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On the Performance of Eigenvalue-Based Cooperative Spectrum Sensing for Cognitive Radio

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Abstract—In this paper, the distribution of the ratio of extreme eigenvalues of a complex Wishart matrix is studied in order to calculate the exact decision threshold as a function of the desired probability of false alarm for the maximum-minimum eigenvalue (MME) detector. In contrast to the asymptotic analysis reported in the literature, we consider a *finite* number of cooperative receivers and a *finite* number of samples and derive the *exact* decision threshold for the probability of false alarm. The proposed exact formulation is further reduced to the case of two receiver-based cooperative spectrum sensing. In addition, an approximate closed-form formula of the exact threshold is derived in terms of a desired probability of false alarm for a special case having equal number of receive antennas and signal samples. Finally, the derived analytical exact decision thresholds are verified with Monte-Carlo simulations. We show that the probability of detection performance using the proposed exact decision thresholds achieves significant performance gains compared to the performance of the asymptotic decision threshold.

Index Terms—Complex Wishart matrix, cooperative spectrum sensing, eigenvalue-based detection.

I. INTRODUCTION

COGNITIVE radio (CR) is defined as an intelligent wireless communication system that provides more efficient communication by allowing secondary users to utilize the unused spectrum segments. In order to make this a reality, spectrum management has to be done efficiently in cognitive radio networks. The spectrum management is composed of four major steps as defined in [1]: sensing, decision making, sharing, and mobility. Among these, the spectrum sensing and decision making are the most important constituents for the establishment of cognitive radio networks. CR users should detect the primary user networks to find the spectrum holes or the unused spectrum to utilize them effectively for cognitive access. At the same time, they should prevent interference to the primary users due to their cognitive access of the channels.

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A number of spectrum sensing algorithms such as the energy detection [2]–[4], the eigenvalue-based detection [5]–[7], the covariance-based detection [8], [9], and cyclostationary-based (or feature-based) detection [10]–[12] are reported in the literature to detect the primary transmitter. Pros and cons of these different techniques are discussed in many studies, for example, see [12]–[14]. One of the most accurate techniques that can simultaneously achieve both high probability of detection and low probability of miss alarms without requiring information of primary user signals and noise power are the eigenvalue-based detection techniques proposed by Zeng and Liang [15]. There are three major eigenvalue-based detection techniques studied in the literature: 1) maximum–minimum eigenvalue (MME) detection; 2) energy with minimum eigenvalue (EME) detection; 3) maximum eigenvalue detection (MED). In [15], it has been shown that in the MME type of eigenvalue-based detection method, the ratio of the maximum eigenvalue to minimum eigenvalue can be used to detect the signal. MME has many advantages over the rest of the sensing methods reported in the literature. This is because, unlike other methods, the decision on presence of the signal can be done irrespective of the knowledge of the signal and the noise properties.

In the eigenvalue-based methods, the expression for the decision threshold has been derived based on random matrix theory to make a hypothesis testing. In most of the eigenvalue-based detection schemes proposed so far in the literature, both the threshold value and the probabilities of detection and false alarm are calculated based on the asymptotical (limiting) distributions of eigenvalues that is mathematically tractable and less complex. In [7], it is mentioned that the largest and the smallest eigenvalues are approximated to deterministic values based on theorems in [16] and [17], respectively. For large number of received signal samples, the probability of false alarm of the MME detection is formulated in terms of Tracy–Widom distribution [18], which is calculated based on limiting distributions of eigenvalues.

