

# An experimental investigation of chatter effects on tool life

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**Abstract:** Tool wear is one of the most important considerations in machining operations, as it affects surface quality and integrity, productivity, and cost. The most commonly used model for tool life analysis is the one proposed by F. W. Taylor about a century ago. Although the extended form of this equation includes the effects of important cutting conditions on tool wear, tool life studies are mostly performed under stable cutting conditions where the effect of chatter vibrations are not considered. This paper presents an empirical attempt to understand tool life under vibratory cutting conditions. Tool wear data are collected in turning and milling on different work materials under stable and chatter conditions. The effects of cutting conditions as well as severity of chatter on tool life are analysed. The results indicate significant reduction in tool life on account of chatter, as expected. They also show that the severity of chatter, and thus the vibration amplitude, greatly reduce the life of cutting tools. These results can be useful in evaluating the real cost of chatter by including the reduced tool life. They can also be useful in justifying the cost of chatter suppression and more rigid machining systems.

**Keywords:** turning, milling, tool life, chatter vibrations

## 1 INTRODUCTION

Tool life is one of the critical factors in machining processes, affecting cost and productivity, and has been investigated in great detail over the past century since the legendary work of Taylor [1–3]. The research on tool wear has improved the understanding of wear mechanisms for different work and tool materials in various machining operations. It has also established the foundations for improved cutting tools and increased productivity. Similarly to tool wear, vibrations, particularly self-excited chatter vibrations, are very critical in machining processes. Although cutting stability has been studied in detail over the past half-century [4–7], chatter vibrations continue to be one of the most important limitations in production operations. In some cases, machining is carried out under chatter conditions owing to the very low dynamic rigidity of the machining system. This means that, in order to reduce the cycle time,

material removal rates higher than the stable limits are used. Tool wear tests, on the other hand, are mainly performed under stable cutting conditions, which cannot explain the wear behaviour in vibratory cutting. The purpose of the present work is to investigate the effects of vibrations on tool wear. This is important for understanding the wear in dynamic cutting conditions. In addition, it is useful in estimating the cost of chatter in terms of reduced tool life in production operations. The information can also be used in justifying the additional cost of rigid tooling and machine tools and of implementing chatter suppression methods.

Tool life is established on the basis of the maximum wear that can be tolerated in a process for the required surface and dimensional quality, surface integrity, etc. There are several types of wear on the cutting tools. Flank wear is an important form of tool wear, as it directly affects the machined surface. Flank wear develops on the flank of the tool along the cutting edge engaged with the work material. The contact zone between cutting tool and workpiece surface becomes larger as the flank wear increases, causing extra rubbing on the surface, which results in poor surface finish and increased cutting temperature. Increased

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cutting forces due to wear may also result in damage to the tool and workpiece [7]. The criterion that is most commonly used sets a limit for flank wear to be reached for the end of tool life.

Self-excited vibrations, or chatter, develop as a result of the dynamic interaction between the structures and the cutting process in machining operations. The wavy surface left by vibrations is removed in the next pass in a dynamic cutting process. The vibrating tool and the wavy surface result in modulated chip thickness, which causes periodically varying cutting forces to excite the machine and workpiece structures. Under certain conditions, the amplitude of the vibrations grows continuously, resulting in instability, i.e. self-excited chatter vibrations. The fundamental mechanism of chatter has been investigated and analysed, starting with work by Tobias [4] and Tlustý and Poláček [5], over the past half-century. Since then, many models have been developed for the analysis of chatter vibrations and prediction of chatter stability limits for different machining processes [6–9]. One of the most significant outcomes of these models is the stability diagram, which is used to determine the chatter-free depth of cuts for different cutting speeds [7–12].

Although effects of various conditions, such as machine and part dynamics, cutting parameters, tool geometry, etc., on chatter have been investigated in detail, research into the effects of chatter on tool life has been limited. However, production engineers and machine tool operators know very well that vibrations reduce tool life and may damage the cutting edge in the case of severe instability. The impacts due to the vibration between the work material and the cutting edge accelerate tool wear; however, the amount of reduction in tool life has not been investigated, and this is the focus of the present analysis. Cutting speed, stiffness, and vibration frequency of the cutting system, and vibration amplitude were selected as the fundamental parameters affecting the wear, and were included in experiments conducted on turning and milling machines using different work materials.

The paper is organized as follows. The basics of tool wear and chatter vibrations are reviewed in the next section. The experimental set-up and test conditions are presented in section 3. The results of the tests are presented and discussed in section 4. The paper is concluded with the main observations and potential future work.

