion.

2 Background: Categorical syllogisms as relational inferences

Categorical syllogisms are inferences concerned with *quantificational* sentences in natural languages. According to the traditional analysis in logic textbooks, such quantificational sentences are analyzed as formulas in first-order logic, i.e., formulas involving quantification over individuals. Thus, *All A are B* is analyzed as $\forall x(Ax \to Bx)$ and *Some A is B* as $\exists x(Ax \land Bx)$, and so on. By contrast, according to the theory of *generalized* *quantifiers* (see Barwise & Cooper [2]), which is dominant in the field of natural language semantics, quantificational expressions such as *every*, *some*, and *no* are analyzed as denoting *relations* between sets. Thus, a universal sentence of the form *All A are B* is semantically analyzed as expressing that $\mathbf{A} \subseteq \mathbf{B}$, where the determiner *all* corresponds to the subset relation. Similarly, *No A are B* is analyzed as expressing that $\mathbf{A} \cap \mathbf{B} = \emptyset$, where the determiner *no* corresponds to the disjointness relation.[8] Proof systems for such a relational semantics of quantificational sentences have been investigated in the modern reconstructions of Aristotelian syllogisms (cf. Łukasiewicz [16]; Corcoran [6]; Smiley [33]) and in the recent development of natural logic (cf. Moss [21]). In these studies, the relational structure of a quantified sentence is taken as a primitive logical form; as a result, syllogistic inferences are formalized as a certain kind of *relational* inference without reference to first-order quantifiers and individual terms.

Mineshima, Okada, and Takemura [20] present a simple proof system based on two primitive relations, i.e., inclusion $\sqsubset$ and exclusion $\vdash\!\dashv$ for syllogistic inferences.[9] The system is called a *generalized syllogistic inference system* and abbreviated as GS. The inference system of GS is simple but expressive enough to represent categorical syllogisms in a perspicuous way. In the rest of this section, we provide a brief overview of the syntax of GS and then see how to formalize categorical syllogisms using inclusion and exclusion relations of GS.

The language of GS is defined as follows. Terms of GS (denoted by $X, Y, Z, \ldots$) are divided into singular terms (denoted by $a, b, c, \ldots$) that correspond to proper names like Socrates, and general terms (denoted by $A, B, C, \ldots$) that correspond to common nouns like philosopher. An atomic formula (denoted by $P, Q, \ldots$) is of the form $X \sqsubset Y$ or $X \vdash\!\dashv Y$, where X and Y are terms. A complex formula (denoted by $\mathcal{P}, \mathcal{Q}, \ldots$) is defined as a set of atomic formulas, $\{P_1, \ldots, P_n\}$. Intuitively, $\{P_1, \ldots, P_n\}$ means the conjunction of atomic formulas, $P_1 \wedge \cdots \wedge P_n$. To simplify the notation, we usually omit the brackets.

The proof system of GS is shown in Fig. 3. A proof in GS has a tree form; it starts with formulas of GS or Axioms (*ax*) and proceeds by one of the inference rules in Fig. 3.[10] As we will see below, the crucial rules for representing categorical syllogisms are the ($\sqsubset$) and ($\vdash\!\dashv$) rules. A set-theoretical semantics of GS can be given in a natural way, but to conserve space we omit it here. See Mineshima, Okada & Takemura [19, 20], where soundness and completeness are established.

Now we turn to categorical syllogisms. A categorial sentence has one of the following forms: *All A are B*, *No A are B*, *Some A are B*, and *Some A are not B*, where A and B are distinct general terms. We assume that readers are familiar with what count as valid inferences in categorical syllogisms.[11] A translation $(\cdot)^\circ$ from a categorical sentence

[8] A strong argument against the traditional first-order analysis of natural language quantifiers comes from the fact that proportional quantifiers such as *most* and *half of* cannot be properly represented in first-order logic. See Barwise & Cooper [2].

[9] The notation of $\vdash\!\dashv$ is due to Gergonne [9], where symbols for some binary relations (the so-called "Gergonne relations") were introduced for the purpose of the abstract representation of Euler diagrams.

[10] The (C) rule allows us to infer $a \sqsubset A$ ("a is A") from $A \sqsubset a$ ("Only a is A") and $a \sqsubset b$ ("a is b") from $b \sqsubset a$ ("b is a").

[11] See Mineshima et al. [20] for discussion on so-called existential import.