All of the above analysis assume that the number of signal samples and the number of cooperative receivers approach infinity. However, in practical situations, these are finite and considerably small to achieve optimal performances and efficient spectrum usage. Therefore, estimating the exact threshold expression for finite number of collected samples and cooperative receivers is of great interest. However, finding the exact distribution of the ratio of extreme eigenvalues of complex random matrix is difficult. The goal of this paper is to derive the exact threshold expressions of the MME detection-based spectrum sensing. Specifically, the proposed novel exact threshold

achieves lower probability of false alarm and higher detection probability in practical scenarios. The exact threshold is derived using a function of the exact distribution of the condition number of complex Wishart matrix which is expressed in terms of complex hypergeometric functions or multiple integral forms [19]. The proposed exact expression works extremely well for finite number of samples and finite number of cooperative receivers as opposed to the asymptotical approaches proposed in the literature. This formulation is further reduced to compact density expression for the systems using two receiving antennas. The asymptotic threshold formula reported in literature turns out to be infinite when the equal number of receiving antennas and samples are used. Therefore, an approximation of the threshold function is derived as an extremely simple formula for decision threshold in terms of complexity for the systems having equal number of receiving antennas and samples. It should be noted that in [20], the authors also have studied a similar problem using a different approach, where the exact eigenvalue ratio probability density function (pdf) is derived using the expression of the joint distributions of an arbitrary subset of ordered eigenvalues of complex Wishart matrices. However, similar to the asymptotic approach which requires a look-up table for computation of inverse CDF of the second-order Tracy–Widom distribution, in the approach given in [20] also, the receiver should be provided with a look-up table in order to calculate the proposed inverse CDF. The proposed exact threshold given in this study does not need a look look-up table.

The rest of the paper is organized as follows. Section II introduces the cooperative signal detection model for spectrum sensing. In Section III, the proposed exact decision threshold is derived for MME spectrum sensing technique. A closed-form approximation for the exact threshold is also presented in this section. Section IV contains the simulation and analytical results of the proposed exact formulation of the decision threshold as well as the probability of detection performances of the same. Finally, the conclusion is given in Section V. The mathematical proofs are given in the Appendix.

II. COOPERATIVE SPECTRUM SENSING

Consider a cooperative spectrum sensing problem with m cognitive users and p ($m > p$) primary users and each user is equipped with single antenna. During the sensing period, the cognitive user received signals, $\boldsymbol{y} = [y_1, \ldots, y_m]^T$, under the two hypothesizes (H_0—absence of primary signal and H_1—presence of primary signal) can be written in a vector form as

$$H_0 : \boldsymbol{y} = \boldsymbol{v} \tag{1}$$
$$H_1 : \boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} \tag{2}$$

where $\boldsymbol{y}, \boldsymbol{v} \in \mathbb{C}^m$, $\boldsymbol{H} \in \mathbb{C}^{m \times p}$, $\boldsymbol{x} \in \mathbb{C}^p$, and $\boldsymbol{v} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_v^2 \boldsymbol{I}_m)$. Noted that the noise $\boldsymbol{v}$ is independent of the primary user signal $\boldsymbol{x}$ and channel matrix $\boldsymbol{H} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_m \otimes \boldsymbol{I}_p)$, where each element h_{ij} denotes the channel coefficient between jth primary transmitter to ith cognitive receiver. Here, we assume $\boldsymbol{x}$ as circularly symmetric complex Gaussian noise, i.e., $\boldsymbol{x} \sim \mathcal{CN}(0, \Sigma)$.

The statistical covariance matrix of the cognitive user received signal can be written under the two hypothesizes as follows:

$$\boldsymbol{R}_{yy} = \mathcal{E}\{\boldsymbol{y}\boldsymbol{y}^\dagger\} = \begin{cases} \sigma_v^2 \boldsymbol{I}_m & H_0 \\ \boldsymbol{H}\Sigma\boldsymbol{H}^\dagger + \sigma_v^2 \boldsymbol{I}_m & H_1. \end{cases} \tag{3}$$

