

# N76-24127

## Implications of Saito's Coronal Density Model on the Polar Solar Wind Flow and Heavy Ion Abundances

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### ABSTRACT

A comparison of polar solar wind proton flux upper limits derived using Saito's coronal density model, with Ly  $\alpha$  measurements of the length of the neutral H tail of comet Bennet at high latitudes, shows that either extended heating beyond  $2 R_{\odot}$  is necessary some of the time or that Saito's polar densities are too low. Whichever possibility is the case, the fact that the solar wind particle flux does not appear to decrease with increasing latitude, indicates that the heavy element content of the high latitude wind may be similar to that observed in the ecliptic. It is then shown that solar wind heavy ion observations at high latitudes allow a determination of the electron temperature at heights which bracket the nominal location of the coronal temperature maximum thus providing information concerning the magnitude and extent of mechanical dissipation in the intermediate corona.

## 1. Introduction

There is, at present, a lack of information on the physical conditions in the polar regions of the solar corona and solar wind. This lack results in a corresponding uncertainty in the global characteristics and extent of that plasma of solar origin which fills interplanetary space and thereby controls the near solar environment. For example, only little is known about the variation with heliographic latitude of so fundamental a flow parameter as the solar wind mass flux. Similarly, hardly anything is known about latitude variations of the solar wind energy and momentum fluxes. Yet, these parameters may be very important in determining the physical state of the polar corona and the size of the solar dominated cavity over the poles which separates the sun from the local interstellar medium. In addition, the existence of heavy elements in the polar solar wind may depend (Allouche, 1967, 1970; Geiss et al., 1970) on whether or not the proton particle flux exceeds a temperature dependent lower limit.

It is therefore useful to consider hypothetical variations of solar wind particle and energy fluxes with heliographic latitude. This task is approached in section 2 of this paper by calculating upper limit values of the polar solar wind particle flux implied by the most comprehensive coronal density model developed to date (Saito, 1970). This model was determined from an average K corona brightness distribution constructed using 15 solar eclipse observations as well as K-coronameter measurements all made at the minimum phases of the solar activity cycle. As a necessary result of the method employed, the model densities (and hence

the upper limit values of the solar wind particle flux derived below) determined for the polar regions are uncertain because it is not possible to uniquely invert the convolution integral which relates the coronal brightness distribution to the average line of sight electron density. Nevertheless, it is shown that if the polar coronal densities are as low as calculated using Saito's model, then without extended heating, the emitted polar particle flux should be substantially less than that observed in the ecliptic plane at 1 AU and less than that necessary to drag coronal heavy elements away from the sun. However, limited evidence based on Ly  $\alpha$  measurements of the neutral hydrogen tail of comet Bennet (Bertaux et al., 1973; Keller, 1973), is consistent with a polar solar wind flux at least as large as that observed in the ecliptic at 1 AU. These observations therefore require either an extended coronal heat source distinct from electron heat conduction or that Saito's polar densities are too low. In any event since the particle flux in the polar wind may be comparable to that observed in the equatorial wind, it is possible that coronal heavy ions at polar latitudes do indeed expand with the protons into interplanetary space.

Since it is reasonable to expect that heavy elements will be observable in the polar solar wind, the range of ionization state "freezing in" distances is estimated in section 3 for selected heavy ion species at polar latitudes. It is found that the polar coronal density may be sufficiently low that the ionization states "freeze in" below the nominal location of the temperature maximum. Hence high latitude heavy ion observations may allow a determination of the thermal state of the intermediate and low corona and provide an estimate of the magnitude and

extent of mechanical dissipation. Section 4 summarizes the main conclusions.

## 2. Latitude Variations of the Solar Wind Particle Flux

It is currently thought that the solar wind evolves from open field regions in the corona (see Hundhausen, 1972 for a review). Such regions are generally distinct from regions of activity and are generally characterized by low density. For these regions, the electron density,  $N$ , as a function of solar distance,  $r$ , and heliographic latitude,  $\theta$ , has been modeled by Saito (1970) with the relation

$$N = \frac{3.09 \times 10^8 (1 - 0.5 \sin\theta)}{R^{16}} + \frac{1.58 \times 10^8 (1 - 0.95 \sin\theta)}{R^6} + \frac{0.0251 \times 10^8 (1 - \sin^{1/2}\theta)}{R^{2.5}} \text{ cm}^{-3} \quad (1)$$

where  $R = r/R_{\odot}$  and  $R_{\odot}$  is the solar radius.

Upper limit values for the polar solar wind particle flux can be derived using relation 1 if various subsets of several reasonable assumptions concerning the state of the intermediate corona are adopted. These assumptions are 1) the coronal gas consists of H, He and electrons only, 2) there is no extended heating other than that due to electron heat conduction much beyond the coronal temperature maximum, 3) the energy equation may be closed with the standard relation  $\bar{Q} = -\kappa_{\odot} T^{5/2} \nabla T$  (Chapman, 1954; Spitzer, 1956) which assumes that binary coulomb interactions limit the mean scattering length of a thermal electron, 4) coronal electron and proton velocity distributions are very

nearly Maxwellian, 5) wave-particle interactions and macroscopic wave pressure effects are negligible above the heating region, and 6) the magnetic field is open but not necessarily radial.

