A Computer Model for the Generation of Visual Compositions

Rafael Pérez y Pérez¹, María González de Cossío¹, Iván Guerrero²

División de Ciencias de la Comunicación y Diseño¹
Universidad Autónoma Metropolitana, Cuajimalpa, México D. F.
Posgrado en Ciencia e Ingeniería de la Computación²
Universidad Nacional Autónoma de México
{rperez/mgonzalezc}@correo.cua.uam.mx; cguerreror@uxmcc2.iimas.unam.mx

Abstract

This paper describes a computer model for visual compositions. It formalises a series of concepts that allows a computer agent to progress a visual work. We implemented a prototype to test the model; it employs letters from the alphabet to create its compositions. The knowledge base was built from examples provided by designers. From these examples the system obtained the necessary information to produce novel compositions. We asked a panel of experts to evaluate the material produced by our system. The results suggest that we are in the right track although much more work needs to be done.

Introduction

This text reports a computer model for visual compositions. The following lines describe the motivation behind it. One of the most important topics that a student in design needs to master is that related to visual composition. By composition we refer to the way in which elements in a graphic work are organised on the canvas. The design process of a composition implies the selection, planning and conscious organisation of visual elements that aim to communicate (Myers 1989; Deepak 2010). Compositions can be very complex with several elements interacting in diverse ways.

Unfortunately, an important number of design texts include what we called "unclear" explanations about composition and its characteristics; in many cases, they are based on personal appreciations rather than on more objective criteria. To illustrate our point, here are descriptions of the concept of visual balance found in some design texts: "Psychologically we cannot stand a state of imbalance for very long. As time passes, we become increasingly fearful, uncomfortable, and disoriented" (Myers 1989: 85); "The formal quality in symmetry imparts an immediate feeling of permanence, strength, and stability. Such qualities are important in public buildings to suggest the dignity and power of a government" (Lauer and Pentak 2012: 92); "exacting, noncasual and quiet, but can

also be boring" (Brainard 1991:96). Similar definitions can be found in Germani-Fabris (1973); Faimon and Weigand (2004); Fullmer (2012); and so on. As one can see there is a need for clearer explanations that can guide designers, teachers and students on these topics.

We believe that computer models of creativity are very useful tools that can contribute to formalize this type of concepts and, hopefully, to make them more accessible and clearer to students and the general public. Therefore, the purpose of this project is to develop a computer model of visual composition and implement a prototype. Particularly, we are interested in representing the genesis of the visual composition process; c.f. with other computer models that represent more elaborated pieces of visual works like ERI-Designer (Pérez y Pérez et al. 2010), The Painting Fool (Colton 2012), DARSY (Norton et al. 2011). Related works also include shape grammars (Stiny 1972) and relational production systems (Vere 1977, 1978). Other interesting approaches are those based in evolutionary mechanism (e.g. Goldberg 1991; Bentley 1999). However, we are interested in understanding each step in the composition process rather than look for optimization processes.

This paper is organised as follows: section 2 describes some characteristics that we consider essential in visual composition; section 3 describes the core aspects of our model; section 4 describes the core characteristics of our prototype and how we used it to test our model; section 5 discusses the results we obtained.

Characteristics of a Composition

Composition is a very complex process that usually involves several features and multiple relations between them. It is out of the scope of this project to attempt to represent the whole elements involved in a composition.

A composition is integrated by design elements and by design principles. The design elements are dots, lines, colours, textures, shapes and planes that are placed on a canvas. The design principles are the way these elements relate to

each other and to the canvas. The principles that we employ in this project are rhythm, balance and symmetry.

Rhythm is the regular repetition of elements. For regular repetition we mean that the distance between adjacent elements is constant. Groups of repeated elements make patterns. The frequency of a pattern describes how many times the same element is repeated within a given area in a canvas. Thus, the frequency depends on the size and distance between elements. A composition might include two or more patterns with the same or different frequencies.

Balance is related to the distribution of visual elements on the canvas. If there is an equal distribution on both sides of the canvas, there is a formal balance. If the elements are not placed with equal distribution, there is an informal balance. Myers describes informal balance as

"Off-centre balance. It is best understood as the principle of the seesaw. Any large, 'heavy' figure must be placed closer to the fulcrum in order to balance a smaller, 'lighter' figure located on the opposite side. The fulcrum is the point of support for this balancing act. It is a physical principle transposed into a pictorial field. The fulcrum is never seen, but its presence must be strongly felt" (1989: 90).