Moreover, the eigenvalues, $\lambda_{\max} = \lambda_1 > \cdots > \lambda_m = \lambda_{\min}$, of $\boldsymbol{R}_{yy}$ under the H_0 is given by $\lambda_{i|H_0} = \sigma_v^2 \ \forall \ i$, and under the H_1 is given by

$$\lambda_{i|H_1} = \begin{cases} \sigma_i^2 + \sigma_v^2 & 1 \le i \le p \\ \sigma_v^2 & p \le i \le m \end{cases} \tag{4}$$

where $\sigma_1^2, \ldots, \sigma_p^2$ represent the primary user received signal powers. It is clear that the ratio of the extreme eigenvalues of the statistical covariance matrix $\boldsymbol{R}_{yy}$ is a good test statistic T to differentiate the two hypothesizes H_0 and H_1, i.e.,

$$T = \frac{\lambda_{\max}}{\lambda_{\min}}. \tag{5}$$

Test statistic $T = 1$ under H_0 and $T > 1$ under H_1. In practice, we only have finite duration of sensing time to detect the primary user signals; hence, we use sample covariance matrix to formulate the test statistic. It should be noted that the sample covariance matrix of the received signal is going to be complex Wishart matrix for the noise only case, i.e.,

$$\widehat{R}_{vv} = \frac{1}{n} \sum_{k=1}^{n} v(k)v(k)^\dagger \tag{6}$$

$$n\widehat{R}_{vv} = \boldsymbol{V}\boldsymbol{V}^\dagger = \boldsymbol{W} \tag{7}$$

where $\boldsymbol{V} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_v^2 \boldsymbol{I}_m \otimes \boldsymbol{I}_n)$, $\boldsymbol{W}$ is the complex central Wishart matrix and its distribution is denoted by $\mathcal{CW}_m(n, \sigma_v^2 \boldsymbol{I}_m)$. The condition number $\mathrm{cond}(\boldsymbol{V})$ of a matrix $\boldsymbol{V}$ is defined as the positive square root of the ratio of the largest to the smallest eigenvalues of the positive definite Hermitian matrix $\boldsymbol{W} = \boldsymbol{V}^\dagger \boldsymbol{V}$. Thus,

$$\mathrm{cond}(\boldsymbol{V}) = \sqrt{\lambda_{\max}/\lambda_{\min}} = \|\boldsymbol{V}\|_2 \|\boldsymbol{V}^{-1}\|_2 \tag{8}$$

and

$$\mathrm{cond}(\boldsymbol{W}) = \mathrm{cond}(\boldsymbol{V})^2$$

where the ℓ_2-norms of the matrix $\boldsymbol{V}$ and the vector $\boldsymbol{x}$ are

$$\|\boldsymbol{V}\|_2 = \sup_{\boldsymbol{x} \ne 0} \|\boldsymbol{V}\boldsymbol{x}\|_2 / \|\boldsymbol{x}\|_2$$

and

$$\|\boldsymbol{x}\|_2 = \left(x_1^2 + x_2^2 + \cdots + x_n^2\right)^{1/2}$$

respectively. We assume that the eigenvalues of $\boldsymbol{W}$ are ordered in strictly decreasing order, $\lambda_{\max} = \lambda_1 > \cdots > \lambda_m = \lambda_{\min} > 0$, since the probability that any eigenvalues of $\boldsymbol{V}$ are equal is zero. The distributions of the test statistic and condition number are given in the next section; see details in [19].

A. Threshold Determination

In the general model of the spectrum sensing, a threshold must be determined to compare with a test (also called the decision) statistics of the sensing metric in order to determine the

presence of a primary user. Depending on the detector, the test statistic T can vary, for example, the decision statistic T is the power of the received signal for energy-based detectors; the spectral correlation density function of the received signal for cyclostationarity-based (or feature-based) detection; the ratio of maximum to minimum eigenvalues of the received signal co-variance matrix for MME-based detection, etc. Depending on the decision statistics, the threshold can be formulated from the formulations of the probability of detection (P_d) or probability of false-alarm (P_{fa}) as follows:

$$P_{fa} = P(T > \gamma | H_0) = \int_\gamma^\infty f_0(t) dt \qquad (9)$$

$$P_d = P(T > \gamma | H_1) = \int_\gamma^\infty f_1(t) dt \qquad (10)$$

where $f_0(t)$ and $f_1(t)$ are the pdf of the test sadistic T under the hypotheses H_0 and H_1, respectively. In energy detector, under both hypotheses, the test statistic is a random variable and can be approximated by a Gaussian distribution using the central limit theorem for large value of sample size [21]. Therefore, it is possible to define the threshold in terms of both P_d and P_{fa}. However, in the case of MME-based detection method, it is difficult to derive the density of the decision statistic for the hypothesis of signal-plus-noise, since the density of ratio of maximum-to-minimum eigenvalues of the covariance matrix of the received signal is unknown. Therefore, the decision threshold is calculated based on P_{fa} rather than P_d since density of ratio of the extreme eigenvalues of the received covariance matrix tractable in noise only case. It should be noted that in the noise-only case, the density of ratio of maximum-to-minimum eigenvalues of a complex Wishart matrix is the required information to calculate the P_{fa}.

III. MAXIMUM-MINIMUM EIGENVALUE DETECTION

Based on the decision statistic given in (5), the sensing threshold, γ, must be estimated for a required probability of false alarm. To define the threshold in terms of P_{fa} or vice versa, the density of test statistic T is required. The density can be found either from asymptotically or by using exact number of samples. In most of the eigenvalue based detection schemes proposed in the literature, both the threshold value and the probabilities of detection and false alarm are derived based on asymptotical (limiting) distributions of eigenvalues that is mathematically tractable and less complicated as explained next.

A. Asymptotic Threshold

An asymptotic formula of sensing threshold in terms of desired probability of false alarm for MME has been proposed in [15]. For a complex signal, the sensing threshold in terms of desired probability of false alarm is calculated by using the results of the theorems, [16] and [17], as follows:

$$\gamma = \frac{(\sqrt{n} + \sqrt{m})^2}{(\sqrt{n} - \sqrt{m})^2} \left(1 + \frac{(\sqrt{n} + \sqrt{m})^{2/3}}{(nm)^{1/6}} F_2^{-1}(1 - P_{fa}) \right) \qquad (11)$$

where F_2^{-1} denotes the inverse of cumulative distribution function (cdf) of the Tracy–Widom distribution of order 2

[18]. The threshold expression in (11) is formulated based on the deterministic asymptotic values of the minimum and maximum eigenvalues of the covariance matrix $\hat{R}_{vv}$. In order to use this formulation in practice, we need large number of collaboratively sensing cognitive receivers. In addition to this, the sample size of the signal has to be significantly large; thus, this will lead to larger sensing time while calculating sample covariance matrices so resulting in higher computational complexity for estimating the thresholds. An alternative asymptotic threshold formula expression is formulated in [22], where the threshold needs a look-up table for the computation of the proposed inverse cdf which outperforms the asymptotic threshold given [5].

In order to estimate the sensing threshold accurately for practical scenarios for limited number of samples and finite collaborative cognitive receivers, the exact threshold expression is needed to be formulated. This threshold should achieve a lower P_{fa} and higher P_d. It should be noted that low probability of false alarm P_f offers more chances for secondary user to utilize the spectrum hole. Thus, low P_f and high P_d should be targeted in order to reliably decide on the two hypotheses. Thus, the main interest of this paper is to derive exact threshold expression as explained next.

B. Exact Threshold

In this section, we formulate the sensing threshold in terms of the desired P_{fa} based on the exact density of the condition number of complex Wishart matrix. Building on authors previous result in [19], where the density of condition number is given in multiple integrations form, here we obtain a simplified expression by solving these multiple integration. We have the following theorem.