The purpose of this section is to show that if the coronal density over the pole drops off as quickly as implied by Saito's analysis then some of the above assumptions may be tested by in situ solar wind observations. We begin with a standard single fluid formulation of the coronal expansion using equation 1 in place of an energy equation and derive upper limits for the particle flux at 1 AU. A separate treatment based on various possible forms of the energy equation is considered next to provide independent estimates of the 1 AU flux upper limit. The results of this analysis are in agreement with those obtained by Durney and Hundhausen (1974). As will be shown in section 3 these upper limits are substantially lower than that observed in the ecliptic at 1 AU and are sufficiently low, that if all of the above assumptions are correct, He<sup>++</sup> and many of the heavier ions should not expand with the protons away from the sun at polar latitudes.

(i) Mass Flux, Momentum Flux and Density Equations

The mass and momentum conservation equations are respectively;

$$NVA(R) = F \quad (2)$$

$$m_p MNV \frac{dV}{dR} = - \frac{d}{dR} (NkT) - \frac{NGM_\odot m_p M}{R^2 R_\odot} \quad (3)$$

Here A(R) is the area of a flux tube which varies as R<sup>2</sup> if the expansion is radial, G is the gravitational constant, M<sub>⊙</sub> is the mass of the sun, m<sub>p</sub> is the proton mass, M is the mean molecular weight = (1 + 4α)/(2 + 5α) where α is the He

abundance by number,  $k$  is Boltzmann's constant,  $N$  is the proton density,  $V$  is the bulk convection speed and  $T$  is the one fluid temperature. Concentrating on the region in the intermediate corona between  $R = r/R_0 = 2$  and 4, equation 1 for  $\theta = 90^\circ$  can be simplified to the form:

$$N(R) \cong \frac{7.9 \times 10^6}{R^6} \text{ cm}^{-3} \quad (4)$$

If it is assumed that  $A(R)$  varies as  $R^S$  then equations 2, 3, and 4 can be integrated analytically to obtain  $T(R)$

$$\frac{T(R)}{T_0} = \left(\frac{R}{R_0}\right)^6 \left\{ 1 - \left(\frac{GM_{\odot} m_p M}{7kR_{\odot} R_{\odot} T_0}\right) \left[ 1 - \left(\frac{R_0}{R}\right)^7 \right] - \left(\frac{m_p M (NV)_0^2}{N_0^2 k T_0}\right) \left(\frac{6-S}{6-2S}\right) \left[ \left(\frac{R}{R_0}\right)^{(6-2S)} - 1 \right] \right\} \quad (5)$$

Here the subscript 0 refers to parameters evaluated at the base radius  $R_0$ .

In the following  $R_0$  is chosen equal to 2.

Equation 5 can be rewritten in simplified form as follows:

$$T(R) = \left(\frac{R}{R_0}\right)^6 \{ T_0 - C_1 \left[ 1 - \left(\frac{R_0}{R}\right)^7 \right] - C_2 (NV)_0^2 \left[ \left(\frac{R}{R_0}\right)^{6-2S} - 1 \right] \} \quad (6)$$

Here  $C_1$  and  $C_2$  are constants which are readily evaluated by comparing equations 6 and 5. Inspection of equation 6 shows that  $T(R)$  depends parametrically on two variables,  $T_0$  and  $(NV)_0$ . Following the analysis of Brandt et al. (1965) it is possible to show that two physically reasonable assumptions imply stringent constraints on the range of realizable values of  $T_0$  and  $(NV)_0$ . These two assumptions are: 1) the derived temperature,  $T(R)$ , must remain positive throughout the range of validity of equation 1; according to Saito (1970),  $R \leq 4$ , 2) there is not sufficient external heating beyond  $R = 2$  to produce a second peak in  $T(R)$ .

It is seen from the third term on the right hand side of equation 5 that for a constant  $T_0$ , increasing  $(NV)_0$  eventually drives  $T(R)$  negative. The radius at which this happens can be increased beyond  $R = 4$  by increasing  $T_0$ . However if  $T_0$  is too large, the  $(R/R_0)^6$  term in front on the right hand side produces a second peak in  $T(R)$  beyond  $R = 2$ . Therefore acceptable ranges of  $T_0$  and  $(NV)_0$  can be determined as follows. The minimum value of  $T_0$ ,  $T_L$ , is calculated for  $(NV)_0 = 0$  under the assumption that  $T(R) \leq 0$  for  $R \geq R_X$  where  $R_X$  is the limiting distance of validity of equation 1. This gives

$$T_L = \left( \frac{GM_p M}{7kR_0 R_0} \right) \left[ 1 - \left( \frac{R_0}{R_X} \right)^7 \right] \quad (7)$$

Given  $R_0 = 2$ ,  $M = 0.547$  (corresponding to a 4% He abundance by number) and assuming  $R_X = 3$  and  $3.5$  then  $T_L = 0.85 \times 10^6 K$  and  $0.89 \times 10^6 K$  respectively.

