Interactive Computational Systems: Rough Granular Approach

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Abstract. The aim of this paper is to present a step toward building computational models for interactive systems. Such computations are performed in an integrated distributed environments on objects of different kinds of complexity, called here as information granules. The computations are progressing by interactions among information granules and physical objects. We distinguish global and local computations. The former ones are performed by the environment (*the nature*) while the local computations are, in a sense, *projections* of the global computations on local systems and they represent information on global computations perceived by local systems. We assume that, the laws of the *nature* are only partially known by the local systems. This approach seems to be of some importance for developing computing models in different areas such as natural computing (*e.g.*, computing models for meta-heuristics or computations models for complex processes in molecular biology), computing in distributed environments under uncertainty realized by multi-agent systems, modeling of computations for feature extraction (constructive induction) for approximation of complex vague concepts, hierarchical learning, discovery of planning strategies or strategies for coalition formation by intelligent systems as well as for approximate reasoning about interactive computations based on such computing models. In the presented computing models, a mixture of reasoning based on deduction and induction is used.

Keywords: interactive computing, interactive systems, multi-agent systems, rough sets, granular computing, wisdom technology.

1 Introduction

Research presented in this paper is aimed at laying foundations for modeling of interactive rough granular computing (IRGC) [13, 30, 32] relevant for interactive computational systems (ICSs) in which computations are progressing through interactions [9]. In IRGC interactive computations are performed on objects called information granules [22] (or infogranules, for short) and both interactive computations and information granules are represented by information systems from the rough set approach [19–21, 34]. Information granules are one of the concepts playing main role in developing foundations for AI, data mining and text mining [22]. They grew up as some generalizations from fuzzy sets [48–50] as well as rough set theory and interval analysis [22].

In order to represent interactive computations (used *e.g.*, in searching for new features) information systems of a new type, namely interactive information systems, are needed [30, 32]. Laying foundations for IRGC is also aimed at giving a unique way of modeling of computations in different various areas such as multi-agent systems, swarm intelligence, metaheuristics, perception based computing, natural computing, membrane computing in addition to data mining and machine learning.

In many areas (*e.g.*, biology, sociology, MAS, robotics, pattern recognition, machine learning or data mining, simulations of complex phenomena, or semantic search engines), the challenge is to discover (induce) complex infogranules from some elementary ones, representing imperfect knowledge about analyzed objects, and concepts, or/and phenomena in such a way that the complex infogranules (*e.g.*, clusters of highly structural objects in data mining, new features obtained by feature construction in machine learning or coalitions in MAS) satisfy the given, often vague, target specification to a satisfactory degree (see, *e.g*, [11, 3, 6, 35, 9, 36, 23]). This idea has been coined, *e.g.*, in rough mereology [24]. Therefore, in addition to theoretical unification of various paradigms mentioned above, research presented in this paper has also a practical objective. It contains an attempt of constructing a basis for tools for creating strategies supporting inducing such complex infogranules satisfying vague, target specification to a satisfactory degree.

To meet these objectives, in the paper several issues are addressed.

We start with a short discussion on ontology for interactive granular computing, *i.e.*, a specification of list of basic concepts for IGC together with their descriptions and interrelations.

We denote by $s(t)$ the *global state* of ICS at time t. The $s(t)$ consists objects called as *agents*. Each agent consists as parts *information bot* (*inbot*, for short) with structure described by infogranules and some *physical objects* (called also as *hunks of matter*, or *hunks*, for short [12]). Among agents is distinguished an agent called as the *environment agent*. The set of agents existing in $s(t)$ different from ag_e) is denoted by $Ag(t)$. For the definition of global transition relation may be necessary some assumptions about the global ontology or at least mereology of hunks in the environment agent. Inbots are used by agents for abstract representation of information perceived about global states, history of interactions with the environment and other agents, etc.. These representations are created using infogranules of different kinds. Special interactions between infogranules and some distinguished hunks are making it possible to represent infogranules in hunks. Note that in general transitions from a given global state may influence inbots too.

Next, we distinguish two kinds of computations realized by ICS, *i.e.*, global computations realized by the ag_e agent and local computations realized by agents from the union of $Ag(t)$ over time t. Due to uncertainty, typically any local computation represent a class of global computations. This causes that control over local computations performed by agents should be robust relative to this class, *i.e.*, all global computations corresponding to the local computation should be of the similar quality.