Axiom (ax): $X \sqsubset X$.
Inference rules:

$$\frac{X \sqsubset Y \quad Y \sqsubset Z}{X \sqsubset Z}\,(\sqsubset) \qquad \frac{X \sqsubset Y \quad Y \vdash\!\dashv Z}{X \vdash\!\dashv Z}\,(\vdash\!\dashv) \qquad \frac{X \sqsubset a}{a \sqsubset X}\,(\mathsf{C})$$

$$\frac{\mathcal{P} \quad \mathcal{Q}}{\mathcal{P} \cup \mathcal{Q}}\,(+) \qquad \frac{\mathcal{P}}{\mathcal{Q}}\,(-)$$

where in $(+)$, $\mathcal{P} \neq \mathcal{Q}$, and in $(-)$, $\mathcal{Q}$ is a proper subset of $\mathcal{P}$.

Fig.3 Proof system of GS

into a GS-formula is defined as follows.

$$\begin{aligned} (\textit{All A are B})^\circ &= A \sqsubset B \\ (\textit{No A are B})^\circ &= A \vdash\!\dashv B \\ (\textit{Some A are B})^\circ &= \{c \sqsubset A, c \sqsubset B\} \text{ for some } c \\ (\textit{Some A are not B})^\circ &= \{d \sqsubset A, d \vdash\!\dashv B\} \text{ for some } d \end{aligned}$$

where c and d are arbitrarily chosen singular terms. The crucial point is that in GS existential sentences are *decomposed* in terms of inclusion and exclusion.[12] Given this translation, all the valid inferences in categorical syllogism can be transformed into the proofs in GS. Let us look at some typical examples. To begin with, syllogisms Barbara (*All A are B*, *All B are C*. Therefore, *No A are C*) and Celarent (*All A are B*, *No B are C*. Therefore, *No A are C*) correspond to the $(\sqsubset)$ and $(\vdash\!\dashv)$ rules, respectively.

$$\dfrac{\substack{\text{All } A \text{ are } B \\ A \sqsubset B} \qquad \substack{\text{All } B \text{ are } C \\ B \sqsubset C}}{\substack{A \sqsubset C \\ \text{All } A \text{ are } C}}\,(\sqsubset) \qquad \dfrac{\substack{\text{All } A \text{ are } B \\ A \sqsubset B} \qquad \substack{\text{No } B \text{ are } C \\ B \vdash\!\dashv C}}{\substack{A \vdash\!\dashv C \\ \text{No } A \text{ are } C}}\,(\vdash\!\dashv)$$

Here, to make clear the translation between categorical sentences and formulas of GS, we attach a categorical sentence with each assumption and conclusion. As a case involving an existential sentence, consider a syllogism Darii (*Some A are B*, *All B are C*. Therefore, *Some A are C*). This inference is simulated in GS as follows:

$$\dfrac{\dfrac{\substack{\text{Some } A \text{ are } B \\ a \sqsubset A, a \sqsubset B}}{a \sqsubset A}\,(-) \qquad \dfrac{\dfrac{\substack{\text{Some } A \text{ are } B \\ a \sqsubset A, a \sqsubset B}}{a \sqsubset B}\,(-) \qquad \substack{\text{All } B \text{ are } C \\ B \sqsubset C}}{a \sqsubset C}\,(\sqsubset)}{\substack{a \sqsubset A, a \sqsubset C \\ \text{Some } A \text{ are } C}}\,(+)$$

As stated above, the formula "$a \sqsubset A, a \sqsubset B$" means the *conjunction* of $a \sqsubset A$ and $a \sqsubset B$. Hence, the $(+)$ and $(-)$ rules can be understood as corresponding to introduction and

12 This translation is similar to Aristotle's alternative way of formulating categorical syllogisms, known as *ecthesis*. See Łukasiewicz [16] for a modern reformulation of *ecthesis*.

elimination rules of conjunction in standard natural deduction systems (i.e., the rule which allows to infer $P \wedge Q$ from P and Q and the rule which allows to infer P as well as Q from $P \wedge Q$). By decomposing existential sentences in terms of inclusion and exclusion, we can represent syllogisms like Darii without using some additional rules specific to existential sentences; if we take existential sentences as primitive formulas or define them from other formulas using sentential negation (e.g. *some A are not B* is defined as *not (all A are B)*), we will need such additional axioms or inference rules. [13] It turns out that all the valid categorical syllogisms (with and without existential import) can be simulated in GS; more specifically, they can be proved using the inference rules $(\sqsubset)$, $(\vdash\!\dashv)$, $(+)$, and $(-)$ only.[14]

If the relational information encoded by categorical sentences was transparent to untrained reasoners, it would be much easier for them to solve categorical syllogisms. However, the cognitive psychological studies of deductive reasoning accumulated so far showed that this is not the case (see Sato et al. [26] and references given there). For example, the fact that logically untrained people often interpret *All A are B* as equivalent to *All B are A* indicates that the relational information $A \sqsubset B$ is not directly available to them. Similarly, the observed difficulties in solving categorical syllogisms involving existential sentences (cf. Evans, Newstead & Byrne [8]) suggest that there is a certain gap between ordinary ways of performing existential inferences and relationally decomposed processes as indicated above.