## 2 TOOL LIFE AND CHATTER

### 2.1 Tool life

The total cutting time corresponding to the maximum allowable wear is called the tool life. The

relation between cutting speed and tool life was first investigated by Taylor [1], who expressed this relation in the following form

$$VT^n = C' \quad (1)$$

where  $V$  is the cutting speed,  $T$  is the tool life, and  $C'$  and  $n$  are experimentally identified constants that depend on the work and tool materials. The effects of other cutting conditions, i.e. chip thickness and depth of cut, were neglected in the basic tool life equation. Their effect on tool life is included in the extended tool life equation [10]

$$VT^n d^x f^y = C \quad (2)$$

where  $f$  is the feed rate,  $d$  is the depth of cut, and  $C$ ,  $x$ , and  $y$  are experimentally identified constants similar to those in equation (1). The effect of vibration on tool wear has not been considered in previous studies on tool life, although it is common knowledge that the wear rate under dynamic conditions is higher, and thus the resulting tool life is usually much shorter than that predicted by equation (1) or (2).

### 2.2 Chatter vibrations

There are several types of vibration that may arise in machining processes. Compared with free and forced vibrations, self-excited chatter vibrations are much more detrimental to finished surfaces and cutting tools owing to their unstable behaviour, which may result in large-amplitude relative displacements between the cutter and workpiece. Chatter develops at one of the natural modes of the cutting system including the tool, workpiece, machine tool, fixture, etc. Under vibrations, the chip thickness becomes modulated, which in turn creates dynamic cutting forces at a frequency close to one of the natural modes, further exciting the system. If the vibration amplitude does not diminish, chatter develops and the system becomes unstable. Chatter research has shown that the depth of cut is the most critical factor affecting the stability of the cutting process. The limiting depth of cut in an orthogonal cutting process can be determined from stability analysis as follows [7]

$$b_{\text{lim}} = -\frac{1}{2K_f G_R(\omega_{\text{ch}})} \quad (3)$$

where  $K_f$  is the cutting force coefficient in the feed direction, and  $G_R(\omega_{\text{ch}})$  is the real part of the transfer function at the chatter frequency  $\omega_{\text{ch}}$  in the chip thickness direction. The chatter frequency varies with cutting speed according to the following relation [7]

$$\frac{\omega_{\text{ch}}}{\omega_s} = 2\pi m + \varepsilon \quad (4)$$

where  $\omega_s$  is spindle speed (rad/s),  $m$  is an integer indicating the number of full waves in one revolution

of the part, and  $\varepsilon$  is the fraction of a full wave, i.e. the phase difference between subsequent vibration waves. Equation (4) shows the variation in chatter frequency with cutting speed. Stability diagrams that show the variation in stability limit with cutting speed can be generated using the above equations. This model has also been used for rotary tool processes such as milling, with some assumptions and approximations resulting in reduced accuracy for stability limit prediction. The main difficulty in modelling chatter stability in milling is due to the rotating tool, which results in varying dynamics because of the time-varying directional factors between the modal directions and the chip thickness direction, in addition to the intermittent engagement of the cutting teeth and the work material. Budak and Altintas [9] developed an analytical chatter stability model using Fourier series expansions for the periodically varying directional coefficients and Floquet's theorem. Using zero-order approximation, i.e. retaining only the first term in the Fourier series expansion, the axial stability limit is obtained as follows

$$a_{\text{lim}} = -\frac{2\pi\Lambda_I}{NK_t} \left( \kappa + \frac{1}{\kappa} \right) \quad (5)$$

where  $N$  is the number of teeth,  $K_t$  is the tangential milling force coefficient,  $\Lambda_I$  is the imaginary part of the eigenvalue, and  $\kappa = \Lambda_R/\Lambda_I$ . The eigenvalue of the milling system is determined from the oriented transfer function matrix, constructed using the frequency response functions in two orthogonal directions and average directional factors [9]. The directional factors depend on the radial depth of cut, which affects the resulting stable axial depth. Similarly to the use of equation (3) for turning, equation (5) can be used to predict the stability limits and diagrams for milling.

### 3 EXPERIMENTAL SET-UP AND CONDITIONS

The investigation of chatter effects on tool wear was conducted entirely experimentally. The main objective in these tests was to determine the change in tool life under vibrations, compared with stable cutting. The effects of different important aspects of the vibrations, such as tool rigidity and chatter severity or vibration amplitude, were considered. The experiments were carried out using different work materials on a lathe and a machining centre. Cutting forces and vibrations were recorded during machining. The tests were performed at different cutting speeds in order to obtain the Taylor tool life parameters for both stable and unstable conditions. The effects of vibration amplitude and rigidity on wear were also investigated by varying the test conditions. The chatter frequency

was altered by varying the clamped length of the holder or tool.