One of the main problem to obtain the robustness for the control strategies is to discover relevant attributes (features) over which the conditions for activation of actions can be induced. In searching for such attributes, hierarchical learning may be used supported by domain knowledge. Among strategies searching for relevant attributes are strategies based on hierarchical structures discovered from data and domain knowledge, interactive computations (in particular, interactions with domain experts are very often required) as well as adaptive judgment strategies embedded in the systems. ICS are introduced as models for solving problems specified by complex vague concepts. Very often ICS is designed for solving a class of problems not a single problem. It is worthwhile mentioning that using the rough set based methods were developed methods for embedding into the system approximate "views" of domain ontologies (see, *e.g.*, [4, 5, 3, 14, 15]). This is a step toward developing methods for perception based computing. Such methods are making it possible to reason from sensory measurements to perception, *i.e.*, understanding of the sensory measurements [49, 50, 29–32].

This article is a step toward realization of the Wistech program (see, *e.g.*, [13]).

We assume the reader is familiar with the basic notions concerning information systems and rough sets [19–21]. In this section, we discuss a generalization of information systems to interactive information systems.

2 Global and Local Computations of ICS

We distinguish two kinds of computations realized by ICS. The first ones are called *global computations* realized by a global transition relation of the environment agent reflecting the dynamics of the nature. This relation is only partially known for agents which are parts of ICS. The second kind of computations create *local computations* relative to a particular agent or group of agents.

The global computations are defined by a global transition relation defined as follows: (i) the environment agent ag_e at a state $s(t)$ gathers information $inf(ag_e, s(t))$ based on perception of the existing in $s(t)$ agents and hunks at time t; in particular, there are perceived active or activated actions by the agents at t, (ii) on the basis of the information $inf(ag_e, s(t)$, ag_e selects for the next moment $t + \Delta$ the next global state $s(t + \Delta)$ from the set of states $State_{inf(ag_e,s(t))}$ corresponding to the gathered information; in the case of unpredictable environment, neither $inf(ag_e, s(t))$ nor $State_{inf(ag_e,s(t)}$ are available for the agents existing in $s(t)$ others than ag_e, (iii) the global computation (of ag_e) is any sequence of global states: $s(t), s(t + \Delta), \ldots, s(t + i\Delta), \ldots$ such that for any two consecutive global states $s(t+i\Delta), s(t+(i+1)\Delta)$ from this sequence, $s(t+(i+1)\Delta)$ is defined from $s(t+i\Delta)$ using the rule described in the previous step.

In this paper, we consider global computations over linear discrete time. However, the approach can be extended to computations over continuous and nonlinear time.

In the following sections, we discuss how such global computations in the environment are perceived by agents performing computations on infogranules. In particular, we discuss a special role of interactive information systems (see, *e.g.*, [32].) Such information systems make it possible to register the sensory measurements over the time also related to the results of performed actions, to record the expected results of actions or plans. Actions or plans are activated on the basis of satisfiability of the complex vague concepts. These complex concepts are approximated using hierarchical information systems [14, 3]. Agents are gathering knowledge discovered over time by their inbots, *e.g.*, in the form of sets of discovered rules. In this way, agents deal with complex objects of different complexity called infogranules. The infogranules should be discovered by using relevant strategies so they can be used for synthesis or inducing more complex infogranules relevant for the solution of the task under consideration. Agents perceive global computations due to interactions with the environment recorded by interactive information systems and modeling of infogranules representing the local history of computation relative to a given agent.

The local computations relative to a given agent $ag \in \bigcup_t Ag(t)$ are, in a sense, "projections" of global computations. These projections relative to $ag \in \bigcup_t Ag(t)$ are defined for any global computation $s(t), s(t+\Delta), \ldots, s(t+i\Delta), \ldots$ as follows: (i) a subsequence of $s(t), s(t+\Delta), \ldots, s(t+i\Delta), \ldots$ is selected using the time scale of ag, where it is assumed that ag exists at all time moments in selected subsequences, [1], (ii) any global state, in the defined above subsequence, is substituted by an infogranule representing an accessible for ag information about this global state.[2]

Any local computation of a given agent represents a class of global computations perceived in the same way by the agent ag_e [30, 31].

The tasks performed by ICS can be characterized as the tasks of searching for (adaptive) control strategy over the local computations for obtaining computations of satisfactory quality relative to the considered tasks.

3 Interactive Framework: Environment and Infogranules of Various Types

Infogranules are purely formal objects, they are specific types of data structures and can be described using sets from different levels of the power-set hierarchy. Agents are acting objects, designed to perform various types of actions

[1] One may consider a more general case by selecting a subsequence of global computation (soft) segments of different sizes.

