Extensions of the multicriteria analysis with pairwise comparison under a fuzzy environment

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Abstract

Multicriteria decision-making (MCDM) problems often involve a complex decision process in which multiple requirements and fuzzy conditions have to be taken into consideration simultaneously. The existing approaches for solving this problem in a fuzzy environment are complex. Combining the concepts of grey relation and pairwise comparison, a new fuzzy MCDM method is proposed. First, the fuzzy analytic hierarchy process (AHP) is used to construct fuzzy weights of all criteria. Then, linguistic terms characterized by L–R triangular fuzzy numbers are used to denote the evaluation values of all alternatives versus subjective and objective criteria. Finally, the aggregation fuzzy assessments of different alternatives are ranked to determine the best selection. Furthermore, this paper uses a numerical example of location selection to demonstrate the applicability of the proposed method. The study results show that this method is an effective means for tackling MCDM problems in a fuzzy environment.
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1. Introduction

An MCDM approach is often used to solve various decision-making and/or selection problems. This approach often requires the decision makers to provide qualitative and/or quantitative assessments for determining the performance of each alternative with respect to each criterion, and the relative importance of evaluation criteria with respect to the overall objective. In the past, numerous studies have used the classical MCDM analysis models to deal with decision or selection problems. Churchman and Ackoff [14] proposed a SAW analysis model to solve MCDM problems. Hwang and Yoon [14] proposed the TOPSIS method to determine a solution with the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. However, the TOPSIS method does not consider the relative importances of these distances. Opricovic [23] proposed the VIKOR method as one applicable technique to implement within MCDM. Wolters and Mareschal [22] considered a new type of stability analysis for additive MCDM methods, including the additive utility function and outranking methods. As the assessment values have various types of vagueness/imprecision or subjectiveness, one cannot always use the classical decision-making techniques for these decision problems. In the past few years, numerous attempts to handle this vagueness, imprecision and subjectiveness have been carried out basically by means of fuzzy set theory, and the application of fuzzy set theory to multiple criteria evaluation methods [2,4,5,34].

Grey theory was proposed by Deng [10] based upon the concept that information is sometimes incomplete and/or unknown. This methodology's intent is the same as factor analysis and cluster analysis, except that these methods often do not work well when the sample size is small and the sample distribution is unknown [19]. The grey relation model is based on developmental trends, so there are no strict sample size requirements. This model is a data analysis technique that can be applied to solve MCDM problems, but this model cannot solve MCDM problems in a fuzzy environment [19]. Therefore, this study will present a simple and efficient fuzzy MCDM model based on the incorporated grey relations [9,19] and pairwise comparison [8] to solve involved multi-judges/MCDM problems in a fuzzy environment. In this model, the results concerning the ranking of the alternatives is based on the pairwise comparison of their corresponding fuzzy utilities or fuzzy preference.

Some previous studies have focused on the stochastic nature of the decision process while other studies concern the uncertainty and imprecise numeric values of decision data (including when information is sometimes incomplete and/or unknown condition) and the subjectiveness and imprecision of humans. For example, Chen [7] proposed a vertex method, which is an effective and simple method to measure the distance between two fuzzy numbers, and extended the TOPSIS procedure to a fuzzy environment. Hsu and Chen [12] discussed an aggregation of fuzzy opinions under group decision-making. Li [17] proposed a simple and efficient fuzzy model to deal with multi-judges/MCDM problems in a fuzzy environment. Li [21] proposed several linear programming models and methods for multiattribute decision-making under "intuitionistic fuzziness", where the concept of intuitionistic fuzzy sets is a generalization of the concept of fuzzy sets. Liang [18] incorporated fuzzy set theory and the basic concepts of positive ideal and negative ideal points, and extended MCDM to a fuzzy environment. Ölçer and Odabaşi [25] proposed a new fuzzy multiattribute decision-making method, which is suitable for multiple attributive group decision-making problems in a fuzzy environment, and this method can

deal with the problems of ranking and selection. Olson and Wu [24] presented a simulation of fuzzy multiattribute models based on the concept of grey relations, reflecting either interval input or commonly used trapezoidal input, and this model is a simulated fuzzy MCDM that can be applied to multiattribute decision-making problems effectively. Yeh et al. [28] proposed a fuzzy MCDM method based on the concepts of positive ideal and negative ideal points to evaluate bus companies' performance. Despite these methods' applicability to many decision-making problems, typical fuzzy multicriteria analyses require the comparison of fuzzy numbers. However, the comparison process can be quite complex and produce unreliable results [6,27,28,30], as it may (1) involve considerable computations, (2) produce inconsistent results from respective fuzzy ranking methods, and (3) generate counter-intuitive ranking outcomes for similar fuzzy utilities.