#### 3.1 Turning tests

Turning tests were performed on medium-carbon steel (AISI 1040). The test set-up is shown in Fig. 1. A laser displacement sensor was used to collect vibration data during the cutting tests. Cutting force and displacement data were collected continuously during the tests, in which uncoated carbide inserts with a  $0^\circ$  rake angle were used. The inserts were always clamped to the toolholder using a torque of 0.8 N m. Two different toolholder lengths were used in order to produce different static and dynamic rigidities, 110 mm and 135 mm, with uncoated carbide inserts. For each case, the chatter limit was determined using stability equation (3). The tool dynamics was obtained using impact testing and modal analysis, where the workpiece was assumed to be rigid. Calibration tests were performed using a force dynamometer to determine the cutting constant in the feed direction,  $K_f$ , which was identified as 1667 MPa for AISI 1040. A constant feed rate of 0.12 mm/rev was used for all tests. In experiments, chatter (C) was produced by using a depth of cut higher than the stability limit. Severe chatter (SC) conditions were also included in the tests by using much higher depth of cuts. The cutting speeds used were 170, 110, and 50 m/min, and the depth of cut ranged from 0.15 to 1 mm. The test conditions and some measurement results are given in Table 1. Impact tests and modal analysis were performed on the toolholder clamped at different lengths to determine the chatter limits and modal frequencies for each case. As a result of the measurements, 110 mm and 135 mm tool lengths were selected, as they provided reasonable chatter stability limits and significantly different dynamic properties. The peak frequency response amplitude was increased by a



Fig. 1 Experimental set-up for the turning tests

factor of about 4, mainly as a result of the greater holder length, and partially because of the reduced damping due to the shorter clamping contact length. Tests with two different holder lengths were used to investigate the effect of tool rigidity on tool wear.

### 3.2 Milling tests

Three different workpiece materials were used in milling tests in order to see the behaviour of different metals: steel (AISI 1040), a nickel alloy (Inconel 718), and a titanium alloy ( $\text{TiAl}_6\text{V}_4$ ) were milled in different conditions. The tests were carried out at two different cutting speeds (170 and 260 m/min) for steel, and at 35 m/min for titanium and Inconel 718, using two different tool lengths (110 and 120 mm). The cutting tool used for the milling tests was a carbide insert with a relief angle of  $15^\circ$  and a PVD coating of three

layers (TiN, TiCN, TiN). The inserts were clamped on a toolholder of 20 mm diameter with two insert seats. A constant feed rate of 0.1 mm/tooth was used for all tests. All milling tests were conducted in downmilling mode with half-radial immersion. The chatter limits were determined using the modal test results in the analytical milling stability model [9]. The test matrix is given in Table 2.

## 4 EXPERIMENTAL RESULTS

Cutting tests were carried out according to the conditions given in Tables 1 and 2. Tool life was determined on the basis of flank wear measurements using a microscope. The wear was measured several times throughout the total life of the tool. Notch wear, which was present in all of the tests, was used as the wear indicator, as it progressed much faster and reduced the cutting time. A maximum wear land of 0.2 mm was used as the tool life criterion.

### 4.1 Turning test results

The wear data are presented in different forms to demonstrate the dynamic effects on tool wear. Figure 2 shows the progress of tool wear with cutting time for the shorter holder length of 110 mm and for cutting speeds of 170, 110, and 50 m/min. In each of the graphs, three curves are shown, corresponding to stable (S), chatter (C), and severe chatter (SC) conditions. The same data are presented for the 135 mm holder length in Fig. 3. These figures show that the tool life was reduced substantially by chatter, and the effect of chatter became more predominant for higher speeds, where the tool life was reduced by several factors. The effect of rigidity on tool life was also analysed, and it proved to be different for stable and unstable cutting conditions. For stable cutting, the lower stiffness ( $L = 135$  mm) resulted in a lower wear rate, which can be seen by comparing the stable