[2] The details explaining how this infogranule is selected by ag can be explained using the Aristotle tetrahedron [13] or its modification discussed elsewhere.

such as information processing, solving problems, planning, making decisions, conducting sensory measurements. ICSs can consists as parts both purely formal objects or physical objects or combinations of formal and physical objects. As examples of the first type of parts one can consider different kinds of algorithms (including interactive algorithms), *e.g.*, classifiers, or search engines. As examples of the second type can serve physical sensors, while as examples of the third type one take into account robots with physical effectors, calculators or computers. We distinguished several components of ICSs such as agents consisting of inbots, and hunks. Inbots in agents are responsible for solving problems, planning, or making decisions. Agents also contain some hunks, in particular sensors (responsible for conducting sensory measurements) and their counterparts in inbots called as *sensor bots* (or *sebots*, for short). For example, sebots can be purely formal objects as elements of interface of search engine querying a given data base. Hunks are physical “counterparts” of infogranules. Hunks can interact and the results of interaction can be perceived by agents. These interactions can by influenced by actions. As examples of hunks can serve dices or chemical particles. Interactions between sebots and sensors are making it possible to represent infogranules in hunks. In the agent structure we distinguish other special infogranules called bots such as *bot for the hierarchy of agent needs* (or *nebot*, for short). The nebot, consisting specification of agent tasks, interacts with the other bots of agents responsible for control, syntactic or semantic issues of infogranules (see Figure 1).

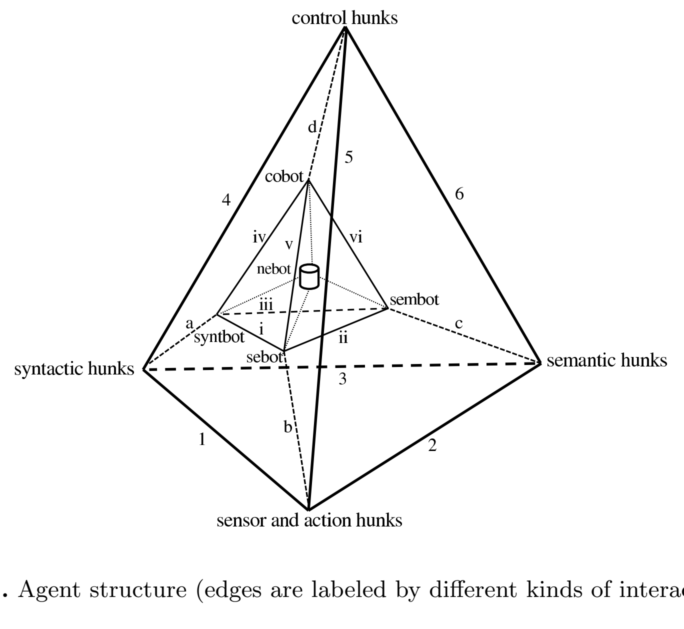

Fig. 1. Agent structure (edges are labeled by different kinds of interactions)

Infogranules and agents are immersed into (artificial) environment or interactive framework for performing interactive granular computations. It is possible that a particular agent through some discovery uses a collection of "law of the nature" or collections of rule of the games, making it possible to better predict the future state of local computation. We assume here that inbots have only partial, limited information about the environment, other agents and about themselves, and analogically, sebots perceive only a part of the environment. Let us note that in particular applications one may assume some properties (laws) the agent ag_e should satisfy.

4 Interactive Information Systems

Interactive information systems, called also interactive tables, are parts of the agent inbots used for representation of agent interaction results with the environment. Agents perceive the environment, construct plans of actions, choose between alternative actions to be performed, make decisions whether to activate or deactivate a given action in the face of the dynamically changing environment and to perform actions (such performance is the way how the environment is influenced by agents). Therefore interactive information systems contain attributes of special kinds, namely *perception attributes*, including *sensory attributes* and *action attributes* [30, 32]. These attributes are *open attributes*, *i.e.*, as functions they are injections from object domains into sets of values.

Perception attributes can be divided into *atomic* and *constructible* attributes. *Atomic attributes* are basic in the sense that their values depend only on some external factors, with respect to a given information system and they are not computed from the values of other attributes of this system.

Constructible attributes are complex attributes which are inductively defined from atomic attributes of a given information system: if b is a constructible attribute, then for any some object x and already defined atomic attributes $a_1, a_2, \ldots, a_k$: $b(x) = F(a_1(x),\ a_2(x), \ldots,\ a_k(x))$, where $F : V_{a_1} \times V_{a_2} \times \ldots \times V_{a_K} \longrightarrow V_b$ and values from V_b are constructed on the basis of values from V_i for $i = 1, \ldots, k$.

