Implementing the Lelièvre-Weiss Strict Convexity Test in Sage

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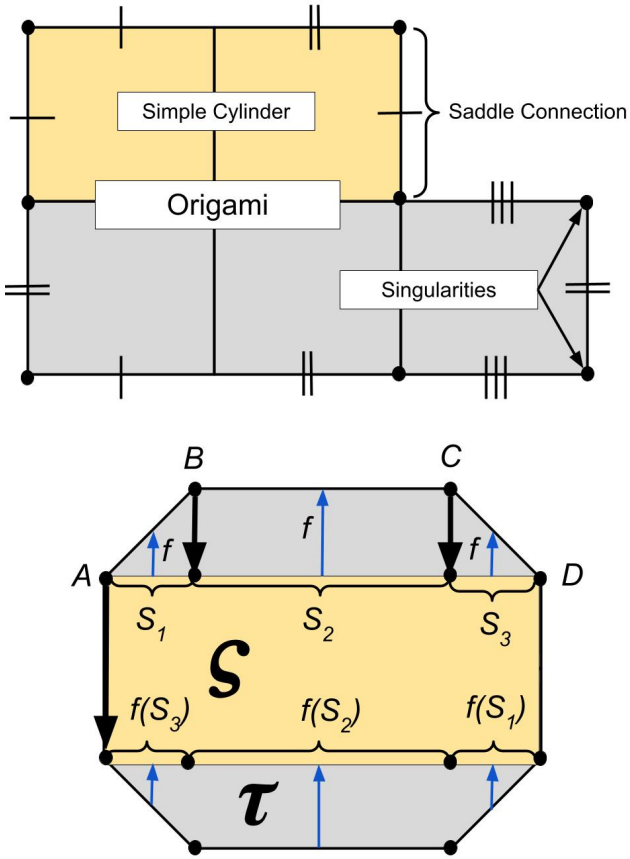
Intro

- Translation surfaces are collections of polygons with sides identified by translation.
- If we can cut and paste to get from one surface to another, they are the same.
- It is not obvious when a translation surface can be cut and pasted into a convex polygon.
- We used Sage to implement a convexity test for "nice" translation surfaces called origamis.

Definitions

- An **origami** is a translation surface made of unit squares.
- A vertex of a translation surface is a **singularity** if the angle around it is more than 2π.
- A **saddle connection** is a line segment joining two singularities that does not contain additional singularities.
- A **cylinder** is a rectangle with one pair of opposite edges glued together.
- A **simple cylinder** is a cylinder for which each boundary is a single saddle connection.

Which **Translation Surfaces** have **Convex Presentations**?



Figures: (top) Example of an **origami**, showing one of its **saddle connections**, two of its **singularities**, and an example of a **simple cylinder**. (bottom) Application of the Lelièvre-Weiss test described in section *Example of Test*.

← See the code

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The Lelièvre-Weiss Test

- Let τ be a translation surface with one singularity.
- A simple cylinder ς exists on τ satisfying certain properties **iff** τ has a strictly convex presentation [LW15]. These properties are demonstrated in the below example.

Example of Test

- Let τ be the octagonal translation surface in the bottom figure.
- ς is a simple cylinder on τ.
- ς has a vertical saddle connection.
- The singularity of τ has 3 southward pointing arrows (black arrows).
- 2 of these southward pointing arrows intersect the top of ς, splitting it into S_1, S_2, S_3.
- The first return map f (blue arrows) reverses the order of S_1, S_2, S_3 on the bottom of ς.
- The points A, B, C, D above ς form a convex region in $\mathbb{R}^2$.

Future Directions

- Use the test to study surfaces in $\mathcal{H}(2g-g)$ for larger genus g.
- Explore non-strict convexity tests for translation surfaces.
- Explore tests for surfaces that are not origamis up to cut and paste.

References

[LW15] Samuel Lelièvre and Barak Weiss. Translation surfaces with no convex presentation. *Geom. Funct. Anal.*, 25(6):1902–1936, 2015.