Fractal Analogies: Preliminary Results from the Raven's Test of Intelligence

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Abstract

The geometric analogy problems of the Raven's Progressive Matrices tests of intelligence appear to require many of the information-processing elements that form the basis of computational theories of general creativity: imagistic representations and reasoning; pattern detection and abstraction; analogical mapping, transfer and instantiation, and so on. In our method of addressing the test, an image is encoded as fractals, capturing its inherent self-similarity. Herein we present preliminary results from using the fractal technique on all 60 problems from the Standard Progressive Matrices version of the Raven's test.

Psychometrics and Creativity

Psychometrics entails the theory and technique of quantitative measurement of intelligence, including factors such as personality, aptitude, creativity, and academic achievement. We propose that *some* psychometric tests of intelligence could also be good tests of *some* aspects of creativity. Consider problems on the Raven's Standard Progressive Matrices test. The task is to pick as the best match one of the several choices for insertion in the empty element of the matrix. Addressing this problem appears to engage many of the information-processing elements that form the basis of computational theories of general creativity (e.g., Casakin & Goldschmidt 1999; Clement 1988; 2008; Croft & Thagard 2002; Davies, Nersesseian & Goel 2005; Goel 1997; Goldschmidt 2001; Hofstadter 1979, 1995; Holyoak & Thagard 1996; Nersessian & Chandrasekharan 2009; Yaner & Goel 2008): imagistic representations and reasoning; pattern detection and abstraction; analogical mapping, transfer and instantiation, etc. Clement (2008) and Nersessian (2008), for example,describe analogical reasoning using imagistic representations as a fundamental process of creative problem solving in science; Goldschmidt (2001) and Hofstadter & McGraw (1995) make a similar point about visual analogies in design creativity.

The Raven's Progressive Matrices tests (Raven, Raven, & Court 1998) are a collection of standardized intelligence tests that consist of visually presented, geometric analogy problems in which a matrix of geometric figures is presented with one entry missing, and the correct missing entry must be selected from a set of answer choices.

The Standard Progressive Matrices (SPM) consists of 60 problems divided into five sets of 12 problems each (sets A, B, C, D & E), roughly increasing in difficulty both within and across sets.

As far as we know, *all* extant computational theories of the Raven's and other similar tests involving geometric analogies, rely on the extraction and use of propositional representations. In contrast, at last year's conference on computational creativity, we had described a proposal to use fractal encodings to address the Raven's test (McGreggor, Kunda & Goel 2010). Our technique is grounded in the mathematical theory of fractal image compression (Barnsley & Hurd 1992) and of general fractal representations (Mandelbrot 1982).

The main goal of our work is to evaluate whether the Raven's Standard Progressive Matrices test could be solved using purely visual representations, without converting the image inputs into propositional descriptions during any part of the reasoning process. We use fractal representations, which encode transformations between images, as our primary non-propositional representation.

Our system operates on inputs that have been scanned directly from a hard copy of the Raven's test and contain the usual rough alignments and pixel-level artifacts. Problem entries are converted to fractal representations, and only relationships among these fractal representations are used to choose the best answer. We stress that at no point are inputs converted into any kind of propositional form (e.g. shapes, colors, lines, edges, or any other visually segmented entity); only the raw RGB pixel values are used.

Fractal Representations and Features

For visual analogy problems of the form *A : B :: C : ?*, each of these analogy elements are a single image. Some unknown transformation T can be said to transform image A into image B, and likewise, some unknown transformation T' transforms image C into the unknown answer image. The central analogy in the problem may then be imagined as requiring that T is analogous to T'. Using fractal representations, we shall define the most analogous transform T' as that which shares the largest number of fractal features with the original transform T.

To find analogous transformations for *A : B :: C : ?*, our algorithm first visits memory to retrieve a set of candidate solution images X to form candidate solution pairs in the form <C, X>. For each candidate pair of images, we generate the fractal encoding of the transformation of candidate image X in terms of image C. From this encoding we generate the fractal features for the transform.

We store each transform in a memory system, indexed by and recallable via each associated fractal feature.

To determine which candidate image results in the most analogous transform to the original problem transform T, we first fractally encode that relationship between the two images A and B. Next, using each fractal feature associated with that encoding, we retrieve from the memory system those transforms previously stored as correlates of that feature (if any). Considering the frequency of transforms recalled, for all correlated features in the target transform, we then calculate a measure of similarity.

Determining Similarity The metric we employ reflects similarity as a comparison of the number of fractal features shared between candidate pairs taken in contrast to the joint number of fractal features found in each pair member (Tversky 1977). In our present implementation, the measure of similarity S between the candidate transform T' and the target transform T is calculated using the ratio model. This calculation determines the similarity between unique pairs of transforms. However, the Raven's test, even in its simplest form, poses an additional problem in that many such pairs may be formed.

Reconciling Multiple Analogical Relationships In 2x2 Raven's problems, there are two apparent relationships for which analogical similarity must be calculated: the horizontal relationship and the vertical relationship. Closer examination of such problems, however, reveals two additional relationships which must be shown to hold as well: the two diagonal relationships. Furthermore, not only must the "forward" version of each of these relationships be considered but also the "backward" or inverse version. Therefore for a 2x2 Raven's problem, we must determine eight separate measures of similarity for each of the possible candidate solutions.