Symmetry, (from the Greek συμμετοείν symmetreín), "with measure", means equal distribution of elements on both sides of the canvas. The canvas is divided into many equal areas as needed. The basic divisions separate the canvas in four areas using a vertical axis and a horizontal axis. Diagonal divisions can also be included. Symmetry can be explained as follows: "Given plane A, a figure is symmetrical in relation to it, when it reflects in A, and goes back to its initial position" (Agostini 1987:97). In other words "symmetry of a (planar) picture [is] a motion of the plane that leaves that picture unchanged" (Field 1995:41). In this project we work with three types of symmetry:

- 1. Reflectional symmetry or mirror symmetry. It refers to the reflection of an element from a central axis or mirror line. If one half of a figure is the mirror image of the other, we say that the figure has reflectional or mirror symmetry, and the line marking the division is called the line of reflection, the mirror line, or the line of symmetry (Kinsey and Moore 2002:129).
- 2. Rotational symmetry. The elements rotate around a central axis. It can be in any angle or frequency, whilst the elements share the same centre. For example, in nature, a sunflower shows each element rotating around a centre.
- 3. Bilateral symmetry or translational symmetry. Refers to equivalent elements that are placed in different locations but with the same direction. "The element moves along a line to a position parallel to the original" (Kinsey and Moore 2002:148).

Description of the Model

For this work we assume that all compositions are generated on a white canvas with a fixed size. Compositions are comprised by the following elements: blank, simple elements and compound elements, also referred to as groups. Blank is the space of the canvas that is not occupied by any element. A simple-element is the basic graphic unit employed to create a visual composition. A compound-element is a group formed by simple-elements (as it will be explained later, all adjacent elements within a group must have the same distance). A compound-element might also include other compound-elements. Once a simple-element is part of a group, it cannot participate in another group as a simple-element.

All elements have associated a set of attributes:

- 1. Blank has an area.
- 2. Simple-elements have a position (determined by the centre of the element), an orientation, an area and an inclination.
- 3. Compound-elements have a position, an area, a shape, a rhythm and a size. The position is calculated as the geometric centre of the element. Compound-elements can have four possible shapes: horizontal, vertical, diagonal and any other. The rhythm is defined as the constant repetition of elements. The size is defined by the number of elements (simple or compound) that comprise the group.

There are three basic primitive-actions that can be performed on simple and compound elements: insert in the canvas, eliminate from the canvas and modify its attributes.

Relations. All elements in a canvas have relations with the other elements. Our model represents three types of relations: distance, balance and symmetry.

Distance. We include four possible distances between elements:

- Lying-on: one element is on top of other element.
- Touch: the edge of one element is touching the edge of other element.
- Close: none of the previous classifications apply and the distance between the centre of element 1 and element 2 is equal or minor to a distance known as Distance of Closeness (DC). It represents that an element is close to another element. The appropriate value of DC depends on cultural aspects and might change between different societies (see Hall 1999).
- Remote: the distance between the centres of element 1 and element 2 is major to DC.

Balance. We employ two different axes to calculate balance: horizontal and vertical. They all cross the centre of the canvas. The balance between two elements is obtained as follows. The area of each element is calculated and then multi-

plied by its distance to the centre. If the results are alike the elements are balanced. Unbalanced relations are not explicitly represented.

Symmetry. We work with three types of symmetry: reflectional (Rf), translational (Tr) and rotational (Rt). We employ two different axes to calculate it: horizontal (H) and vertical (V). So, two different elements in a canvas might have one of five different symmetric relations between them: horizontal-reflectional (H-Rf), vertical-reflectional (V-Rf), horizontal-translational (H-Tt), vertical-translational (V-Tt) and rotational (Rt). Asymmetrical relations are not explicitly represented.

Creation of Groups. Inspired by Gestalt studies in perception (Wertheimer 2012) in this work, groups are created based on the distance between its elements. The minimum distance (MD) is the smallest distance between two elements (e.g. if the distance between element 1 and element 2 is 1 cm, the distance between element 2 and element 3 is 3 cm, and the distance between element 1 and element 3 is 4 cm, MD is equal to 1 cm). Its value ranges from zero (when the centre of element 1 is lying on top of the centre of element 2) to DC.

$0 \le MD \le DC$

That is, inspired by Gestalt studies that indicate that the eye perceives elements that are close as a unit, a group cannot include elements with a remote distance.