**Table 1** Test matrix for turning experiments\*

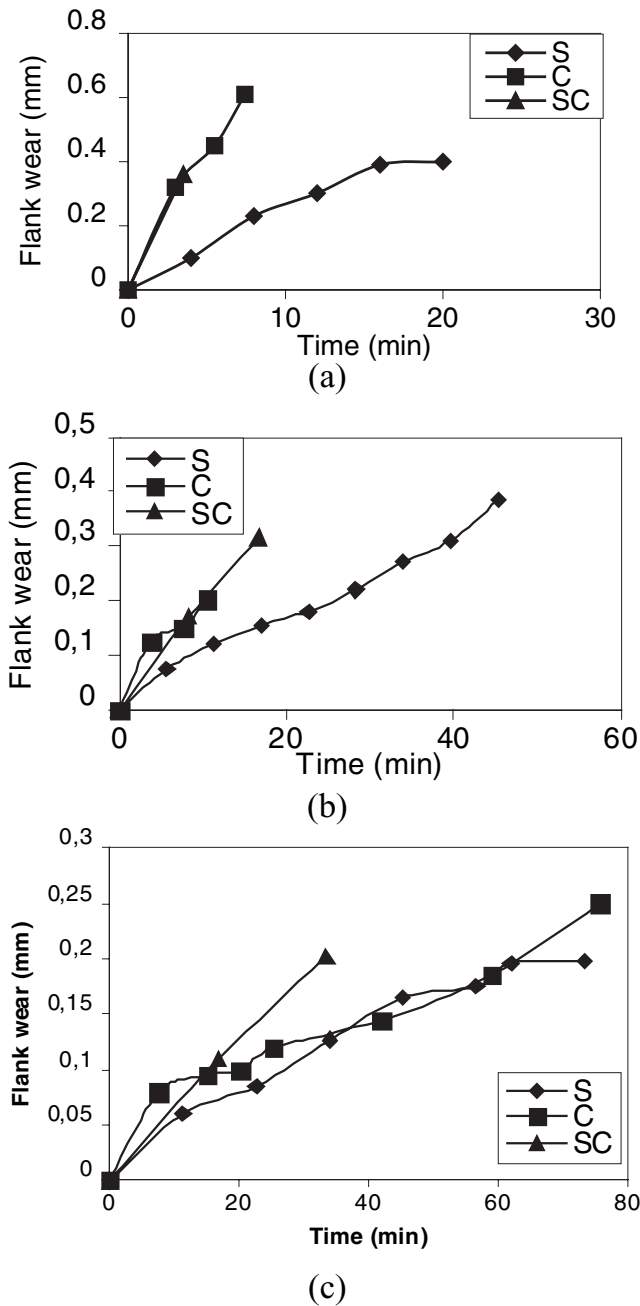
Test number	Holder length (mm)	Cutting speed (m/min)	Axial depth of cut (mm)	Chatter condition	Cutting time (min)	Vibration amplitude ( $\mu\text{m}$ )
1	110	166	0.3	S	7	0
2	110	110	0.3	S	25.5	0
3	110	50	0.3	S	73.3	0
4	110	175	0.65	C	1.8	7
5	110	114	0.65	C	10.5	22
6	110	50	0.65	C	63	12
7	110	180	1	SC	2	11
8	110	107	1	SC	9.5	27
9	110	52	1	SC	33	15
10	135	167	0.15	S	19	0
11	135	126	0.15	S	44	0
12	135	51	0.15	S	73	0
13	135	170	0.5	C	2.2	7
14	135	116	0.5	C	7.2	9
15	135	53	0.5	C	55	9
16	135	167	0.75	SC	2	16
17	135	108	0.75	SC	4	23
18	135	52	0.75	SC	29.5	2

\*S = stable, C = chatter, SC = severe chatter. Radial depth of cut is 10 mm for all tests.

**Table 2** Test matrix for milling tests (half-immersion downmilling)

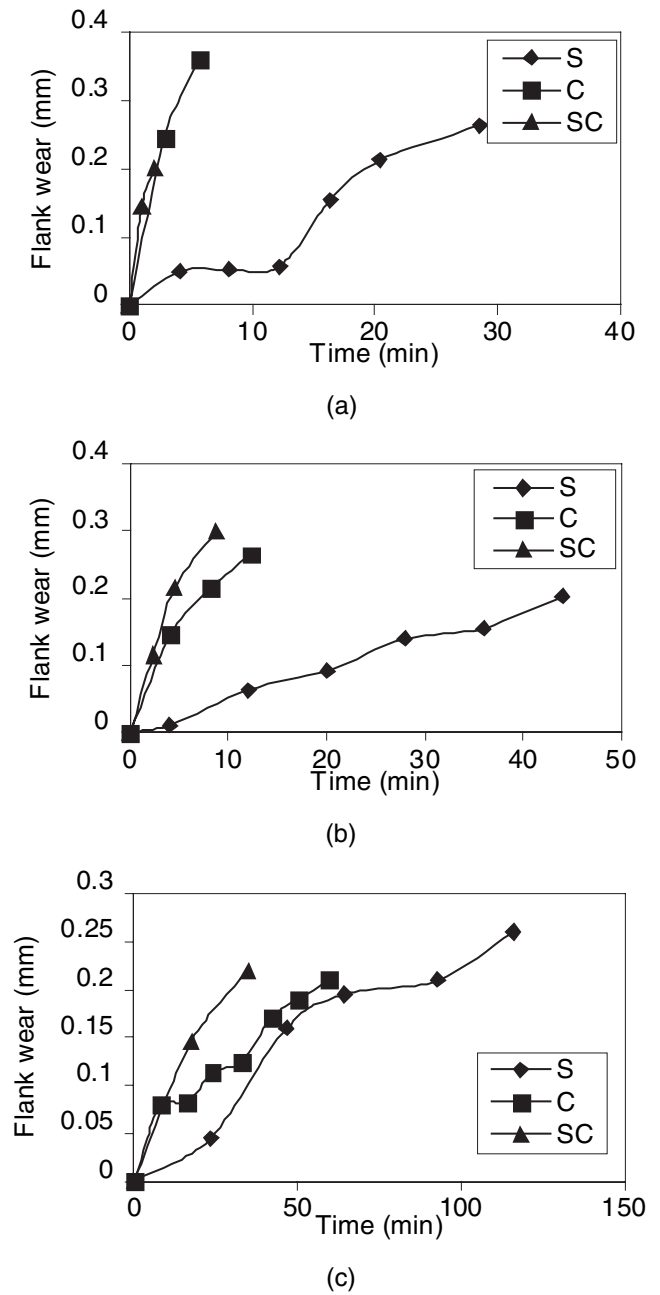
Test number	Tool length (mm)	Cutting speed (m/min)	Axial depth of cut (mm)	Material	Chatter condition	Tool life (min)
1	110	170	1	AISI 1040	S	49
2	110	260	1	AISI 1040	S	30
3	110	170	2	AISI 1040	C	30
4	110	260	1.8	AISI 1040	C	16
5	120	170	0.7	AISI 1040	S	56
6	120	260	0.7	AISI 1040	S	33
7	120	170	1	AISI 1040	C	41
8	120	260	0.85	AISI 1040	C	27
9	120	35	1	$\text{TiAl}_6\text{V}_4$	S	46
10	120	35	2	$\text{TiAl}_6\text{V}_4$	C	31
11	120	35	2.5	$\text{TiAl}_6\text{V}_4$	SC	15
12	120	15	0.6	Inconel 718	S	13
13	120	15	1.2	Inconel 718	C	9.5
14	120	20	0.6	Inconel 718	S	12
15	120	20	1.2	Inconel 718	C	2.5