The 3x3 matrix problems from the SPM introduce not only more pairs for possible relationships but also the possibility that elements or subelements within the images exhibit periodicity. Predictably, the number of potential analogical relationships blooms. At present, we consider 48 of these relationships concurrently.

Relationship Space and Maximal Similarity For each candidate solution, we consider the similarity of each potential analogical relationship as a value upon an axis in a large "relationship space." To specify the overall fit of a candidate solution, we construct a vector in this multidimensional relationship space and determine its Euclidean distance length. The candidate with the longest vector length is chosen as the solution to the problem.

Results on the Raven's Test

To create inputs for the fractal algorithm, each page from the SPM test booklet was scanned, and the resulting grayscale images were rotated to roughly correct for page alignment issues. Then, the images were sliced up to create separate image files for each entry in the problem matrix and for each answer choice. These separate images were the inputs to the fractal algorithm for each problem. The fractal algorithm attempted to solve each SPM problem independently, i.e. no information was carried over from problem to problem.

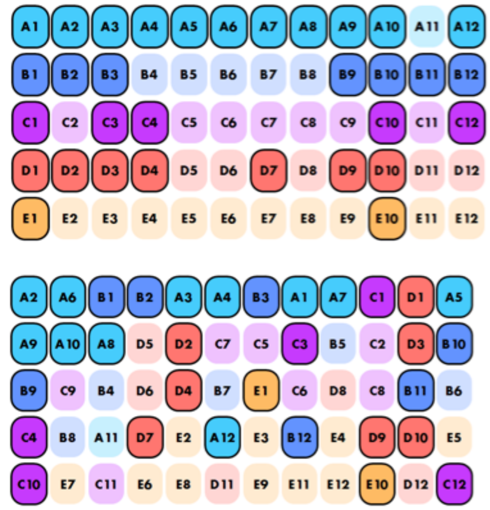

Figure 1. Fractal performance on SPM problems ordered by test ordering (a, top) and difficulty (b, bottom). Correct answers are in bold.

Our fractal algorithm obtained a total score of 32 correct out of 60 problems. Figure 1a illustrates the performance of the algorithm on all 60 problems according to test problem order; Figure 1b shows the performance with problems ordered by difficulty, as determined by normative data (Raven, Raven, & Court 1998).

There are three main assessments that can be made following the administration of the SPM to an individual: the total score, which is given simply as the number of correct answers; an estimate of consistency, which is obtained by comparing the given score distribution to the expected distribution for that particular total score; and the percentile range into which the score falls, for a given age and nationality (Raven et al. 1998).

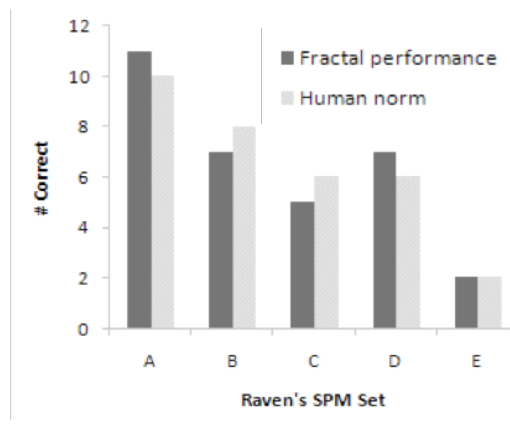

Figure 2. SPM scores ordered by set for fractal algorithm (dark) and human norms for given total score (light).

The score breakdown by set, along with the expected score composition for a total score of 32, are shown in Figure 2. A score is deemed “consistent” if the difference between actual and expected scores for any given set is no more than ± 2 (Raven et al. 1998). The score differences for the fractal algorithm on each set were no more than ±1. This score pattern illustrates that the results achieved by the algorithm fall well within typical human consistency norms on the SPM. Using norms from the United States, we find that a total score of 32 corresponds to the 50th percentile for children around 9-10 years old (Raven, Raven, & Court 1998).

Conclusions

As mentioned earlier, at ICCC-10, we presented a proposal to use fractal encodings to address the Raven’s test. Here, we have described preliminary results from this work. Many problems on these intelligence tests appear to engage cognitive processes that form the building blocks of human creativity, e.g. visual analogy. Our fractal technique works directly on visual inputs, without any need to extract propositional representations from them. The performance of our program would place it at the 50th percentile for 9-10 year olds. We believe that this technique can be enhanced significantly and we anticipate improved results in the near future.

Fractal representations are analogical representations in that they have a structural correspondence to the images they represent: the collage theorem (Barnsley & Hurd, 1992) provides a rigorous characterization of this structural isomorphism. Similarity and analogy often have been viewed as central to theories of intelligence. Hofstadter (1995), among others, has posited that analogy forms the core of human cognition. Fractal representations add the powerful idea of self-similarity.

While the use of fractal representations is central to our technique, the emphasis upon visual recall in our solution afforded by features derived from those representations is also important. We take the position that placing candidate transformations into memory, indexed via fractal features, affords a new method of discovering image similarity. That images, encoded either in terms of themselves or other images, may be indexed and retrieved without regard to shape, geometry, or symbol, suggests that the fractal representation bears further exploration not only as regards solutions to problems akin to the RPM, but also to those of general visual memory and recall.

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