The process of grouping works as follows. All simple-elements that are separated from other simple-elements by the same distance are grouped together, as long as such a distance is minor to the remote distance. If as a result of this process at least one group is created, the same process is performed again. The process is repeated until it is not possible to create more groups. Notice that this way of grouping produces that all groups have associate a rhythm, i.e. all groups include the constant repetition of (at least one) elements. We refer to the groups created during this process as Groups of Layer 1. Figure 1 layer 0 shows simple elements on a canvas before the system groups them; Figure 1 layer 1 shows the groups that emerge after performing this process: group 1 (the blue one), group 2 (the purple one) and group 3 (the yellow one); d1 represents the distance between elements in group 1; d2 represents the distance between elements in group 2; d3 represents the distance between elements in group 3. The following lines describe the algorithm:

First iteration, Layer 1

- 1. Considering only simple-elements find the MD value.
- 2. If there are not at least two simple-elements whose MD is equal or minor to DC then finish.

- 2. All simple-elements that are separated from other simple-elements by a distance MD form a new group.
- 3. Go to step 1.

Now, employing a similar mechanism, we can try to create new groups using the Groups of Layer 1 as inputs (see Figure 1 Layer 2). We refer to the groups created during this second process as Groups of Layer 2. Groups at layer 2 are comprised by simple-elements and/or compound-elements. The algorithm works as follows:

If at least one group was created during Layer 1 then perform Layer 2.

Second iteration, Layer 2

- 1. Considering simple and compound elements, that have not formed a group in this layer yet, find the value of the MD.
- 2. If there are not at least two elements whose MD is equal or minor to DC then finish.
- 2. All elements that are separated from other elements by a distance MD form a new group.
- 3. Go to step 1.

Notice how the blue group and the purple group merge; the reason is that the distance between purple group and the blue group (d21) is smaller than the distance between the blue group and the yellow group (d13), or the distance between the purple group and the yellow group (d23). Because there is no other group to merge, the yellow group has to wait until the next cycle (next layer) to be integrated (see Figure

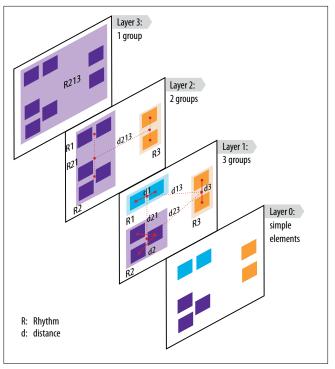


Figure 1. A composition represented by 3 layers.

1 layer 3). This process is repeated until no more layers can be created. All groups created during the first iteration are known as Groups at Layer 1; all groups created during the second iteration are known as Groups at Layer 2; all groups created during the nth iteration are known as Groups at Layer n. A composition that generates n layers is referred to as nth Layers Composition.

Calculating rhythms. The process to calculate rhythms within a composition works as follows. Each group at layer 1 has its own rhythm (see Figure 1 layer 1). So, the blue group has a rhythm 1 (R1), the purple group has a rhythm 2 (R2) and the yellow group has a rhythm 3 (R3). When the system blends the blue and purple groups, the new group includes three different rhythms (see Figure 1 Layer 2): R1, R2 and a new rhythm R21. Rhythm R21 is the result of the distance between the centre of the blue group and the centre of the purple group. We can picture groups as accumulating the rhythms of its members. So, in Figure 1 Layer 2 we can observe four rhythms: R1, R2, R21 (inside the purple group) and R3 in the yellow group. A group that includes only one rhythm is classified monotonous; a group that includes two or more rhythms is classified as varied. So, the purple blue has a varied rhythm while the yellow group has a monotonous rhythm.

Analysis of the composition. Our model represents a composition in terms of all existing relations between its elements. This representation is known as Context.

Because each layer within a composition includes different elements, and possible different relations between them, the number of contexts associated to one composition depends on its number of layers. Thus, a 3 layers composition has associated three contexts: context-layer 1, context-layer 2 and context-layer 3.

Besides relationships, a context-layer also includes information about the attributes of each element, and what we refer to as the attributes of the layer: Density of the layer, Balance of the layer, Symmetry of the layer and Rhythm of the layer. The Density of the Layer (DeL) is the relation between the blank's area and all elements' area:

The Balance of the layer and Symmetry of the layer indicate if the layer as a whole is symmetrical and is balanced. The Rhythm of the layer indicates the type of rhythm that the layer has as a whole. Like in the case of the groups it can have the following values: Monotonous or Varied (see Figure 2).