Upper limits for  $T_0$  and  $(NV)_0$  are determined from equation 5 by finding the largest value of  $T_0$  and  $(NV)_0$  such that  $T(R) \geq 0$  and  $dT/dR \leq 0$  for  $R$  in the range  $R_0 \leq R \leq R_X$ . Thus for each  $R$  the following two relations must be satisfied;

$$T_0 \geq \left( \frac{GM_p M}{7kR_0 R_0} \right) \left[ 1 - \left( \frac{R_0}{R_X} \right)^7 \right] + \left( \frac{m M (NV)_0^2}{N_0^2 k} \right) \left( \frac{6 - S}{6 - 2S} \right) \left[ \left( \frac{R}{R_0} \right)^{6-2S} - 1 \right] \quad (8)$$

and

$$T_o \leq \left( \frac{GM_e m_p}{7kR_o R_e} \right) \left[ 1 + \frac{1}{6} \left( \frac{R_o}{R} \right)^7 \right] + \left( \frac{m_p M (NV)_o^2}{N_o^2 k} \right) \left( \frac{6-S}{6-2S} \right) \left[ \left[ \frac{6-S}{3} \right] \left( \frac{R}{R_o} \right)^{6-2S} - 1 \right] \quad (9)$$

Inspection of equations 8 and 9 shows that for  $(NV)_o = 0$  both conditions can be satisfied simultaneously. However for each R both conditions cannot be satisfied if  $(NV)_o$  is larger than some maximum value. This maximum is obtained by equating the right hand sides of equations 8 and 9.

$$(NV)_o \leq \frac{N_o \left( \frac{GM_e}{7R_o R_e} \right)^{1/2} \left[ \frac{1}{6} \left( \frac{R_o}{R} \right)^7 + \left( \frac{R_o}{R_X} \right)^7 \right]^{1/2}}{\left\{ \left( \frac{6-S}{6-2S} \right) \left[ \left( \frac{R_X}{R_o} \right)^{6-2S} - \left( \frac{6-S}{3} \right) \left( \frac{R}{R_o} \right)^{6-2S} \right] \right\}^{1/2}} \quad 2 \leq S < 6 \quad (10)$$

Setting  $S = 2$  (radial flow) and  $(NV)_e = (NV)_o (R_o/R_e)^2$  (the subscript e refers to parameters evaluated at the orbit of the earth), equation 10 is plotted in Figure 1 for  $R_X = 3$  and 3.5. The minimum value of the right

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hand side of equation 10 for  $R$  in the range  $R_0 \leq R \leq R_X$  is the maximum flux at 1 AU consistent with the assumptions  $dT/dR \leq 0$  for  $R \geq 2$  and  $T \geq 0$  for  $R \leq R_X$ . Thus for  $R_X = 3.0$ ,  $(NV)_e \leq 0.4 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$  with  $0.85 \times 10^6 \text{ K} \leq T(R=2) \leq 1.1 \times 10^6 \text{ K}$  and for  $R = 3.5$ ,  $(NV)_e \leq 0.2 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$  with  $0.89 \times 10^6 \text{ K} \leq T(2) \leq 0.98 \times 10^6 \text{ K}$ . The curves for  $T(R)$ , corresponding to values of  $(NV)_e$  and  $T(2)$  determined from equations 9 and 10 evaluated near the minimum of the curves in Figure 1, are drawn in Figure 2. Drawn also for comparison are the polar scale height temperature,  $T_H(R) = (GM_\odot m_p M)/(kr^2 d\ln i/dr)$  and the curve  $T \propto R^{-2/7}$ .

Since it is likely that the flow is more divergent than  $R^{-2}$  inside of some radius,  $R_D$ , it is necessary to consider how this possibility affects the upper limit of  $(NV)_e$ . This may be accomplished by assuming the area of a flux tube increases as  $R^S$  out to  $R_D$  and then as  $R^2$  from there to 1 AU,  $R_e$ . Using this model,  $(NV)_D = (NV)_0 (R_0/R_D)^S = (NV)_e (R_e/R_D)^2$  and hence  $(NV)_e$  can be determined from equation 9 using the relation

$$(NV)_e = (NV)_0 \left(\frac{R_0}{R_e}\right)^2 \left(\frac{R_0}{R_D}\right)^{S-2} \quad R_0 \leq R \leq R_X \leq R_D \quad (11)$$

Investigations of equations 10 and 11 for  $S$  in the range  $2 \leq S \leq 4$ ,  $(R_D/R_0) = 2$ ,  $(R_X/R_0) = 1.75$  and  $R_0 \leq R \leq R_X$  show that the maximum flux at 1 AU consistent with a single temperature maximum below  $R_0$  is not significantly changed from its value for  $S = 2$  ( $(NV)_e \leq 0.2 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ ). However, if  $S$  is sufficiently large and/or  $(R_D/R_0)$  is sufficiently small, this upper limit is raised. For example choosing  $S = 5$  with  $(R_D/R_0) = 2$  (which is equivalent to expansion from a polar region defined by  $60^\circ \leq \theta \leq 90^\circ$  at

$R_0$  to the full hemisphere at  $R_D$ ),  $(NV)_e \leq 0.28 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ . It should be noted though that for all cases of nonradial expansion  $(NV)_0$  is significantly raised over that obtained for  $S = 2$ .