Based on the above concepts from the literature, this paper discusses fuzzy MCDM problems, and a new fuzzy MCDM method, which may reflect both subjective judgment and objective information in real-life situations, is proposed. Through the pairwise comparison method [8], the proposed model will obtain preference relations and ranking orders of the alternatives, and it can also avoid the direct ranking of fuzzy numbers regarding the alternatives. Therefore, the proposed model can overcome the problems of comparing fuzzy numbers and inconsistent ranking of alternatives, and it can also efficiently grasp the ambiguity existing in the available information as well as the essential fuzziness in human judgment and preference. Finally, this paper will use the example of an international logistics (IL) location selection problem to illustrate the proposed method, as this problem is complex and difficult in a real-life environment. Through this case, we will demonstrate that the proposed fuzzy MCDM method for selecting the IL location is a good means of evaluation, and it appears to be more appropriate than other methods.

The remainder of this paper is organized as follows. The basic definitions and notations of the fuzzy numbers and linguistic variables and the concept of the grey relation model are introduced in Section 2. The new method of decision-making analysis based on the incorporated grey relations and pairwise comparison is proposed in Section 3. In Section 4, an illustrative example applying the proposed fuzzy MCDM method to select feasible IL locations is presented, after which we discuss and show how the new fuzzy MCDM method is effective. Finally, conclusions are presented in Section 6.

2. The concepts of the grey relation model and fuzzy set theory

In this section, the concepts of the grey relation model and fuzzy set theory utilized in this paper are briefly introduced.

2.1. Grey relation model for MCDM

The concept of grey relational space [10] is based on the combined concepts of system theory and space theory. The grey relation model can be used to capture the correlations between the reference/aspiration-level (desired) factors and other compared (alternative) factors of a system [9,19]. This model's feature is that both qualitative and quantitative relationships can be identified among complex factors in a system. Therefore, this model is used to examine the extent of connections between two alternatives by applying a distance measurement. Some relevant concepts and the calculation process for the grey relation model are briefly reviewed.

Let $X = \{x_0, x_1, x_2, \ldots, x_i, \ldots, x_m\}$ be a sequence (alternative) set. x_0 denotes the referential sequence (referential alternative), and x_i is a comparative sequence. Let x_{0j} and x_{ij} represent the respective values at point/factor j, $j = 1, 2, \ldots, n$, for x_0 and x_i. The grey relation coefficient $\gamma(x_{0j}, x_{ij})$ of these alternatives at point j [19], can be calculated by

$$\gamma(x_{0j}, x_{ij}) = \frac{\min_i \min_j \Delta_{ij} + \zeta \max_i \max_j \Delta_{ij}}{\Delta_{ij} + \zeta \max_i \max_j \Delta_{ij}},$$

where $\Delta_{ij} = |x_{0j} - x_{ij}|$, and ζ is the resolving coefficient $\zeta \in [0,1]$,

$$i \in I = \{1, 2, \ldots, i, \ldots, m\}, \quad j \in J = \{1, 2, \ldots, j, \ldots, n\}.$$

After obtaining all grey relation coefficients, the grade of grey relation $\gamma(x_0, x_i)$ between x_0 and x_i can be calculated:

$$\gamma(x_0, x_i) = \sum_{j=1}^{n} w_j \gamma(x_{0j}, x_{ij}), \quad \sum_{j=1}^{n} w_j = 1,$$

where w_j denotes the weight of point/factor j.