Components of a context-layer
Relation between elements
Attributes of the elements
Attributes of the layer

Figure 2. Components of a context layer.

Composition process

We can describe a composition as a process that consists on sequentially applying a set of actions, which generate several partial or incomplete works... until the right composition arises or the process is abandoned (Pérez y Pérez et al. 2010)

Thus, if we have a blank canvas and perform an action on it, we will produce an initial partial composition; if we modify that partial composition by performing another action, then we will produce a more elaborated partial composition; we can keep on repeating this process until, with some luck, we will end producing a whole composition. Thus, by performing actions we progress the composition (see Figure 3).

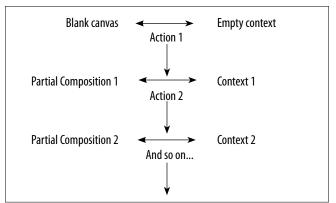


Figure 3. A composition process.

The model allows calculating for each partial composition all its contextual-layers. This information is crucial for generating novel compositions.

Producing new works

Our model includes two main processes: the generation of knowledge structures and the generation of compositions.

Generation of knowledge structures

The model requires a set of examples that are provided by human experts; we refer to them as the previous designs. So, each previous design is comprised by one or more partial compositions; each of these partial compositions is more elaborated than the previous one. At the end we have the final composition.

As explained earlier, we can picture a composition process as a progression of contexts mediated by actions until the last context is generated. In the same way, if we have the sequence of actions that leads towards a composition (and that is the type of information we can get from the set of examples), we can analyse and register how the composition process occurred. The goal is to create knowledge structures that group together a context and an action to be performed. In other words, the knowledge base is comprised by contexts (representing partial compositions) and actions to transform them in order to progress the composition.

Because the previous designs do not represent explicitly their associated actions, it is necessary to obtain them. The following lines explain how this process is done. We compare two contexts and register the differences between them. Such differences become the next action to perform. For example, if Context 1 represents an asymmetrical composition and Context 2 represents a horizontal symmetrical one, we can associate the action "make the current composition horizontally symmetrical" to Context 1 as the next action to continue the work in progress.

Once this relation has been established, it is recorded in the knowledge base as a new knowledge structure. We do the same with all the contexts in all the layers of a given partial composition. The actions that can be associated to a context are: make (reflectional, rotational or translational) symmetrical the current composition; balance (horizontally or vertically) the current composition; insert, delete or modify a simple or compound element; make (reflectional, rotational or translational) asymmetrical the current composition; unbalance (horizontally or vertically) the current composition; end the process of composition. The following lines describe the algorithm to process the previous designs.

- 1. Obtain the number of all the partial compositions of a given example (NumberPC)
- 2. Calculate all the contexts for each partial composition
- 3. For n = 1 to (NumberPC 1)
 - 3.1 Compare the differences between Context n and Context n+1
 - 3.2 Find the action that transform Context n into Context n+1
 - 3.3 Create a new knowledge structure associating Context n and the new Action
 - 3.4 Record in the knowledge base this new knowledge structure.
- 4. The context of the last partial composition gets the action "end of the process of composition".

We repeat the same process for each example in the set of previous designs. All the knowledge structures obtained in this way are recorded in the knowledge base. The bigger the set of previous designs the richer our knowledge base is.

Generation of compositions: The composition process follows the E-R model described in (Pérez y Pérez and Sharples 2001). The following lines describe how it works.

The E-R model has two main processes: engagement and reflection. During engagement the system generates material; during reflection such material is evaluated and, if necessary, modified. The composition is a constant cycle between engagement and reflection. The model requires an initial state, i.e. an initial partial composition to start; then, the process is triggered. The following lines describe how we defined engagement and reflection:

Engagement:

- 1. The system calculates all the Contexts that can be obtained from the current partial composition.
- 2. All these contexts are employed as cues to probe memory.
- 3. The system retrieves from memory all the knowledge structures that are equal or similar to the current contexts. If none structure is retrieved, an impasse is declared and the system switches to reflection.
- 4. The system selects one of them at random and performs its associated action. As a consequence the current partial composition is updated.
- 5. And the cycles repeats again (step 1).

Reflection:

- 1. If there is an impasse the system attempts to break it and then returns to the generation phase.
- The system checks that the current composition satisfies the requirements of coherence (e.g. the system verifies that all the elements are within the area of the canvas; that elements are not accidentally on top of each other; and so on).
- 3. The system verifies the novelty of the composition in progress. A composition is novel if it is not similar to any of the compositions in the set of previous designs.