ii) Energy Flux Supply to the Polar Wind

An alternative approach to the polar particle flux problem is possible by considering the energy equation. Here, a limit on  $(NV)_e$  may be established if the velocity at 1 AU is known and if the usual assumptions about the state of the intermediate corona are made. Using a one-fluid, steady-state, spherically symmetric model, the energy equation

$$\frac{1}{A(R)} \frac{d}{dR} (A(R)Q) = -kTV \frac{dN}{dR} + \frac{3}{2} NkV \frac{dT}{dR} \quad (12)$$

may be combined with equations 2 and 3 and integrated to yield

$$A(R)Q + F \left[ \frac{1}{2} m_p MV^2 + \frac{5}{2} kT - \frac{GM_{\odot} m_p M}{R R_{\odot}} \right] = \epsilon_0 = \text{constant}. \quad (13)$$

If a supersonic solar wind exists at 1 AU but not at  $F_0$  then the dominant term at 1 AU is  $F \left[ \frac{1}{2} m_p MV_e^2 \right]$  while at  $R_0$ , the dominant terms are  $A(R_0)Q + F \left[ \frac{5}{2} kT_0 - \frac{GM_{\odot} m_p M}{R_0 R_{\odot}} \right]$ . Therefore

$$\frac{1}{2} m_p MV_e^2 \cong \frac{Q_0}{(NV)_0} + \frac{5}{2} kT_0 - \frac{GM_{\odot} m_p M}{R_0 R_{\odot}} \quad (14)$$

Further progress is not possible without an additional closure relation which gives  $Q_0$  in terms of the lower velocity moments. Usually the Spitzer conductivity is assumed valid so that

$$\bar{Q} = -\kappa_0 T^{5/2} \bar{\nabla} T \quad (15)$$

with  $\kappa_0 = 7.7 \times 10^{-7} \text{ erg cm}^{-1} \text{ sec}^{-1} \text{ K}^{-7/2}$  (Chapman, 1954; Spitzer, 1956).

However, it is also possible that the density is sufficiently low over the poles that equation 15 is not obeyed. In particular, it is possible that the polar density is so low that the dimensionless third moment,  $q = Q/[1.5 NkT(kT/m_e)^{1/2}]$ , becomes impossibly large at a low altitude (Parker, 1964). For example if  $N_0 = 1.23 \times 10^5 \text{ cm}^{-3}$  (Saito, 1970),  $T_0 = 0.98 \times 10^6 \text{ K}$  (see section (i) above) and  $T \propto R^{-2/7}$  then  $q = (0.15)(R/R_0)^{(31/7)}$  or  $q \geq 1$  when  $R \geq 1.5 R_0$ . It is therefore probable that below this altitude instabilities develop (Forslund, 1970) which will limit  $Q$  to a value less than the Spitzer upper limit. In other words, the heat flux will be limited within  $1.5 R_0$  thus effectively producing an isothermal region at lower altitudes and a region of steeper than  $R^{-2/7}$  temperature decrease at higher altitudes.

It is thus not clear how to estimate the value of  $Q_0$  in equation 14. For the sake of concreteness two alternate approaches are adopted below. The first assumes equation 15 to be valid with  $T \propto R^{-2/7}$  and the second adopts an exospheric approach. In both cases the solar wind He abundance is assumed to be a free parameter since its value is observed to be highly variable in the ecliptic at 1 AU and is not known at high polar latitudes. Such an assumption is necessary since, in contrast to the analysis presented in section (i) where upper limits for  $(NV)_0$  were independent of  $M$  (see e.g. equation 10), the magnitude of the He abundance may be significant here. This fact results because most of the energy needed to drive the solar wind expansion goes into gravitational potential and kinetic energy which are both mass dependent. However this effect is more than compensated for by the fact that maximum values of

$T_o$  derived from the analysis in section (i) scale linearly with  $M$  (see e.g. equation 9).

Assuming first that equation 15 is valid,  $T \propto R^{-2/7}$  and  $T_o = (0.98 \times 10^6)(M/0.547)$  K (see Figures 1 and 2) then upper limit values for  $(NV)_e$  can be calculated from equation 14 for chosen values of  $M$  and  $V_e$ . The results are summarized in Table 1 under the label  $(NV)_e$  (Spitzer) for  $V_e = 320, 450$  and  $750$  km/sec and  $M$  values corresponding to a He abundance of 0, 0.04 and 0.08 by number.

In the appendix, an analysis is presented which shows that it is not clear whether or not electrons are collisionless below  $R = R_X$ . If indeed coronal electrons are collisionless near to but outside of  $2 R_\odot$  then  $Q$  must be calculated using exospheric theory (Jockers, 1970; Lemaire and

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Scherer, 1971a,b; Schulz and Eviatar, 1972; Hollweg, 1974; Eviatar and Schulz, 1975). In this approach, only those electrons above the electric potential barrier,  $|e\Delta\phi|$  with velocities directed away from the sun can carry heat. The three fluid energy equations may be combined and integrated to yield

$$|e\Delta\phi| = M \Delta \left[ \left( \frac{1}{2} m_p v^2 \right) - \left( \frac{GM_\odot m_p}{R R_\odot} \right) \right] \quad (16)$$

where the  $\Delta$  symbol signifies a difference between any two radial distances and  $e$  is the electronic charge. Choosing  $R_\odot$  and  $R_e = 1 \text{ AU}$  as the two reference distances then

$$|e\Delta\phi| \cong M \left[ \frac{1}{2} m_p v_e^2 + \frac{GM_\odot m_p}{R_\odot R_\odot} \right] \quad (17)$$