Theorem 1: (Ratnarajah [19]) Let an $n \times m$ complex Gaussian random matrix $\boldsymbol{A}$ be distributed as $\boldsymbol{A} \sim CN(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_n \otimes \boldsymbol{I}_m)$ with mean $\mathcal{E}\{\mathcal{A}\} = \boldsymbol{0}$ and covariance $\text{cov}\{\boldsymbol{A}\} = \sigma^2 \boldsymbol{I}_n \otimes \boldsymbol{I}_m$. Then the complex central Wishart matrix and its distribution is denoted by $\boldsymbol{W} = \boldsymbol{A}^H \boldsymbol{A} \sim CW_m(n, \sigma^2 \boldsymbol{I}_m)$. The condition number of a Wishart matrix $\boldsymbol{W}$ is defined as $\text{cond}(\boldsymbol{W}) = \lambda_{\max}/\lambda_{\min} = T$ and the density of $y = 1 - 1/\text{cond}(\boldsymbol{W}) = 1 - 1/T$ is given by

$$f_y = \int_0^\infty f_{\lambda_{\max},y} d\lambda_{\max}, \quad 0 < y < 1 \qquad (12)$$

where $f_{\lambda_{\max},y}$ is the joint density of $\lambda_{\max}$ and y given by

$$f_{\lambda_{\max},y} = \frac{\pi^{m(m-1)} (\sigma^2)^{-nm}}{C\Gamma_m(m) C\Gamma_m(n)} \lambda_{\max}^{mn-1} e^{-\frac{\lambda_{\max}(m-y)}{\sigma^2}}$$
$$\times y^{m^2-2} (1-y)^{n-m} \det(\boldsymbol{\Delta}) \qquad (13)$$

where $C\Gamma_m(a)$ is the complex multivariate gamma function defined as

$$C\Gamma_m(a) = \pi^{m(m-1)/2} \Pi_{k=1}^m \Gamma(a-k+1) \qquad (14)$$

and $\boldsymbol{\Delta}$ is defined as

$$[\boldsymbol{\Delta}]_{i,j} = \sum_{k=0}^{n-m} \binom{n-m}{k} (-y)^k (A_1 - 2A_2 + A_3) \qquad (15)$$

with

$$A_1 = \frac{_1F_1(i+j+k+1;i+j+k+2;\beta)}{i+j+k+1} \quad (16)$$

$$A_2 = \frac{_1F_1(i+j+k+2;i+j+k+3;\beta)}{i+j+k+2} \quad (17)$$

and

$$A_3 = \frac{_1F_1(i+j+k+3;i+j+k+4;\beta)}{i+j+k+3} \quad (18)$$

where $\beta = (1/\sigma^2)\lambda_{\max}y$ and $_1F_1(.;.;.)$ is confluent hypergeometric function of the first kind.

Proof: The key step is to solve the multiple integration in the expression of the joint density of $\lambda_{\max}$ and y given in [19]

$$\varrho(\psi, m-2, 2, 0, 1) = \frac{1}{\Gamma(m-1)} \int_0^1 \prod_{k=1}^m (x_k^T \psi(x_k))$$
$$\times \left(\det\left(x_i^{j-1}\right)\right)^2 \bigwedge_{k=1}^m dx_k. \quad (19)$$

where

$$\psi(x) = (1-x)^2(1-yx)^{n-m}e^{\beta x}, \quad 0 < \lambda_{\max} < \infty \quad (20)$$

and $0 < y < 1$. This is done with the technique proposed in [23], which gives

$$\varrho(\psi, m-2, 2, 0, 1) = \det(\Delta) \quad (21)$$

where Δ is an $(m-2) \times (m-2)$ matrix with entries defined by

$$[\Delta]_{i,j} = \int_0^1 x^{i+j}\psi(x)dx \quad (22)$$

the integration can be further simplified as

$$[\Delta]_{i,j} = \int_0^1 x^{i+j}(1-2x+x^2)$$
$$\times \left(\sum_{k=0}^{n-m}\binom{n-m}{k}(-y)^k x^k\right)e^{\beta x}dx$$
$$= \sum_{k=0}^{n-m}\binom{n-m}{k}(-y)^k$$
$$\times \int_0^1 x^{i+j+k}(1-2x+x^2)e^{\beta x}dx. \quad (23)$$