3 Solving categorical syllogisms using diagrams

As mentioned in Section 1, there are two aspects in which diagrams can externally support ordinary reasoning. Given the relational analysis of categorical syllogisms in the last section, we can summarize the effectiveness of Euler diagrams in syllogistic reasoning as follows.

1. *Interpretation.* Euler diagrams that are externally given to reasoners make explicit the *relational* information contained in categorical sentences.
2. *Inference.* Then the process of combining premise information to draw a valid conclusion can be replaced by the process of manipulating diagrammatic objects and extract the relevant relational information.

In what follows, we will concentrate on the inferential aspect in (ii). We start by explaining the representation system of Euler diagrams used in the experiment of Sato et al. [26, 27], called the EUL system. The formal properties of this system are studied in Mineshima et al. [19]. The exposition in this section is informal. More technical material as well as a detailed discussion on the motivation behind the relational approach to formalizing Euler diagrams can be found in Mineshima et al. [19].

---

[13] See Łukasiewicz [16], Corcoran [6], Smiley [33], and Moss [21] for such proposals.

[14] For a proof, see Mineshima et al. [20]. Conversely, all the proofs in GS that have syllogistic formulas in premises and conclusion can be simulated in categorical syllogism. This means that categorical syllogisms are *faithfully* embeddable into GS. In other words, although GS is more expressive than categorical syllogism, the syllogistic fragment of GS proves all and only the valid inferences in categorical syllogism.

The EUL system is a simple representation system; diagrams are composed only of circles and points and no syntactic device to express negation, such as "shading" in Venn diagrams, is introduced. Following traditional Euler diagrams, the EUL system represents quantificational sentences in terms of the spatial relationships between circles, in particular, inclusion and exclusion relations (see Fig. 1 in Section 1). In what follows, we refer to diagrams in the EUL system simply as Euler diagrams.

In the EUL representation system, an Euler diagram D is abstractly defined as a set of relations holding between objects in D. Based on this idea, a proof system for Euler diagrams, called GDS, is developed in Mineshima et al. [19]. An alternative, standard approach to formalization of diagrams is a "region-based" approach, where diagrams are defined as a set of regions (e.g. Shin [31]; Hammer [11]).[15] In our approach, there are three kinds of relations to be distinguished:

(i) a circle or a point X is located inside a circle A, symbolically written as $X \sqsubset A$;
(ii) a circle or a point X is located outside a circle A, written as $X \vdash\!\dashv A$;
(iii) a circle A and a circle B partially overlap each other, written as $A \bowtie B$.

In this symbolic notation, we use the same binary symbols as in GS for the relations in (i) and (ii). Indeed, the abstract representations of diagrams can be naturally translated into formulas of GS.

A deductive reasoning task generally requires us to combine the information contained in premise sentences. Given a correspondence between Euler diagrams and categorical sentences, such a process of combining the premise information can naturally trigger the process of unifying premise diagrams and extracting the relational information. We will explain, by some typical examples, how our Euler diagrams can be used in representing and reasoning about categorical sentences.

First, consider the case of the syllogism of the form: *All A are B, No C are B; therefore No C are A*.

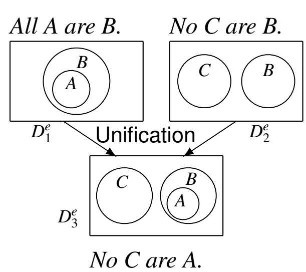


Fig.4 Solving a syllogism with Euler diagrams

Here the premise *All A are B* is associated with diagram D_1^e, where the relation $A \sqsubset B$ holds, and the premise *No C are B* is associated with diagram D_2^e, where the relation

[15] A comparison of these two approaches from a logical point of view is found in Mineshima et al. [18], where the region-based inference system is formalized as resolution calculus, in contrast to the relation-based system formalized as a natural deduction system.