If both  $v_e$  and the shape of the electron distribution at  $R_\odot$ ,  $f(v)$ , are known then  $Q_\odot$  is readily evaluated using the relation

$$Q_\odot = \int_{v_B}^{\infty} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \left( \frac{1}{2} m_e v^3 \cos\theta \cos\phi \right) f(v) v^2 dv \cos\theta d\theta d\phi \quad (18)$$

where  $\frac{1}{2} m_e v_B^2 = |e\Delta\phi|$ . Assuming a Maxwellian shape for  $f(v)$  then

$$Q_\odot = \frac{N_\odot kT_\odot}{\sqrt{2\pi}} \left( \frac{kT_\odot}{m_e} \right)^{1/2} e^{-\left( \frac{|e\Delta\phi|}{kT_\odot} \right)} \left[ \left( \frac{|e\Delta\phi|}{kT_\odot} \right)^2 + 2 \left( \frac{|e\Delta\phi|}{kT_\odot} \right) + 2 \right] \quad (19)$$

Equations 14, 17, and 19 can be combined to give a self-consistency condition for  $\beta = Q_0 / [(NV)_0 kT_0]$  and hence an upper limit for  $(NV)_0$  if the bulk convection speed at 1 AU is to be greater than or equal to  $V_e$ .

$$(NV)_0 \leq N_0 \left( \frac{kT_0}{2m_e} \right)^{1/2} e^{-(\beta+2.5)} \left[ \beta + 7 + \frac{13.25}{\beta} \right] \quad (20)$$

where

$$\beta = \frac{1}{2} \frac{M_m V_e^2}{kT_0} + \frac{GM_0 M_m}{R_0 R_0 kT_0} - \frac{5}{2} \quad \text{and } V_e \geq 0 \quad (21)$$

Using equations 20 and 21 along with the assumptions that  $R_0 = 2$ ,  $N_0 = 1.23 \times 10^5 \text{ cm}^{-3}$  (Saito, 1970) and  $T_0 = 0.98 \times 10^6 (M/0.547) \text{ K}$  (see e.g. Figures 1 and 2), upper limit values for  $(NV)_e$  have been calculated for various values of  $M$  and  $V_e$  and are also listed in Table 1.

A comparison of the exospheric upper limits with the Spitzer upper limits for  $(NV)_e$  shows that if both the wave-particle collision frequency and the coronal electron density are low enough over the solar pole so that an exospheric formalism is appropriate, very severe upper limits can be placed on the solar wind flux at 1 AU whether or not the He particles expand with the plasma. These upper limits fall well below that calculated from the mass and momentum equations alone. However, since it is likely that the corona is sufficiently turbulent that an exospheric formalism is not appropriate, the true upper limits for  $(NV)_e$  may be less than but closer to that calculated using the Spitzer conductivity.

A comparison of the upper limit values for  $(NV)_e$  derived using the form of the energy equation which incorporates the Spitzer conductivity, with that

derived using the mass and momentum equations, requires a knowledge of  $M$  and  $V_e$ . Reasonable choices for values of these quantities are made as follows. First, inspection of Table 1 shows that by choosing the solar wind He abundance to be 4% by number, the error made in estimating  $(NV)_e$  is probably less than 20%. Therefore, for the purposes of this comparison,  $M$  is chosen to be 0.547. Concerning the speed of the polar solar wind at 1 AU several pieces of evidence have recently indicated that  $V_e$  over the pole is higher than that observed in the ecliptic (Cole, 1974) and may be close to 750 km/sec (Gosling et al., 1976; Feldman et al., 1976). If this is the case then from Table 1, energy considerations require that  $(NV)_e$  be less than approximately  $0.4 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ .<sup>\*</sup> This value compares favorably with that derived using the mass and momentum equations ( $(NV)_e \lesssim (0.2 \text{ to } 0.4) \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ ). It is therefore concluded that if heat conduction is the dominant mode of energy transport at about  $2 R_\odot$ , and if Saito's polar density model is correct then the particle flux of the polar solar wind should be less than about  $0.5 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ . This upper limit is about a factor of 7 times less than the solar wind particle flux observed in the ecliptic at 1 AU (Feldman et al., 1976).

### 3. Latitude Variations of Heavy Ion "Freezing In" Distances

Allouche (1967) derived an approximate criterion necessary for a heavy ion of mass  $A m_p$  and charge  $Z$  to diffuse upward in an expanding corona.

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<sup>\*</sup>It should be noted that this value is an upper limit. If the heat flux,  $Q$ , is regulated below  $1.5 R_\odot$  as suggested earlier, then the region below  $1.5 R_\odot$  becomes more nearly isothermal thereby reducing  $\nabla T$ ,  $Q$ , and hence the upper limit value derived for  $(NV)_e$ .