The system starts in engagement; after three actions it switches to reflection and then goes back to engagement. If during engagement an impasse is declared, the system switches to reflection to try to break it and then switches back to engagement. The cycle ends when an unbreakable impasse is triggered or when the action "end of the process of composition" is performed.

Example of a composition: For space reasons, it is impossible to describe in detail how the system creates a whole new design. Instead, in Figure 4 we show some partial compositions generated by our program and their associated contexts. To create the partial-composition in Figure 4A, the system starts

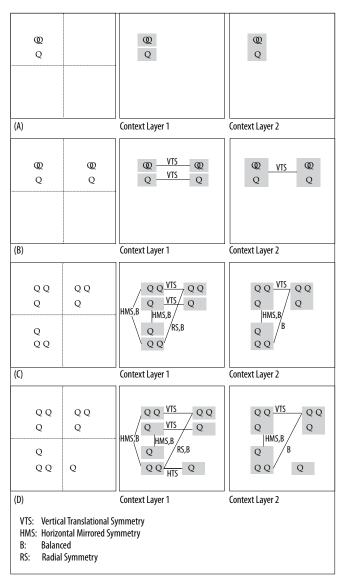


Figure 4. Partial compositions and their contexts.

with a blank canvas and then inserts three elements at random (the three elements on the top-left). This partial composition has two layers: the context of each layer is depicted on the right side of Figure 4A. For the sake of clarity the figure does not include the attributes of the elements; then, during engagement, it takes the current contexts as cues to probe memory and retrieves some actions to progress the work. Between the retrieved actions one is selected at random. So, it inserts three new elements that produce a vertical translational symmetry (see Figure 4B). The context in each layer clearly shows the relation between all elements in the canvas. In this case, in Layer 1 we have two Vertical Translational Symmetry (VTS) and in Layer 2 we have one VTS symmetry.

The system switches to reflection and realises that some elements are on top of others. Employing some heuristics to analyse the composition, the program decides that is better to separate them. The system switches back to engagement, takes the current contexts as cues to probe memory and retrieves actions to be performed. In this occasion, the system inserts in the third quadrant a new group with a horizontal mirrored symmetry (see Figure 4C). The right side of the figure shows the context at each layer. The process is repeated again generating the partial composition in Figure 4D and its corresponding contexts.

Tests and Results

We implemented a prototype to test our model. Because of the technical complexity of implementing the whole model we decided to include some constraints. In our prototype all simple-elements have the same size, colour and shape: in this work, simple elements are letters of the alphabet. Because of the technical difficulty of implementing relationships, in this prototype we only use symmetry and balance.

Like the model, the prototype has two main parts: creation of knowledge structures and generation of new compositions. The prototype has an interface that allows the user to create her own compositions. She can insert, delete or modify letters in the canvas. By clicking one button she can also build new symmetrical or balanced elements, or generate random groups. The program automatically indicates all the existing groups in all layers; it also shows all the relationships that currently exist between the elements in the canvas. In the same way, the attributes of all elements are displayed as well as their rhythms. So, the user only has to create her composition on the canvas (the program includes a partial-composition button that allows the user to indicate when a partial composition is ready). In this way, the system automatically creates the file of previous designs. Once the knowledge base is ready, the user can trigger the E-R cycle to generate novel compositions.

We provided our prototype with five previous designs; Figures 5 and 6 show two works generated by our program.

In order to obtain an external feedback we decided to ask a panel of experts their opinion about our program's work. The

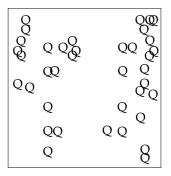


Figure 5. A composition created by our agent. It is Composition 2 in the questionnaire.

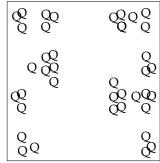


Figure 6. A second composition created by our agent. It is Composition 3 in the questionnaire.



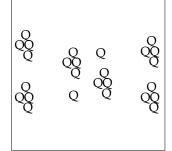


Figure 7: Human generated composition. Corresponds to composition 1 in the questionnaire.