**Fig. 2** Tool wear versus cutting time for different chatter conditions and cutting speeds for turning tests ( $L=110$  mm): (a) 170 m/min; (b) 110 m/min; (c) 50 m/min. S = stable, C = chatter, SC = severe chatter

(S) wear data in Figs 2 and 3. The same behaviour was also observed in milling tests, as will be discussed in the next section. The opposite was true for unstable cutting, where higher flexibility resulted in shorter tool life. This was mainly due to the increased vibration amplitudes as a result of the reduced dynamic rigidity of the toolholder. The results have been compiled in other figures set out below to demonstrate these effects more clearly.



**Fig. 3** Tool wear versus cutting time for different chatter conditions and cutting speeds for turning tests ( $L=135$  mm): (a) 170 m/min; (b) 110 m/min; (c) 50 m/min. S = stable, C = chatter, SC = severe chatter

From the turning data, tool life curves developed for 110 and 135 mm tool lengths using a tool life criterion of 0.2 mm flank wear are shown in Fig. 4. It can be seen that the tool life decreased significantly as the chatter severity increased. Figure 5 shows the effect of holder length on tool life for severe chatter conditions. It can be concluded from these results that the tool life was greatly affected by the holder length, i.e. the dynamic rigidity or vibration amplitude. The effect of chatter on tool life is summarized in Fig. 6 for

all conditions tested. This figure shows the drastic reduction in tool life as a result of vibrations for all cases. Tool life was up to 5 times lower on account of chatter. From Fig. 4, the Taylor tool life parameters in equation (1) can be determined through regression analysis for stable and chatter conditions. These are given in Table 3 for short and long holder lengths

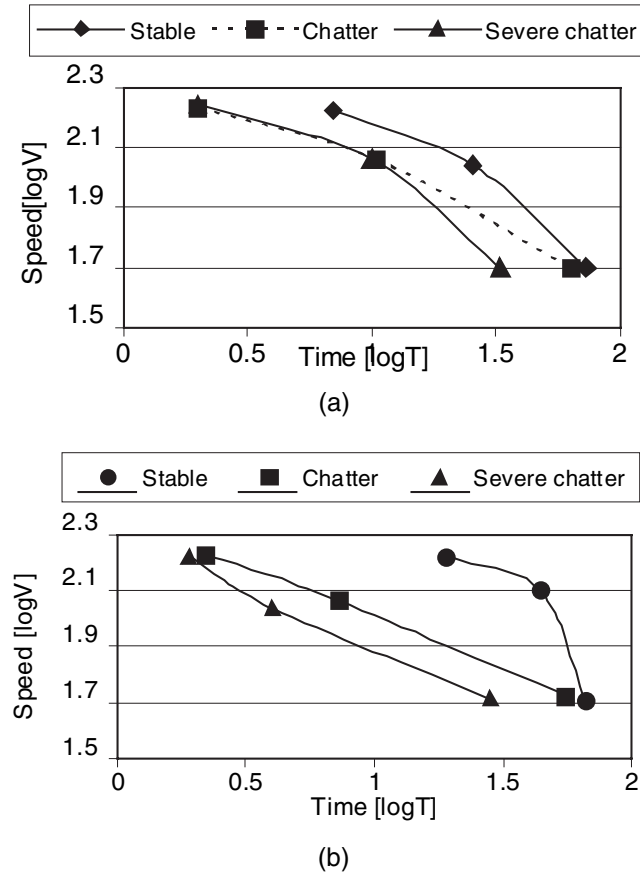


Fig. 4 Variation in tool life with cutting speed and chatter conditions for (a) 110 mm and (b) 135 mm tool-holder length