To this end, the desired result can be obtained with the help of the following integration identity:

$$\int_0^1 x^{a-1}e^{\beta x} = \frac{_1F_1(a;1+a;\beta)}{a}. \quad (24)$$

∎

The required density of the test statistics T and the density of $y = 1 - 1/T$ given in Theorem 1 have one-to-one relation-

ship. Using the density of y, we compute the exact value of the threshold for the target P_{fa} from the following equation:

$$P_{fa} = P(T \geq \gamma)$$
$$= P\left(\frac{1}{T} \leq \frac{1}{\gamma}\right)$$
$$= P\left(1 - \frac{1}{T} \geq 1 - \frac{1}{\gamma}\right)$$
$$= P\left(y \geq 1 - \frac{1}{\gamma}\right)$$
$$= \int_{\frac{\gamma-1}{\gamma}}^{\infty} f_y dy. \quad (25)$$

As shown from (25), the exact threshold can be calculated from the density given in (12). So, the P_{fa} can be tuned in order to find the exact threshold γ. The exact value of the threshold can be computed for any finite number of receiver (m) and the number of samples (n). The numerical values of the exact threshold values are found from the numerical calculation of the integral given in (25), in the following section.

C. Exact Closed Form Expression for $m = 2$

From Theorem 1, we have the following corollary.

Corollary 1: If $m = 2$, then the density of the condition number can be expressed as

$$f_T = \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)}\left(1-\frac{1}{T}\right)^2\left(\frac{1}{T}\right)^n\left(1+\frac{1}{T}\right)^{-2n}.$$

Proof: See Appendix. ∎

D. Approximate Closed Form Expression for Large $m = n$

We have the following proposition.

Proposition 1: If the number of cooperative receivers (m) and the number of samples (n) are equal (i.e., $n = m$) and both m and n are large, then the density of the ratio of maximum to minimum eigenvalues can be approximated as

$$f_T = \frac{4n^2}{T^2}e^{-\frac{4n^2}{T}}. \quad (26)$$

Proof: The pdf of the decision statistic T is derived from the given density of ratio of condition number to number of samples in [24, p. 71]. ∎

This expression is not only extremely simple, it is also very accurate, especially for large m and n as shown in the simulation results. Based on this simple expression, we derive a closed-form expression for the threshold given the false alarm rate in the following. First, the cumulative distribution function can be obtained as

$$F_T = e^{-\frac{4n^2}{T}}. \quad (27)$$

Then, the false alarm rate is given in closed-form by

$$P_{fa} = P(T \geq \gamma)$$
$$= 1 - F(\gamma)$$
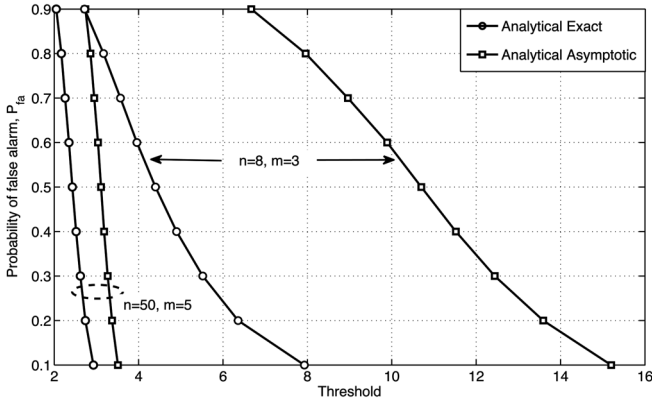$$= 1 - e^{-\frac{4n^2}{\gamma}}. \quad (28)$$

Fig. 1. Probability of false alarm versus decision threshold for analytically and empirically estimated exact threshold values and asymptotically estimated threshold values. Symbols "∘" and "⋄" denote the empirical results.

Fig. 2. Probability of detection versus SNR at probability of false alarm 0.1 ($P_{fa} = 0.1$) using exact and asymptotic thresholds.