$C \vdash\!\dashv B$ holds. These diagrams make explicit the relational information contained in the premise sentences. The operation of combining two diagrams D_1^e and D_2^e in Fig. 4 is an instance of an application of the *unification* rule.[16] In this case, the unification process consists in identifying circle B and keeping all the relations holding on the premise diagrams. The resulting diagram, D_3^e, has three relations: $A \sqsubset B$, $C \vdash\!\dashv B$ and $A \vdash\!\dashv C$. The first two are inherited from the premise diagrams D_1^e and D_2^e, and the last one, the exclusion relation $A \vdash\!\dashv C$, is created as a by-product of the unification process. As is seen in Fig.4, this new relation $A \vdash\!\dashv C$ corresponds to the sentence *No C are A*, and hence, one can arrive at the valid conclusion of this syllogism.

An important characteristic of the unification process is that by combining the two premise diagrams, one can almost automatically determine the semantic relation holding between the objects in question, without any additional operation. Such information that is automatically inferred from the result of a diagrammatic operation is what Shimojima [32] calls a "free ride".

For the process of unifying diagrams, there are two constraints that determine the spatial relationship between objects in the conclusion diagrams. Namely, for any circle or point X and for any circle Y and Z,

(C1) if X is inside Y in one diagram D_1 and Y is inside Z in another diagram D_2, then X is inside Z in the combined diagram $D_1 + D_2$;
(C2) if X is inside Y in one diagram D_1 and Y is outside Z in another diagram D_2, then X is outside Z in the combined diagram $D_1 + D_2$.

In the example in Fig. 4, the relation $A \vdash\!\dashv C$ is obtained using (C2). Note that these two constraints have counterparts in inference rules in GS: (C1) corresponds to the ($\sqsubset$) rule and (C2) to the ($\vdash\!\dashv$) rule.

The constraints (C1) and (C2) seem so natural and intuitive that even users who do not have explicit training on diagrammatic reasoning can exploit them to draw a correct conclusion without much effort. Theoretically, the inference rules ($\sqsubset$) and ($\vdash\!\dashv$), which are crucial for deriving valid syllogisms, are simulated in terms of the spatial constraints, (C1) and (C2). Such a simulation can happen in actual syllogistic reasoning with external diagrams. For example, a procedure using the ($\vdash\!\dashv$) rule, which licenses us to derive $A \vdash\!\dashv C$ from $A \sqsubset B$ and $B \vdash\!\dashv C$, can be made manifest by perceiving the spatial relationships between diagrammatic objects as seen in Fig. 4. We can then argue that sentential (linguistic) premises themselves do not provide untrained reasoners with specific procedures of solving syllogisms in terms of ($\sqsubset$) and ($\vdash\!\dashv$), such as the ones we saw in the last section; by contrast, Euler diagrams externally given provide the reasoners with a concrete problem-solving procedure based on intuitive understanding of such constraints as (C1) and (C2).

As a second example, let us look at a syllogism having no valid conclusion, which is known to be particularly difficult for untrained reasoners (cf. Evans et al. [8]) and hence deserves special attention.

[16] The rule of unification plays a central role in the inference system for Euler diagrams developed in Mineshima et al. [20]. The system has another rule called *deletion* rule, which allows to delete an object from a given diagram. For discussion on the relevance of deletion rule to the cognitive process of information extraction, see Sato, Mineshima and, Takemura [28].

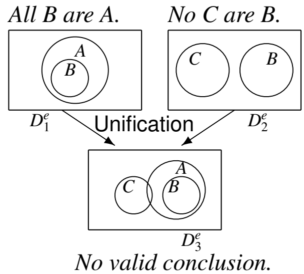


Fig.5 Solving a syllogism with no valid conclusion using Euler diagrams

In the syllogism in Fig.5, sentence *All B are A* is associated with diagram D^e_1, where the relation $B \sqsubset A$ holds, and sentence *No C are B* is associated with diagram D^e_2, where the relation $C \vdash\dashv B$ holds. Again, by unifying these two diagrams, one can obtain the conclusion diagram D^e_3. Note that in this case, neither constraint (C1) nor (C2) can be applied. That is, none of the the inclusion and exclusion relations between circles A and C (i.e., $A \sqsubset C$, $C \sqsubset A$, and $A \vdash\dashv C$) is inferable from the information conveyed by the two premises. In such a case, one needs to put circles A and C in such a way that they partially overlap each other, that is, $A \bowtie C$ holds. Note that such a convention of partially overlapping circles is common to Venn diagrams; it enables us to handle partial or indeterminate information in a relatively simple way.