His result is:

$$(NV)_o > \frac{3kT_o \left( \frac{GM_o m_p}{R_o^2 R_\odot^2} \right) \left[ \frac{kT_o}{m_p} \right]^{1/2}}{8\sqrt{\pi} e^4 \ln\lambda (Z^2/A)} \quad (22)$$

where the symbols are as previously defined and  $\ln\lambda$  is the coulomb logarithm. Choosing  $N_o = 1.23 \times 10^5 \text{ cm}^{-3}$ ,  $T_o = 0.98 \times 10^6 \text{ K}$  and expressing 22 in terms of  $(NV)_e$  we get:

$$(NV)_e > \frac{2.0 \times 10^8}{(Z^2/A)} \left( \frac{R_o}{R_D} \right)^{S-2} \text{ cm}^{-2} \text{ sec}^{-1} \quad (23)$$

Since for a radial expansion ( $S = 2$ ) this limit is approximately a factor of 4 to 10 times greater than the upper limit for  $(NV)_e$  derived above, it is reasonable to conclude that if Saito's model is correct and if the polar corona is not externally heated above  $r = 2R_\odot$ , He may not expand with the solar wind. This conclusion remains valid for  $S \leq 4$  and  $(R_D/R_o) = 2$  as well.

However, observation of the length of the neutral hydrogen tail of Comet Bennet (Bertaux et al., 1973; Keller, 1973) as a function of heliographic latitude indicates that the polar solar wind flux is at least as large as  $2 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$  at 1 AU. This value is in disagreement with the upper limits deduced in section 2. It is therefore concluded that at least one of the assumptions made in the above analysis is not correct and that it should indeed be possible to observe solar wind heavy ions at polar latitudes at 1 AU. If true then measurements of the population densities of individual heavy ion ionization states will yield information concerning the temperature



structure of that region in the polar solar corona where the various ionization states "freeze in."

It is possible to determine the "freezing in" distances of the various heavy ion species as a function of heliographic latitude using equation 1 if the following assumptions are made: 1) the flow is radial; 2) the velocity distribution is Maxwellian; and 3) the electron temperature,  $T$ , depends on the radius,  $r$ , as  $T = T_{\odot} (r/R_{\odot})^{-\gamma}$ . Following previous work (Hundhausen et al., 1968a, b; Bamert et al., 1974) these distances are defined as those for which the expansion rate,  $\tau_e^{-1} = (V \ln N / dr)$  becomes larger than the ionization state changing rate,  $\tau_{ri}^{-1} + \tau_{ci}^{-1} = N(R_i + C_i)$ . Here  $T_{\odot}$  is the electron temperature at the base of the corona,  $V$  is the solar wind speed,  $R_i$  is the rate of recombination from state  $i$  to state  $i-1$  and  $C_i$  is the rate of collisional ionization from state  $i$  to state  $i+1$ .\*

Changes in the "freezing in" distances with latitude of a sample of the most abundant ions are shown schematically in Figure 3 superimposed on scale height temperatures calculated using equation 1 for  $\theta = 0^\circ$  and  $90^\circ$ . For purposes of illustration an isothermal corona with  $T_F = 1.0 \times 10^6$  K and a 1 AU particle flux of  $2.5 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$  were assumed for evaluating  $\tau_{ri}^{-1} + \tau_{ci}^{-1}$ . The scale height temperature for a static corona is given by  $T_H = (GM_{\odot} / r_p) / (kr^2 \ln N / dr)$ . Inspection of Figure 3 shows that the region in the corona for which temperature values can be determined from solar wind heavy ion data moves inward from above to below the temperature maximum as  $\theta$  varies between  $0^\circ$  and  $90^\circ$ . Thus at some intermediate latitude, coronal temperatures bracketing the maximum can be sampled allowing the magnitude and extent of mechanical dissipation in the intermediate corona to be estimated (see e.g. the analysis of Brandt et al., 1965).

\*Collisional ionization and radiative recombination (including dielectronic recombination) coefficients for O, Si, and Fe were kindly supplied by Dr. A. Dupree.

#### 4. Summary and Conclusions

In this paper two related aspects of the physical state of the interplanetary plasma at high solar latitudes were explored. In the first part upper limits for the polar solar wind particle flux were derived using a set of reasonable assumptions concerning the base coronal conditions along with Saito's (1970) coronal density model. In the second part, it was determined whether this flux was sufficient to drag the heavier ions away from the sun into interplanetary space.

From the analysis in the first part it was concluded that if Saito's model is correct, the polar electron density is sufficiently low that in the absence of extended heating the solar wind flux at high latitudes should be at least a factor of from 4 to 10 times less than that observed in the ecliptic at 1 AU. Such a low particle flux was shown in the second part to be small enough that most heavy ions would not be expected to expand with the protons into interplanetary space.