Sensory attributes represent sensor measurements. They are atomic attributes whose values are results of measurements conducted by sensors thus they depend only on the environment and are independent from values of other attributes. *Perception attributes* are sensory attributes or constructible attributes defined on the basis of sensory ones. The latter are also called *complex perception attributes*. Complex perception attributes represent higher order result of perception, *e.g.*, some identified patterns or created perceptual infogranules.

Let us consider some features of interactive information systems important from the point of view modeling interactive computations: (i) information systems consist of attributes together with some relational structures on them such as the linear time order which allows us to represent the status of sensory measurements in time in different rows of information system, (ii) sensor attributes make it possible to record a time moment of initialization/finalization of the sensor measurement, (iii) for some period of time the value of sensory measurement may be unknown, *e.g.*, because the measurement process is not finalized, (iv) new sensor or action attributes may be added in time, (v) new rows may be added or changed in the following moments of time to the current information system.

Formally, *interactive information systems* are decision systems where condition attributes contain sensory attributes as well as some complex perception attributes and decision attributes contain action attributes. It should be noted that interactive information systems are dynamically evolving systems where classes of condition attributes and decision attributes can be expanded: new perception attributes can be constructed on the basis of values returned previously by sensors or some previously existing complex perception attributes. Therefore they are not only tables with dynamically changing values of attributes. Using this properties interactive information systems can represent interactive computations with intrastep interactions [10, 30] and thus they are much more general information systems than the studied dynamic information systems (see, *e.g.*, [7, 17]) which make it possible to consider incremental changes in information systems but they do not contain the perception and action attributes necessary for modeling interactive computations, in particular for modeling intrastep interactions [30].

Interactive information systems are used for discovery new knowledge, *e.g.*, in the form of some rule sets. Such rule sets may be treated as theories of information systems [2]. They make it possible to express, *e.g.*, interactions in the context of considered infogranule types.

Operations on information systems can be used for generation of new infogranules relevant for the considered task.

5 Hierarchies of Interactive Information Systems

Infogranules can have an elementary structure (such as elementary neighborhoods of objects see: [26], [37], [43]) or a complex structure (such as cognitive agents [42], autonomous software bots, teams of software bots [36], [41], complex patterns or classifiers in data mining, [34], [1]). Infogranules of higher order are constructed in hierarchical way from simpler infogranules. For example, infogranules in agents can have a complex structure consisting of many components responsible for, *e.g.*, perceiving the environment, planning actions, or sending messages to other agents [30, 32]. Coalitions of agents lead to a special kind of infogranules in layered granular networks. Note that autonomous software bots can use complex vague concepts as guards of actions performed during the interaction with the environment [30, 32]. For the approximation of such concepts the rough set approach can be used (see, *e.g.* [19], [20], [21], [43]). Interactions among infogranules and approximations of granules are two basic concepts related to interactive computations on infogranules. In layered infogranular networks we represent the (hierarchical) structure of granules as well as links between interacting infogranules. Granular layered networks are built over information systems representing infogranules and their interactions.

Complex perception attributes defined by means of relational structures can be used to represent some structural properties of objects, for example time windows in information systems where objects are time points. In hierarchical modeling, object signatures at a given level of hierarchy can be used for constructing structural objects on the next level of hierarchy. On the basis of complex perception attributes previously existing in the interactive information systems, new complex perception attributes can be created. This can be done by creating new attribute values on the basis of values previously existing in the system. For example, relational structures being values of complex perception attributes can be fused. Let $\{(V_{a_i}, \tau_{a_i})\}$ be a family of tolerance spaces, *i.e.* relational structures where V_{a_i} is a value domain of an attribute a_i and $\tau_{a_i} \subseteq V_{a_i} \times V_{a_i}$ is a tolerance relation (relation that is reflexive and symmetric) for $i = 1, \ldots, k$. Their fusion is a relational structure over $V_{a_1} \times \ldots \times V_{a_k}$ consisting of a relation $\tau \subseteq (V_{a_1} \times \ldots \times V_{a_k})^2$ such that for any $(v_1, \ldots, v_k), (v'_1, \ldots, v'_k) \in V_{a_1} \times \ldots \times V_{a_k}$ we assume $(v_1, \ldots, v_k)\tau(v'_1, \ldots, v'_k)$ if and only if $v_i \ \tau_{a_i} \ v'_i$ for $i = 1, \ldots, k$. Note that τ is also a tolerance relation. Intuitively, a vector $(v_1, \ldots, v_k)$ represents a set of objects possessing values $v_1, \ldots, v_k$ for attributes $a_1, \ldots, a_k$ respectively. Thus some vectors from $V_{a_1} \times \ldots \times V_{a_k}$ (not necessarily all) represent infogranules consisting of objects (some vectors from $V_{a_1} \times \ldots \times V_{a_k}$ correspond to the empty set). Therefore a relation τ corresponds to a relation between infogranules.