To be more specific, the relevant rule is the following. For any circles X and Y,

(C3) if none of the relations $X \sqsubset Y$, $Y \sqsubset X$, or $X \vdash\dashv Y$ holds in the combined diagram, put X and Y in such a way that $X \bowtie Y$ holds.

Using this rule, one can see that the relations holding on the conclusion diagram D^e_3 in Fig. 5 are $B \sqsubset A$, $B \vdash\dashv C$, and $A \bowtie C$. The fact that $A \bowtie C$ holds in the conclusion diagram indicates that no specific semantic information about terms A and C can be drawn from the premises. This amounts to saying that there is no valid conclusion with respect to A and C (except trivial ones such as $A \sqsubset A$) in this syllogism. Here again, we can see that Euler diagrams associated with sentential premises play a dual role in the process of checking the *invalidity* of a syllogism: first, they make explicit the relational information underlying categorical sentences; second, the unification of premise diagrams using the constraints (C1) and (C2) leads us to understanding what relational information can be obtained in a given inference; when no particular inclusion or exclusion relation is newly introduced by the unification, that is, when the situation is as described in (C3), the reasoner can conclude that there is no valid conclusion of the inference.

The procedure of checking invalidity of inferences sketched here is remarkably distinguished from the standard procedure in model-theoretic semantics, according to which an inference is judged to be invalid if one can construct a counter-model in which all the premises are true but the conclusion is false. Note that some existing proposals using diagrams are also based on such an idea of counter-model constructions. Thus in Lewis Carroll's version of logic diagrams [5], an inference is invalid if it is impossible to superpose all the premises and the *negation* of the conclusion; see Lear [15] for a discusion. The diagrammatic procedure based on (C3) is distinctive in that it does not depend on any process of negating the conclusion; the information that there is no valid conclusion with respect to the two terms in question can be obtained in a direct way, via a process of unifying premise diagrams. An interesting point to note is that the process of manipulating premise diagrams, more specifically, the process of unification, is common to the tasks of checking validity and invalidity. In other words, not only *proof* but also *refutation* is realized as a syntactic process of manipulating diagrams, rather than as a process of constructing counter-models.

4 Comparisons with the mental model theory of reasoning

As mentioned in Section 1, it has been noticed in cognitive study of deductive reasoning that falsification tasks are often difficult when inference materials are presented in natural languages (cf. Evans et al. [8]). As we argued above, our logic diagrams can contribute to solving such falsification tasks by making available to users syntactic processes of unifying diagrams. An interesting feature of such unification processes is that premise diagrams themselves impose a constraint on the possible ways of unification, so that by simply trying to unify the premise diagrams, the user can observe what relations hold between the objects in the resulting diagram. In this respect, it is worth noting that there is a difference between the underlying mechanism behind unification processes of diagrams discussed in the last section and the reasoning mechanism behind the mental model theory (e.g. Johnson-Laird & Byrne [13]), which is a dominant model of linguistic (sentential) deductive reasoning in cognitive psychology.

According to the mental model theory, mental models are made up of tokens (i.e., elements of a set) and supposed to represent states of the world (cf. e.g., Bara, Bucciarelli, & Lombardo [1]). For instance, sentence *All A are B* corresponds to a model in which each token of set A is connected to a token of set B. Similarly for sentence *All B are C*, in which case each token of set B is connected to a token of set C. As a crucial step, such two premise models are integrated into a single mental model. In the present example, we can finally obtain a model in which each token of set A is connected to a token of set C. This model corresponds to the categorical sentence *All A are C*. Note that not all tokens of set C are necessarily connected to some token of set A. By this fact, we can confirm that *All C are A* cannot be a valid conclusion of this syllogism. In general, difficulties in drawing a valid conclusion are measured by the number of models that can be constructed from the integrated model.

As is suggested by this brief exposition, there is a certain similarity between processes of solving syllogisms using Euler diagrams and reasoning processes with mental models. Specifically, a unification process of Euler diagrams is very similar to an integration process of mental models, and both processes play a crucial role in deriving a valid conclusion from given premises.