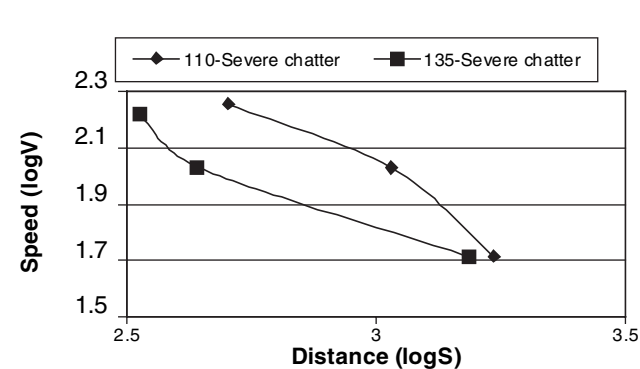


Fig. 5 Cutting distance versus speed for different holder lengths in turning tests under severe chatter conditions

respectively. These results clearly demonstrate the great effect of chatter on tool life.

Vibration amplitudes were also measured in cutting tests using a laser displacement sensor. The peak amplitude varied from 5 to 25  $\mu\text{m}$  depending on the severity of chatter. The effect of vibration amplitude on tool life is shown in Fig. 7. As can be seen from this figure, again tool life was reduced significantly with increasing vibration amplitude at all cutting speeds. However, as mentioned before, the reduction in tool life at higher speeds was much higher, which may not be clear from Fig. 7 owing to scaling.

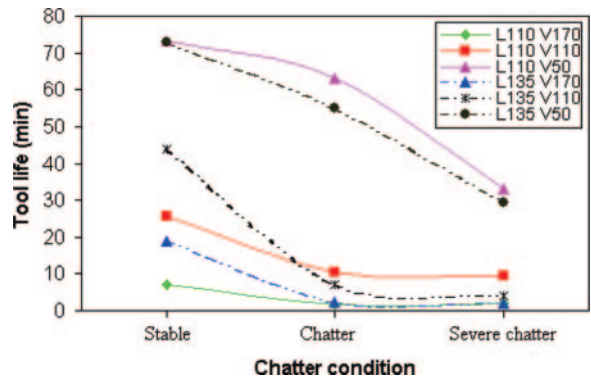


Fig. 6 Tool life under different chatter conditions and cutting speeds

Table 3 Constants  $C'$  and  $n$  identified from wear data for different chatter conditions and tool lengths in the turning of steel AISI 1040\*

	$L = 110 \text{ mm}$			$L = 135 \text{ mm}$		
	S	C	SC	S	C	SC
$C'$	2.678	2.344	2.499	3	2.363	2.323
$n$	0.533	0.354	0.5257	0.6	0.365	0.427

\*S = stable, C = chatter, SC = severe chatter.

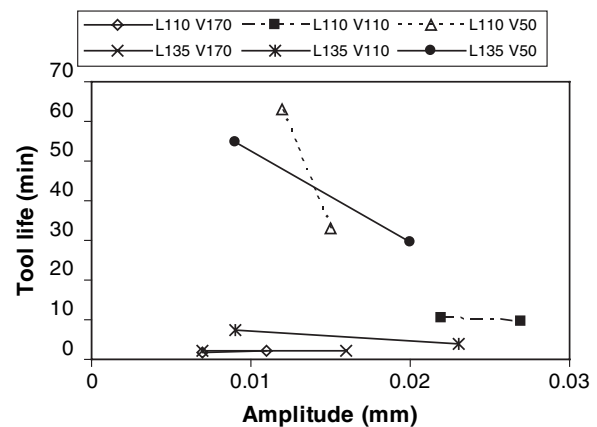


Fig. 7 Effect of chatter amplitude on tool life

4.2 Milling test results

Similar results were observed in milling tests. As an example, the variation in tool wear for stable and unstable conditions is shown in Fig. 8 for the milling of titanium alloy  $TiAl_6V_4$  at 35 m/min cutting speed. This figure clearly shows that chatter also had a significant effect on tool wear in milling. The tool life was reduced

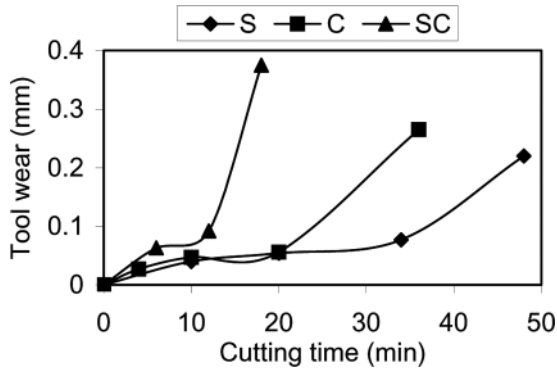


Fig. 8 Tool wear versus cutting time for stable (S), chatter (C), and severe chatter (SC) conditions. The work material for the milling tests was titanium alloy  $TiAl_6V_4$