Figure 8: Human generated composition. Corresponds to composition 4 in the questionnaire.

panel consisted of twelve designers: four men and eight women. All of them had studied a bachelor's degree in design and half of them got a postgraduate degree. We developed a questionnaire that included four compositions: two were created by our system (compositions 2 and 3, Figures 5 and 6) and two were created by a designer (composition 1 and 4, Figures 7 and 8). The human compositions had to follow similar constraints to those of our program's compositions: they had to be in black and white, the designer can only employ one letter to develop her work, and so on. The participants were not told that some works had been done by a computer program. Subjects were asked to assess in a range from 1 (lowest) to 5 (highest) four characteristics for each composition: a) whether they liked the composition, b) whether they considered that the composition had symmetry, c) whether the composition had balance and, d) what kind of rhythm the composition had. They were also invited to comment freely on each composition regarding balance and symmetry. In the last part of the questionnaire, participants were asked to rank the compositions from the best to the worst. Figure 9 shows the results of the questionnaire.

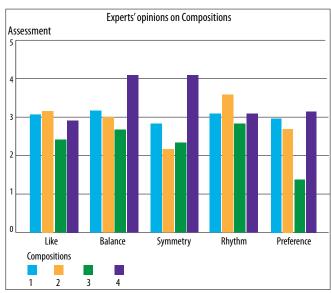


Figure 9: Results of the questionnaire.

Experts liked composition 1 and 2. This was an interesting result because it suggested that our model was capable of generating designs with an acceptable quality. It was also clear that most experts disliked composition 3 (Figure 6); although it is fair to say that their evaluation was only one point lower than the highest evaluation.

Compositions 1 and 4 (made by the human designer) had a better evaluation regarding balance and symmetry than compositions 2 and 3 (made by our program). We could have forced our program to generate symmetrical or balanced designs, but that was exactly what we wanted to avoid. Our system had the capacity of detecting such characteristics and nevertheless attempted something different. Expert's assessment on symmetry was neither clear nor unanimous. We were surprised to find this out, since symmetry does not depend on subjective judgment. Something similar occurred with balance and to some extent with rhythm. These results seemed to suggest that experts had different ways of evaluating these characteristics. Experts considered that the rhythm in Composition 2 was the best.

Overall subjects preferred composition 4; compositions 1 and 2 got similar results, with a slightly preference for composition 1; composition 3 got the lowest rank.

Discussion and Conclusions

This project describes a computer model for visual composition. The model establishes:

- A clear criteria to define simple-elements and groups.
- A set of attributes for simple-elements, groups and layers.
- Relationships between elements and a mechanism to identify such relationships.
- A method to analyse a visual composition based on layers, relationships and attributes.
- A mechanism based on the E-R model to produce novel compositions.

As far as we know, there is no other similar model. Although we are aware that many important features of compositions are not considered yet, we claim that our model allows a computer agent to produce novel visual designs.

We tested our model implementing a computer agent. The system was capable of producing compositions. None of them are alike to any of the previous designs, although some of its characteristics resemble the set of examples.

A panel of experts evaluated two compositions generated by our system and two compositions generated by a human designer. We decided to ask a small group of experts, who we believe share core concepts about design, to evaluate our prototype's compositions rather than to ask lots of people with different backgrounds. The results suggest two interesting points: 1. In most cases, the opinions of the experts were not unani-

- mous. That is, some experts found more interesting some of the characteristics of the computer-generated composition than those produced by humans.
- 2. Experts seem to have different ways of perceiving and evaluating compositions.

Point 1 suggests that our model is capable of generating interesting compositions. That is, it seems that we are moving in the right direction.

Point 2 seems to confirm the necessity of clearer mechanisms to evaluate a composition. Of course, we are not suggesting that personal taste and intuition should be eliminated from design. We are only recommending the use of clearer definitions and mechanisms for evaluations. We are convinced that they will be very useful, especially in teaching and learning graphic composition.

One of the reviewers of this paper suggested comparing our work with shape grammars (Vere 1977, 1978). Our proposal is far of being a grammar; it does not include features like terminal shape elements and non-terminal shape elements. In the same way, we do not work with shapes but with relations between the elements that comprise the composition. Those relations drive the generation of new compositions. We believe that our approach is much more flexible than the grammars approach. A second reviewer suggested comparing our work with relational productions (Stiny 1972). It is true that our work also employs the "before and after" situations described by Stiny. However, we are not interested in modelling inductive (or any other type of) learning; our purpose is to record the actions that the user performs in order to progress a composition. Later, the system employs this information to develop its own composition. None of these two approaches include characteristics like a flexible generation process intertwined to an evaluation process, analysis by layers of the relations between the elements that comprise a composition, and other characteristics that our approach does. Thus, although some of the features that our model employs remind us of previous works, we claim that our approach introduces interesting novel features.

We hope this work encourage other researches to work on visual composition generation.

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