However, by taking a closer look at processes of invalidity judgements, we can find that an important difference exists between the two conceptions. In the case of syllogistic reasoning by mental models, processes of integrating mental models can be performed without determining the relation between the tokens in question. That is, alternative models are to be searched for *after* performing the process of integrating the premise models. Thus, according to the theory presented by Bucciarelli and Johnson-

Laird [3], the process of constructing alternative models from an integrated model is constrained by a representational convention such as [a]. Bracketed token [a] indicates that the set containing it is represented by this individual; no new tokens can be added to the sequence with bracketed tokens. On the other hand, in the case of tokens without a square bracket, new tokens can be added so that alternative models are constructed. As an illustration, consider the case of a syllogism having no valid conclusion, as shown in Table 1 (cf. Bucciarelli and Johnson-Laird [3], p. 260). Here, there are two premise models corresponding to *All A are B* and *All C are B*. In the integrated model, then, the relationship between set A and C is indeterminate; the integration process does not require us to resolve such indeterminacy, in sharp contrast to the case of unification in diagrammatic reasoning. Processes of adding the token "b" to this integrated model are performed after the integration process. As a result, the integration process itself does not constrain the ways of constructing alternative models; see Stenning & Oberlander [35] for a related discussion.

Table 1 Representations by mental model theory for the syllogistic task from premises *All A are B* and *All C are B* to the conclusion that there is no valid conclusion.

[a] b	[c] b	[a] b [c]	[a] b	[a] b
[a] b	[c] b	[a] b [c]	b [c]	b [c]
			[a] b [c]	[a] b
				b [c]
All A are B	*All C are B*			
1st premise	*2nd premise*	*Integrated model*	*Alternative model 1*	*Alternative model 2*

On the other hand, the unification process of Euler (EUL) diagrams forces a user to decide what relation holds between the terms in question. That is, the configurations of diagrams constrain what relations (i.e., $\sqsubset$, $\vdash\dashv$, or $\bowtie$) are created in unifying the premise diagrams. As is well known, such a characteristic of diagrammatic representations is called *specificity* by Stenning and Oberlander [35]. In the case of syllogistic reasoning with our Euler diagrams, the relevant constraints are (C1) and (C2); these constraints are almost self-evident given intuitive understanding of inclusion and exclusion relations. In the case of falsification tasks, in particular, such a constraint can increase the chance of finding that the indeterminacy relation $\bowtie$ holds between the relevant objects, say, A and C, that is, none of the relations $A \sqsubset C$, $C \sqsubset A$, or $A \vdash\dashv C$ hold in the unified diagram.

It should be noted here that diagrams themselves are not models in the sense of model theory but certain syntactic representations that are subject to model-theoretic (set-theoretic) interpretations. In particular, if $A \bowtie C$ holds in a unified diagram, one can readily construct a counter-situation to any of the relations $A \sqsubset C$, $C \sqsubset A$, and $A \vdash\dashv C$. We may say that the *potential* to construct alternative models from diagrams are presupposed in the process of unification, in particular, in the process of entertaining a diagram containing the $\bowtie$-relation. It has been observed that the specificity of diagrammatic representations often impedes reasoning; in particular, Shimojima [32] aptly characterizes such a negative aspect of representations in terms of the notion of *over-specificity*. Interestingly, in the present case, the specificity of diagrams has a *positive* effect on the process of checking the invalidity of a syllogistic inference. That is, the failure to apply constraints such as (C1) and (C2), i.e., the failure to create a meaningful relation ($\sqsubset$-relation or $\vdash\!\dashv$-relation), can trigger the recognition that a given set of premises does not have a (non-trivial) valid conclusion.

We can summarize that in the case of reasoning with Euler diagrams, the process of entertaining alternative possibilities is presupposed, and implicitly triggered, in the process of unifying premise diagrams, whereas in the case of reasoning with mental models, such a process is only conducted after the process of integration, without appealing to visual constraints. In this respect, the two conceptions of combining the premise information stand in striking contrast to each other.

5 Concluding remark

In the cognitive psychology of deduction, it has long been known that solving categorical syllogisms is a difficult task for those who are untrained in logic (cf. [8]). The experimental results in Sato et al. [26] were consistent with this traditional view in that they show the performance of the Linguistic group as much lower than that of the Euler and Venn groups. The question we asked is: how can diagrams that are externally provided improve the performance of syllogism solving even for untrained people? To answer this question, in Section 1, we distinguished between interpretational and inferential efficacy of diagrams in the overall process of solving syllogisms. Now, given the relational analysis of categorical syllogisms and Euler-style diagrammatic inferences presented so far, we can elaborate and summarize the distinction in the following way.