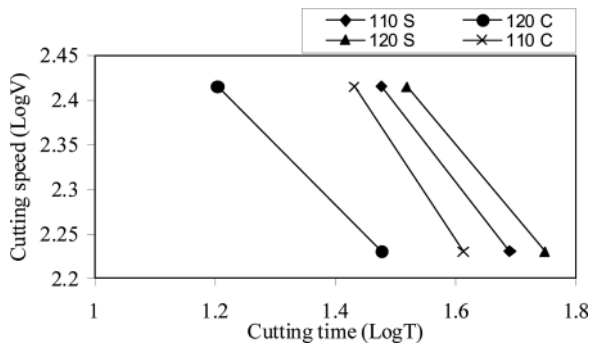


Fig. 9 Cutting time versus speed for the milling of steel with 110 and 120 mm toolholder lengths under stable (S) and chatter (C) conditions

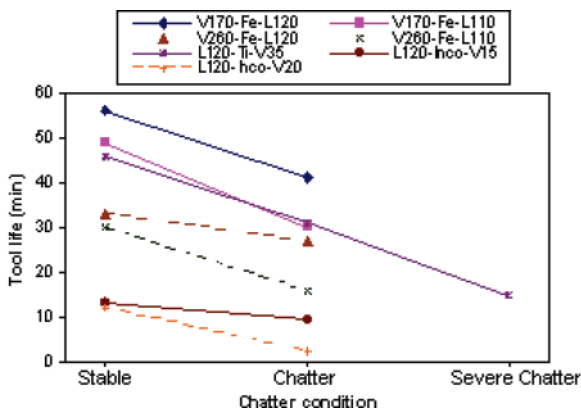
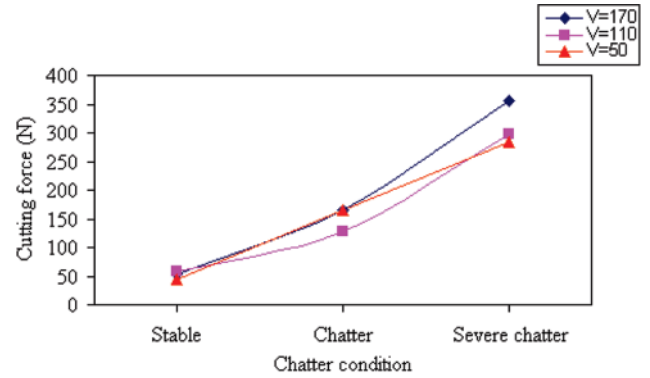
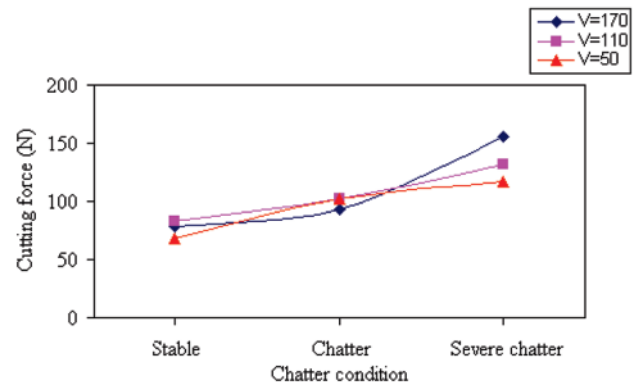


Fig. 10 Tool life versus chatter condition in milling

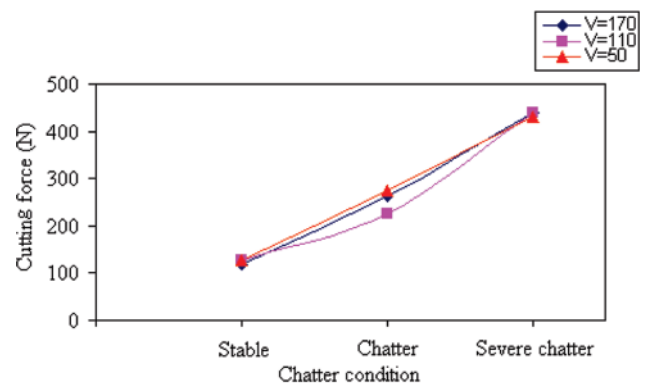
by up to one-third under chatter conditions and by up to two-thirds under severe chatter conditions. The effect of tool rigidity was also tested (Fig. 9). Similarly to the turning tests, low rigidity resulted in higher tool life under stable cutting conditions, whereas it yielded substantial reductions in tool life under chatter conditions. Like the turning tests, the severity of chatter, and thus the vibration amplitude, had a very great effect on tool wear, as shown in Fig. 10 for all three



(a)



(b)



(c)

Fig. 11 Values of the forces under different chatter conditions in the turning of steel AISI 1040 ( $L = 110$  mm): (a) radial force  $F_x$ ; (b) feed force  $F_y$ ; (c) cutting force  $F_z$

materials tested. This graph shows that tool life was reduced by up to 50 per cent as a result of chatter.