In the process of searching for infogranules relevant for the considered task, a very important role play operations on information systems called sums (joins) with constraints [27]. Roughly speaking such operations allow us to generate a wide spectrum of new infogranules from given information systems as arguments of operations. As the result of such operation we obtain infogranules of defined by attribute value vectors of rows of arguments satisfying some constraint. Hence, objects in new information systems are relational structures. The attributes in the new information systems are defined over such structural objects. This very general scheme may be widely used for generation of complex infogranules in searching for relevant infogranules for a given task.

More formally, for given information systems $\mathcal{A}_1, \ldots, \mathcal{A}_k$, we consider *constraints* $W \subseteq INF(A_1) \times \ldots \times INF(A_k)$, where $INF(A_i) = \{Inf_{A_i}(x) : \ x \in U_i\}$, A_i is the set of attributes of $\mathcal{A}_k$, and U_i is the set of objects in $\mathcal{A}_i$, for $i = 1, \ldots, k$. A *join of* $\mathcal{A}_1, \ldots, \mathcal{A}_k$ *relative to* W (or W*-join*, for short) is any information system $\mathcal{A} = (U, A)$, where $U \subseteq W$ and $A = \{(a, i); \ a \in A_i \ \& \ i \in \{1, \ldots, k\}\}$, where $(a, i)(x_1, \ldots, x_k) = a(x_i)$, for $i = 1, \ldots, k$ [27, 2].

Let us consider some examples: (i) interactive information systems with clusters of similar/indiscernible objects, (ii) interactive information systems with time windows over objects from the lower level, (iii) interactive information systems with time windows over groups of interacting objects from the lower level, (iv) interactive information systems with sets of time windows as objects obtained as indiscernibility/similarity classes, (v) interactive information systems with sequences of time windows (or sets of time windows) as objects; attributes in such systems may have as values models of concurrent systems (*e.g.*, Petr nets) consistent with sets of sequences of time windows, (vi) interactive information systems with objects representing a set of concurrent models with constraints; attributes in such systems may have as values concurrent systems (*e.g.*, Petri nets) consistent with the specification given by objects what can allow us to understand the structure of interactions between processes.

In Figure 2 arrows are showing possible interactions of the interactive hierarchical structure with the environment as well as between different layers of the hierarchy. On differen layers are illustrated (see parallelograms) interactive information systems and (see parallelograms with rounded corners) induced from them knowledge (*e.g.*, in the form of sets of rules consistent with the information systems). Observe that searching for such complex infogranules may be neces-

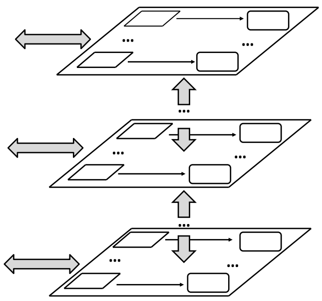

Fig. 2. Interactive hierarchical structure

sary when task specifications require to deliver complex dynamic infogranules. For example, one can consider the challenging task of modeling interactions of cells on the basis of of interactions of biochemical reactions in cells and their environment [8]:

> [...] One of the fascinating goals of natural computing is to understand, in terms of information processing, the functioning of a living cell. An important step in this direction is understanding of interactions between biochemical reactions. [...] the functioning of a living cell is determined by interactions of a huge number of biochemical reactions that take place in living cells.

It is worthwhile mentioning that our approach to ICSs differs substantially from the other existing ones, *e.g.*, in MAS or CAS. We assume that the control structure of agents should be discovered using some adaptive or/and evolutionary strategies. In particular, the approximations of complex vague concepts are adaptively changing when the interactive computations are progressing. These concepts are used, *e.g.*, for activating actions or plans responsible for the agent behaviors.

6 Conclusions

We discussed some aspects of computational models for ICS such as global and local computations, interactions between different components of ICS or interactive information systems. We emphasized the problem of controlling local computations by relevant selection of actions activated on the basis of satisfiability degrees of complex vague concepts. In particular, such concepts may be related to emotional or ethical concepts. This requires methods for hierarchical learning supported by domain knowledge based, *e.g.*, on ontology approximation [14, 3].

The presented approach is also a step toward solving of challenging problems related to communication language evolution (see, *e.g.*, [47, 46, 16]) or representation of interactions in ICSs (see, *e.g.*, [9]).

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