First, concerning the interpretational side, the relational semantic information associated with quantificational sentences is often not directly accessible to reasoners. Thus, there is a tendency to interpret the sentence *All A are B* as equivalent to *All B are A*, and *Some A are not B* as implying *Some B are not A* (called *conversion error* in Sectoin 1). Euler and Venn diagrams can then help reasoners realize the underlying semantic relations implicit in categorical sentences in virtue of their spatial properties, more specifically, in virtue of inclusion and exclusion relations between objects. Hence, such external representations allow us to fix the intended relational interpretations of categorical sentences in syllogistic reasoning tasks, resulting in interpretational efficacy.

Second, concerning the inferential side, the manipulation of diagrams in the inferential process is triggered without effort, if the spatial relations holding on external diagrams are governed by *natural* constraints, i.e., constraints that depend solely upon spatial properties of diagrams and hence are accessible even to untrained users. Furthermore, given the fact that a deductive reasoning task in general requires the reasoner to assemble the information contained in the premises, the syntactic manipulations of diagrams could be spontaneously triggered when those diagrams are externally presented. The essential steps involved in the manipulations of Euler diagrams are unification processes, that is, those processes in which the inclusion and exclusion relations between objects in the unified diagrams are effectively determined using the constraints (C1), (C2), and (C3). Such a unification process is composed of steps in matching an object (a circle or a point) with another object and determining the diagrammatical relationships between the other objects. Users can exploit the natural constraints of diagrams and extract the correct procedure to apply from Euler diagrams themselves.

If these claims are correct, it would be expected that any diagram that can make explicit the relational information of a categorical sentence in a suitable way would be effective in supporting syllogistic reasoning. Sato and Mineshima [29] examines the case of a linear variant of Euler diagrams, where set-relationships are represented by one-dimensional lines, rather than by circles in a plane. The experimental results obtained there indicated that the linear diagrams for syllogistic reasoning work as effectively as Euler diagrams. This provides partial evidence that the effectiveness of external diagrams in syllogistic reasoning does not depend upon particular shapes such as circles that are specific to Euler diagrams. Rather, what is crucial is the fact that diagrams can effectively represent relational structures and aid reasoning about them.[17]

Such a comparison between reasoning with various forms of diagrams would provide further evidence to specify the semantic primitives of sentences used in reasoning tasks, and thus contribute to making progress in understanding the nature of both linguistic and diagrammatic inferential processes in human deductive reasoning. There are various ways of extending our basic fragment of syllogistic logic; e.g., relational syllogisms [24], syllogisms involving proportional quantifiers like *most* [21], and syllogisms involving conjunctive and disjunctive terms [23]. Applications of our framework to such extended syllogistic and diagrammatic inferences are left for future research.

References

1. Bara, B.G., Bucciarelli, M., & Lombardo, V. (2001). Model theory of deduction: A unified computational approach. *Cognitive Science*, 25, 839–901.
2. Barwise, J. & Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy*, 4, 159–219.
3. Bucciarelli, M. & Johnson-Laird, P.N. (1999). Strategies in syllogistic reasoning. *Cognitive Science*. 23(3), 247–303.
4. Calvillo, D.P., DeLeeuw, K. & Revlin, R. (2006). Deduction with Euler circles: Diagrams that hurt. In *Proceedings of Diagrams 2006*, *LNAI 4045* (pp.199–203), Heidelberg: Springer.
5. Carroll, L. (1896). *Symbolic Logic*. New York: Dover.
6. Corcoran, J. (1974). Aristotle's natural deduction system. In J. Corcoran (ed.), *Ancient Logic and its Modern Interpretations* (pp. 85–131), Dordrecht: Reidel.
7. Euler, L. (1768). *Lettres à une Princesse d'Allemagne sur Divers Sujets de Physique et de Philosophie*. Saint-Pétersbourg: De l'Académie des Sciences.
8. Evans, J.St.B.T., Newstead, S.E. & Byrne, R. (1993). *Human Reasoning: The Psychology of Deduction*. Hove: Lawrence Erlbaum.
9. Gergonne, J. D. (1817). Essai de dialectique rationelle. *Annuales de Mathematiques pures et appliqukes*, 7, 189–228.
10. Gurr, C.A. (1999). Effective diagrammatic communication: syntactic, semantic and pragmatic issues. *Journal of Visual Languages & Computing*, 10(4), 317–342.
11. Hammer, E. (1995). *Logic and Visual Information*. Stanford, CA: CSLI Publications.
12. Hammer, E. & Shin, S. (1998). Euler's visual logic. *History and Philosophy of Logic*, 19, 1–29.

[17] This claim is consistent with Gurr's [10] "well matchedness" theory.