### 4.3 Cutting forces

The maximum cutting forces were higher owing to the increased dynamic chip thickness caused by vibrations and to the increased dynamic contact between the tool and the work material. The cutting forces can increase by several factors under normal chatter conditions and by as much as a factor of 10 under severe chatter conditions. The force measurements in three directions are shown in Fig. 11 for the turning of steel AISI 1040 under stable and chatter conditions. Similar results were obtained in milling and with other materials. Therefore, the increase in cutting forces as a result of chatter was much higher than the percentage increase in depth of cut. This increase in cutting forces may be one of the contributing factors to the higher wear rate. In order to investigate this hypothesis, wear versus cutting force graphs were prepared and analysed for the turning and the milling tests. Figure 12 shows the

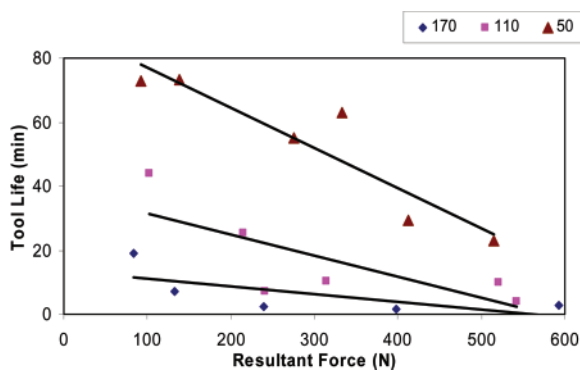


Fig. 12 Tool life versus resultant cutting force for turning tests of steel AISI 1040 (the graph contains data for 50, 110, and 170 mm holder lengths)

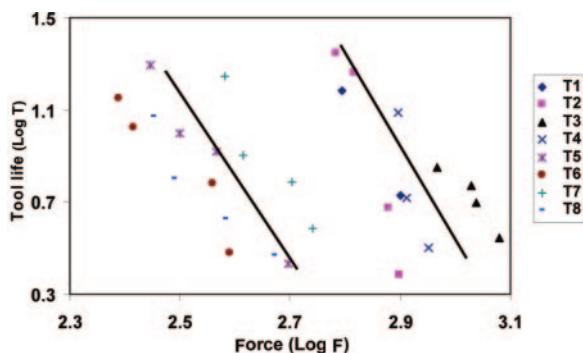


Fig. 13 Tool life versus resultant cutting force for milling tests of steel AISI 1040 (T1, etc. are the test numbers)

variation in tool life with resultant cutting force for the turning tests. Normally, i.e. for stable cuts, the depth of cut would have a very slight effect on tool life, and thus almost horizontal lines would be expected in Fig. 12. However, the depths used in the tests were not selected arbitrarily, but to control the stability. As a result, an increased depth of cut, yielding chatter and much higher cutting forces, may cause a significant decrease in tool life. A similar result was observed in the milling data. Figure 13 shows the variation in tool life with resultant cutting force in the milling of steel AISI 1040 (tests 1 to 8 in Table 2). The data can be approximately represented by two lines with similar slopes. Each data cluster in the figure corresponds to a tool length, one for 110 mm and the other for 120 mm. This is because their different dynamic rigidities result in different chatter limits, i.e. different axial depths and cutting forces. These data show that the increased cutting forces due to chatter may be one of the reasons for shorter tool life.

## 5 CONCLUSIONS

An experimental investigation into the effects of chatter vibrations on cutting tool life has been presented. The observations from the tests can be summarized as follows.

1. Chatter vibrations result in a drastic reduction in tool life in turning tests: by about 50 per cent in most cases and by more than 80 per cent in some cases (higher cutting speeds). Tool life in milling decreases by an average of 30–50 per cent under chatter conditions, and by up to 70 per cent under severe chatter conditions.
2. The chatter effects on tool life are more pronounced at higher cutting speeds in turning.
3. The vibration amplitude has a direct influence on tool life. The higher vibration amplitudes result in substantially reduced tool life.
4. The rigidity of the cutting system has a great influence on tool life. Under stable cutting conditions, a less rigid toolholder results in higher tool life both in turning and in milling. The opposite is true for unstable cutting, where high flexibility reduces tool life. When the cutting depth is increased beyond the chatter limit, the percentage increase in vibration amplitude is much higher for a less rigid tool assembly, resulting in a greater reduction in tool life.
5. There are significant cutting force differences between stable and chatter conditions. Also, tool life and cutting forces seem to be related both in turning and milling.



6. The reduction in tool life as a result of chatter is believed to be due to two main factors. First of all, high-frequency impacts on the cutting edge cause faster material loss on the flank face, similar to the surface wear observed in fretting contacts. They also cause fatigue and micro-chipping, which can be deduced from the fact that higher-amplitude vibrations result in a much shorter tool life. Higher cutting forces resulting from chatter are believed to be the other reason for reduced tool life, although this observation requires further experimental investigation.

These results show the important effects of chatter on tool life, which cannot be ignored in machining process planning, parameter selection, and cost analysis. The results may also be used in justifying the additional cost of chatter suppression and more rigid machining systems.

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