However, indirect and limited evidence available at present is consistent with a polar solar wind that has at least as large a velocity (Coles et al., 1974; Brandt et al., 1974) and as large a particle flux (Bertaux et al., 1973; Keller, 1973) as that observed in the ecliptic at 1 AU. From the analysis presented in section 2, these observations then require either that extended heating distinct from that provided by electron heat conduction is necessary some of the time above  $2R_{\odot}$  or that Saito's polar densities are too low. Whichever is the case, the fact that the solar wind particle flux does not appear to decrease with increasing heliographic latitude (Bertaux et al., 1973; Keller, 1973) indicates that coronal heavy ions may be expected to expand with the protons away from the sun. If true

then measurements of the population densities of individual heavy element ionization states in the polar wind will provide information at 1 AU concerning the thermal state of that region in the intermediate corona where the respective ionization states freeze in. It turns out that the latitude variation of these freezing in distances calculated using Saito's model is such that the region in the corona for which temperature values can be determined moves inward from above to below the nominal location of the temperature maximum as  $\theta$  varies between  $0^\circ$  and  $90^\circ$ . Therefore, measurements of heavy ions at high solar latitudes may provide valuable information concerning the magnitude and extent of mechanical dissipation in the intermediate polar corona.

### Acknowledgments

I wish to thank Drs. L. Biermann, J. Gosling, A. Hundhausen and M. Montgomery for many useful discussions.

This work was performed under the auspices of the U.S. Energy Research and Development Administration.

### References

- Alloucherie, Y. J., Heavy Ions in the Solar Corona, Ph.D. Thesis, The University of Maryland, 1967.
- Alloucherie, Y. J., Diffusion of Heavy Ions in the Solar Corona, J. Geophys. Res., 75, 6899, 1970.
- Bame, S. J., J. R. Asbridge, W. C. Feldman and P. D. Kearney, The quiet corona: temperature and temperature gradient, Solar Phys., 35, 137, 1974.
- Bame, S. J., J. R. Asbridge, W. C. Feldman, M. D. Montgomery, and P. D. Kearney, Solar wind heavy ion abundances, to be published in Solar Phys., 1975.
- Bame, S. J., Spacecraft observations of the solar wind composition, in Solar Wind, C. P. Sonett, P. J. Coleman, Jr., and J. M. Wilcox, ed., NASA SP 308, p. 535, 1972.
- Bertaux, J. L., J. E. Blamont, and M. Festou, Interpretation of hydrogen Lyman-alpha observations of comets Bennett and Encke, Astron. and Astrophys. 25, 415, 1973.
- Brandt, J. C., R. W. Michie, and J. P. Cassinelli, Interplanetary gas X. Coronal temperature, energy deposition and the solar wind, Icarus, 4, 19, 1965.
- Brandt, J. C., R. S. Harrington, and R. G. Roosen, Interplanetary gas XX. does the radial solar wind speed increase with latitude?, Astrophys. J., 196, 877, 1975.
- Chapman, S., The viscosity and thermal conductivity of a completely ionized gas, Astrophys. J. 120, 151, 1954.

Coles, W. A., B. J. Rickett, and V. E. Rumsey, Interplanetary Scintillations, in Solar Wind Three, C. T. Russell, ed., Inst. of Geophys. and Planet. Phys., U.C.L.A., Publ., p. 351, 1974.

Durney, B. R. and A. J. Hundhauser, The expansion of a low-density solar corona: a one-fluid model with magnetically modified thermal conductivity, J. Geophys. Res., 79, 3711, 1974.

Eviatar, A. and M. Schulz, Quasi-exospheric heat flux of solar wind electrons, Report SAMSO-TR-75-139, 1975.

Feldman, W. C., J. R. Asbridge, S. J. Bame, and J. T. Gosling, High speed solar wind flow parameters at 1 AU, submitted to J. Geophys. Res., 1976.

Forslund, D. W., Instabilities associated with heat conduction in the solar wind and their consequences, J. Geophys. Res., 75, 17, 1970.

Geiss, J., P. Hirt, and H. Leutwyler, On acceleration and motion of ions in corona and solar wind, Solar Phys., 12, 458, 1970.

Gosling, J. T., J. R. Asbridge, S. J. Bame and W. C. Feldman, A study of solar wind speed variations: 1962-1974, submitted to J. Geophys. Res., 1976.

Hirayama, T., The abundance of helium in prominences and in the chromosphere, Solar Phys., 19, 38<sup>1</sup>, 1971.

Hollweg, J. V., On electron heat conduction in the solar wind, J. Geophys. Res., 79, 3845, 1974.

Hundhausen, A. J., Coronal Expansion and solar Wind, Springer-Verlag, Berlin-Heidelberg, 1972.

Hundhausen, A. J., H. E. Gilbert, and S. J. Bame, The state of ionization of Oxygen in the solar wind, Astrophys. J., 152, L3, 1968a.

Hundhausen, A. J., H. E. Gilbert, and S. J. Bame, Ionization state of the interplanetary plasma, J. Geophys. Res., 73, 5485, 1968b.

- Jockers, K., Solar wind models based on exospheric theories, *Astron. and Astrophys.*, 6, 219, 1970.
- Keller, H. U., Hydrogen production rates of comet Bennett (1969i) in the first half of April, 1970, *Astron. and Astrophys.*, 27, 51, 1973.
- Lemaire, J. and M. Scherer, Simple model for an ion-exosphere in an open magnetic field, *Phys. of Fluids*, 14, 1683, 1971a.
- Lemaire, J. and M. Scherer, Kinetic models of the solar wind, *J. Geophys. Res.*, 76, 7479, 1971b.
- Nakada, M. P., A study of the composition of the solar corona and solar wind, *Solar Phys.* 14, 457, 1970.
- Parker, E. N., Dynamical properties of stellar coronas and stellar winds, 2, integration of the heat flow equation, *Astrophys. J.*, 139, 93, 1964.
- Saito, K., A non-spherical axisymmetric model of the solar K corona of the minimum type, *Annals Tokyo Astron. Observ.*, 12, 53, 1970.
- Schulz, M. and A. Eviatar, Electron-temperature asymmetry and the structure of the solar wind, *Cosmic Electrodyn.*, 2, 402, 1972.
- Spitzer, L., Jr., *The Physics of Fully Ionized Gases*, Interscience, N. Y., 1956.
- Yeh, T., A three-fluid model of solar winds, *Planet. Space Sci.*, 18, 199, 1970.