13. Johnson-Laird, P.N., & Byrne, R. (1991). *Deduction*. Hillsdale, NJ: Erlbaum.
14. Larkin, J. & Simon, H. (1987). Why a diagram is (sometimes) worth 10,000 words. *Cognitive Science*, 11, 65–99.
15. Lear, J. (1980) *Aristotle and Logical Theory*. Cambridge, UK: Cambridge University Press.
16. Łukasiewicz, J. (1957). *Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic*, Second edition, Oxford: Oxford University Press.
17. Mineshima, K., Okada, M., Sato, Y & Takemura, R. (2008). Diagrammatic reasoning system with Euler circles: theory and experiment design. In *Proceedings of Diagrams 2008*, *LNAI 5223* (pp.188–205), Berlin, Heidelberg: Springer.
18. Mineshima, K., Okada, M., & Takemura, R. (2010). Two types of diagrammatic inference systems: Natural deduction style and resolution style. In *Proceedings of Diagrams 2010*, *LNAI 6170* (pp. 99–114), Berlin, Heidelberg: Springer.
19. Mineshima, K., Okada, M., & Takemura, R. (in press-a). A diagrammatic reasoning system with Euler circles. *Journal of Logic, Language and Information*, to appear.
20. Mineshima, K., Okada, M., & Takemura, R. (in press, b). A generalized syllogistic inference system based on inclusion and exclusion relations. *Studia Logica*, to appear.
21. Moss, L.S. (2008). Completeness theorems for syllogistic fragments. In F. Hamm & S. Kepser (eds.), *Logics for Linguistic Structures* (pp.143–173), Berlin : Mouton de Gruyter.
22. Newstead, S.E. & Griggs, R. (1983). Drawing inferences from quantified statements: a study of the square of opposition. *Journal of Verbal Learning and Verbal Behavior*, 22, 535–546.
23. Nishihara, N. & Morita, K. (1988). An extended syllogistic system with conjunctive, disjunctive and complementary terms, and its completeness proof (in Japanese). *Trans. IEICE Japan*, Vol. J71-D, No.4. 693–704, 1988.
24. Pratt-Hartmann, I. & Moss, L.S. (2009). Logics for the relational syllogistic. *Review of Symbolic Logic*, 2, 647–683.
25. Rizzo, A. & Palmonari, M. (2005). The mediating role of artifacts in deductive reasoning. In *Proceedings of 27th Annual Conference of the Cognitive Science Society* (pp. 1862–1867).
26. Sato, Y., Mineshima, K., & Takemura, R. (2010a). The efficacy of Euler and Venn diagrams in deductive reasoning: Empirical findings. In *Proceedings of Diagrams 2010*, *LNAI 6170*, (pp. 6–22), Berlin, Heidelberg: Springer Verlag.
27. Sato, Y., Mineshima, K., & Takemura, R. (2010b). Constructing internal diagrammatic proofs from external logic diagrams. In *Proceedings of the 32nd Annual Conference of the Cognitive Science Society* (pp. 2668–2673). Austin, TX: Cognitive Science Society.
28. Sato, Y., Mineshima, K., & Takemura, R. (2011). Interpreting logic diagrams: a comparison of two formulations of diagrammatic representations. In *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (pp. 2182–2187). Austin, TX: Cognitive Science Society.
29. Sato, Y., & Mineshima, K. (2012). The efficacy of diagrams in syllogistic reasoning: A case of linear diagrams. *Proceedings of Diagrams 2012*, to appear.
30. Scaife, M. & Rogers, Y. (1996). External cognition: how do graphical representations work? *International Journal of Human-Computer Studies*, 45, 185–213.
31. Shin, S.-J.(1994). *The Logical Status of Diagrams*. New York: Cambridge University Press.
32. Shimojima, A. (1996). *On the Efficacy of Representation*. PhD thesis, Indiana University.
33. Smiley, T. (1974). What is a syllogism? *Journal of Philosophical Logic*, 1, 136–154.
34. Stapleton, G. (2005). A survey of reasoning systems based on Euler diagrams. *Proceedings of Euler Diagrams 2004*, *ENTCS 134* (pp. 127–151). Amsterdam: Elsevier.
35. Stenning, K., & Oberlander, J. (1995). A cognitive theory of graphical and linguistic reasoning. *Cognitive Science*, 19, 97–140.
36. Stenning, K., Cox, R. & Oberlander, J. (1995) Contrasting the cognitive effects of graphical and sentential logic teaching: reasoning, representation and individual differences. *Language and Cognitive Processes*, 10, 333–354.