Hence, we have $-(4n^2/\gamma) = \ln(1 - P_{fa})$ and

$$\gamma_{app} = \frac{-4n^2}{\ln(1 - P_{fa})}. \tag{29}$$

With the expression given above, the threshold value can be calculated very easily for a desired P_{fa}. Moreover, γ_{app} converges almost surely to exact threshold value as $n \to \infty$. This convergence can be observed from the simulation results given in Fig. 4.

IV. SIMULATION RESULTS

In the simulations, single and multiple transmitter systems having different number of receivers ($m = 3, 5, 10, 100$) are used to evaluate the performances of probability of detection and false-alarms when $n = 8, 10, 50,$ and 100 number of samples are used. The results are averaged over minimum 10^5 tests using Monte-Carlo simulations written in Matlab. Simulation results are obtained using BPSK modulated random primary signal and independent and identically distributed (i.i.d.) noise samples with Gaussian distributed.

Fig. 1 shows the exact analytical and asymptotic threshold versus probability of false alarm. We also plotted the empirical results for each cases and denoted as "∘" and "⋄," respectively. As shown in this figure, one may infer that the threshold values of the asymptotic approach given in (11) significantly deviate from the exact threshold values when m and n are small. It is also highlighted that the analytically and empirically estimated exact threshold values are very close to each other in all of the cases considered.

Fig. 2 illustrates the performance of probability of detection P_d for the scenarios considered in Fig. 1. The detection performances with the exact thresholds are significantly higher than that of the asymptotically estimated threshold values. For comparison, Fig. 3 shows the exact and asymptotic probability of detection versus SNR for different number of transmitters. The results are taken for $n = 50$, $m = 5$ and $P_{fa} = 0.1$. The corresponding exact and asymptotic threshold values for theses parameters are 2.93 and 3.51, respectively. The plot reveals the following fact that higher number of transmitter (or primary users) reduce the detection performance.
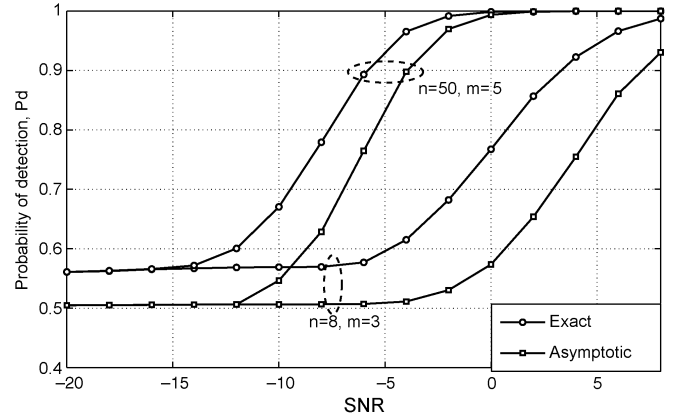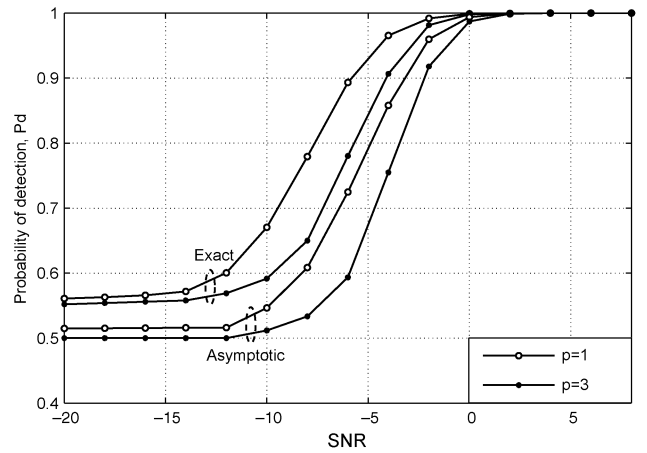