Table 1

Upper Limit Values of  $(NV)_e$   
 Consistent With the Energy Equation

km/sec	%	$\text{cm}^{-2} \text{sec}^{-1}$	$\text{cm}^{-2} \text{sec}^{-1}$
$V_e$	He/H	$(NV)_e$ (Spitzer)	$(NV)_e$ Exospheric
320	0	$1.02 \times 10^8$	$1.24 \times 10^6$
450	0	$0.70 \times 10^8$	$4.96 \times 10^4$
750	0	$0.33 \times 10^8$	0.41
320	4	$1.28 \times 10^8$	$1.3 \times 10^6$
450	4	$0.88 \times 10^8$	$5.2 \times 10^4$
750	4	$0.41 \times 10^8$	0.4
320	8	$1.54 \times 10^8$	$1.40 \times 10^6$
450	8	$1.06 \times 10^8$	$5.59 \times 10^4$
750	8	$0.49 \times 10^8$	0.46



## Appendix

### Comparison of Expected Electron-Electron Collision Lengths

#### with Scale Lengths in the Polar Corona

The magnitude of the electron conductivity in the polar corona depends critically on the electron-electron collision length,  $\lambda_c$ . If  $\lambda_c$  is small enough then the Spitzer conductivity is applicable but if it is too large, then an exospheric approach is needed to evaluate the polar electron heat flux. It turns out that, according to Saito,  $N_0$  is sufficiently low over the pole that it is not clear whether or not thermal electrons are collisionless above  $R = 2R_0$ . For example the self scattering time for a thermal electron at  $2R_0$  is  $\tau_c = (1.1 \times 10^{-2}) T_0^{3/2} / N_0 = 87$  sec (Spitzer, 1956) whereas at that distance the expansion time (assuming a radial magnetic field) is  $\tau_e = [(kT_0/m_e)^{1/2} d \ln N / dR]^{-1} = 60.5$  sec. Furthermore the coulomb scattering length,  $\lambda_c$ , defined by

$$l = \frac{R_0}{(kT_0/m_e)^{1/2}} \int_{R_0}^{R_0 + \lambda_c} \frac{dR}{\tau_c} \quad (A1)$$

may be either larger than or smaller than the temperature scale length,  $\lambda_T = [-d \ln T / dR]^{-1} = 3.5 R_0$ , depending on the value of the maximum altitude at which Saito's density model is valid,  $R_X$ . This is readily shown by assuming a density model consistent with Saito's results (1970):

$$N = N_0 \left( \frac{R}{R_0} \right)^{-6} \quad 2 \leq R \leq R_X$$

$$N = N_0 \left( \frac{R_X}{R_0} \right)^{-6} \left( \frac{R}{R_X} \right)^{-2.5} \quad R_X < R \quad (A2)$$

with  $R_X \leq 4$ . Combining A1 and A2 and using  $R_0 = 2$  with  $N_0$  from equation 1 evaluated at  $\theta = 90^\circ$  it is found that  $l_c/R_0 = 1.33, 2.48$  and  $6.69$  for  $R_X = 3.0, 3.5$  and  $4.0$  respectively.

Since the actual value of  $l_c$  may be either less than or greater than  $l_T$  depending on the value of  $R_X$ , it is not known whether the Spitzer conductivity or a conductivity calculated using exospheric theory is most valid in the polar solar corona. However, the fact that  $l_c$  is of the same order of magnitude as  $l_T$  suggests that neither of the above is correct and that to obtain an accurate determination of the true conductivity a kinetic approach may be necessary.

### Figure Captions

- Figure 1. Plots of equation 9 for  $R_L = 3$  and 3.5. Radial flow is assumed and hence  $(NV)_e = (NV)_o (R_o/R_e)^2$ . The minimum value of each curve corresponds to the maximum flux at 1 AU consistent with the assumptions  $dT/dR \leq 0$  for  $R \geq 2$  and  $T \geq 0$  for  $R \leq R_X$ .
- Figure 2. Coronal temperatures,  $T(R)$  for values of  $(NV)_e$  and  $T(2)$  determined from equations 9 and 10 evaluated at the minima of the curves in figure 2. Drawn also for comparison are the polar scale height temperature,  $T_H(R)$  and the curve  $T \propto R^{-(2/7)}$ .
- Figure 3. Variations with latitude of the "freezing in" distances of a sample of the most abundant heavy ions. Scale height temperatures are calculated using equation 1 for  $\theta = 0^\circ$  and  $90^\circ$  and a constant "freezing in" temperature of  $T_F = 1 \times 10^6$  K is assumed.