Fig. 3. Exact and asymptotic probability of detection versus SNR at $P_{fa} = 0.1$ for different number of transmitters (or primary users) with $n = 50$ and $m = 5$.
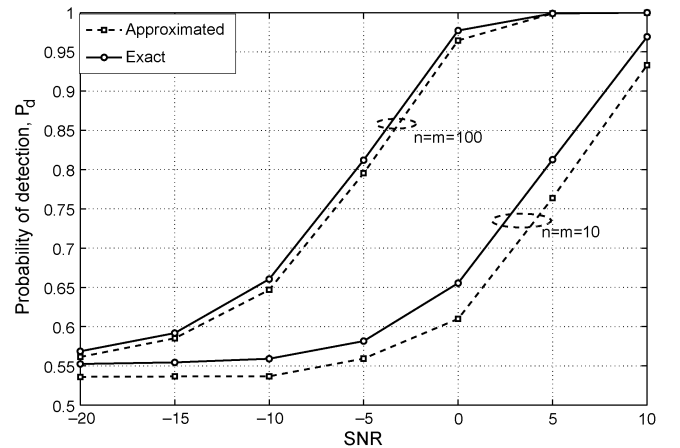
Fig. 4. Probability of detection versus SNR at $P_{fa} = 0.1$.

As it is derived in Proposition 1, decision thresholds of the systems having $n = m$ can be approximated by using a very simple formula given in (29). Fig. 4 shows the probability of detection versus SNR for the approximated threshold for different values of n. One may observe that for a given $P_{fa} = 0.1$, the probability of detection increases with SNR. It is clear that the detection performance with the approximated threshold value

performs extremely well and performs very close to the performance using the exact threshold for higher n and m. It should be noted that the approximated formula for the exact threshold value can be calculated extremely simply compared to that of the exact threshold value. At the same time, it provides improved performance compared to that of the asymptotic threshold based performance.

V. Conclusion

In this paper, we derived the exact decision threshold as a function of the desired probability of false alarm for MME-based cooperative spectrum sensing in cognitive radio. This is based on the actual distribution of the ratio of the extreme eigenvalues of the complex Wishart matrix. The expression for the decision threshold was simplified for two receiving antenna or for the case of two user collaborative sensing. We also derived a simpler closed-form threshold function using an asymptotic distribution with equal numbers of receive antennas and signal samples, that is $m = n$ and large n. Simulations using i.i.d. Gaussian noise and BPSK signals were presented in order to verify the derived threshold values based on the probability of detection performance. It has been shown that analytical and empirical results are coincide with each other. Moreover, the probability of detection performance using the proposed exact decision thresholds achieve significant performance gains compared to the performance using the asymptotic decision threshold reported in the literature, which leads to efficient spectrum usage.

Appendix

When $m = 2$, the density of the function y given in (12) can be reduced to

$$f(y) = \int_0^\infty \frac{1}{\prod_{k=1}^m (m-k)! \prod_{k=1}^m (n-k)!} \lambda_{\max}^{2n-1}$$
$$\times e^{-\lambda_{\max}(2-y)} y^2 (1-y)^{n-2} d\lambda_{\max} \quad (30)$$

which can be simplified as

$$f(y) = \frac{y^2(1-y)^{n-2}}{\Gamma(n)\Gamma(n-1)} \int_0^\infty \lambda_{\max}^{2n-1} e^{-\lambda_{max}(2-y)} d\lambda_{\max}. \quad (31)$$

Noticing that the integration can be solved with the help of the following identity

$$\int_0^\infty x^t e^{-\mu x} dx = t! \mu^{-t-1}, \quad [Re\, \mu > 0] \quad (32)$$

we obtain the density expression of y as

$$f_y = \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)} y^2 (1-y)^{n-2} (2-y)^{(-2n)}. \quad (33)$$

To this end, the density of T can be derived from (33) as

$$f_T = \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)} \frac{1}{T^2} \left(1 - \frac{1}{T}\right)^2 \left(\frac{1}{T}\right)^{n-2} \left(1 + \frac{1}{T}\right)^{-2n}.$$

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