For the referential sequence x_0 and all comparative sequences x_i, $i = 1, 2, \ldots, m$, if $\gamma(x_0, x_1), \gamma(x_0, x_2), \ldots, \gamma(x_0, x_m)$ satisfy $\gamma(x_0, x_1) > \gamma(x_0, x_2) > \cdots > \gamma(x_0, x_m)$, then we obtain the grey relational order as $x_1 \succ x_2 \succ \cdots \succ x_m$.

2.2. Fuzzy numbers

If X is a collection of objects then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs:

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \in X\},$$

where $\mu_{\widetilde{A}}$ is the membership function and $\mu_{\widetilde{A}}(x)$ is the grade of membership of x in $\widetilde{A}$. The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. If $\sup_{x \in X} \mu_{\widetilde{A}}(x) = 1$, then the fuzzy set $\widetilde{A}$ is called normal (see Fig. 1).

Definition 1. The fuzzy number is of the L–R type if its membership function is of the form [1,5,32], as shown in Fig. 2.
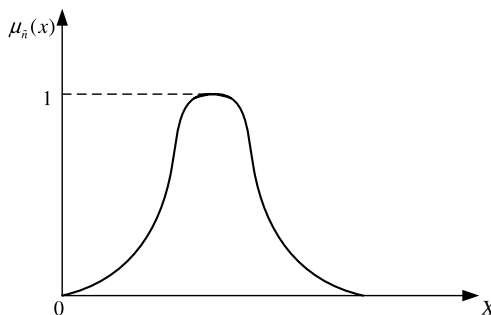
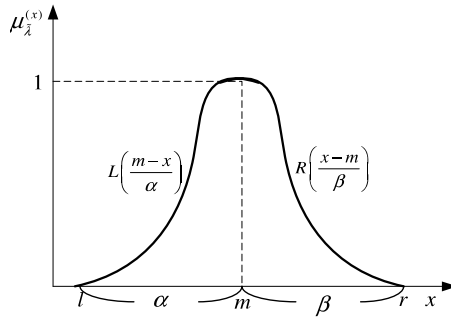

Fig. 1. A fuzzy number $\tilde{n}$.

Fig. 2. A L–R fuzzy number $\widetilde{A}$.

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) & \text{for } x \leqslant m, \alpha > 0, \\[2mm] R\left(\dfrac{x-m}{\beta}\right) & \text{for } x \geqslant m, \beta > 0, \\[2mm] 0 & \text{otherwise}, \end{cases}$$

where m, α and β denote the mean value, the left and right spreads of fuzzy number $\widetilde{A}$, respectively. $L(\cdot)$ and $R(\cdot)$ are two reference functions mapping from $\Re$ (real line) onto $[0,1]$, and satisfy the following properties [1,5,32]:

(1) $L(x) = L(-x)$, $R(x) = R(-x)$,
(2) $L(0) = 1$, $R(0) = 1$,
(3) L and R are two strictly increasing function on $[0,\infty)$.

Definition 2. The α-cut of a fuzzy number $\tilde{n}$ is defined as

$$\tilde{n}^{\alpha} = \{x | \mu_{\tilde{n}}(x) \geqslant \alpha, \ x \in X\},$$

where $\alpha \in [0,1]$.

$\tilde{n}^{\alpha}$ is a non-empty bounded closed interval contained in X and it can be denoted by $\tilde{n}^{\alpha} = [n_{\mathrm{l}}^{\alpha}, n_{\mathrm{r}}^{\alpha}]$, n_{l}^{α} and n_{r}^{α} are lower and upper bounds of the closed interval, respectively [1,5,32]. The fuzzy number $\tilde{n}$ and α-cuts are shown in Fig. 3. For $\tilde{n}^{\alpha_1} = [n_{\mathrm{l}}^{\alpha_1}, n_{\mathrm{r}}^{\alpha_1}]$ and $\tilde{n}^{\alpha_2} = [n_{\mathrm{l}}^{\alpha_2}, n_{\mathrm{r}}^{\alpha_2}]$, if $\alpha_2 \geqslant \alpha_1$, then $n_{\mathrm{l}}^{\alpha_2} \geqslant n_{\mathrm{l}}^{\alpha_1}$ and $n_{\mathrm{r}}^{\alpha_1} \geqslant n_{\mathrm{r}}^{\alpha_2}$.
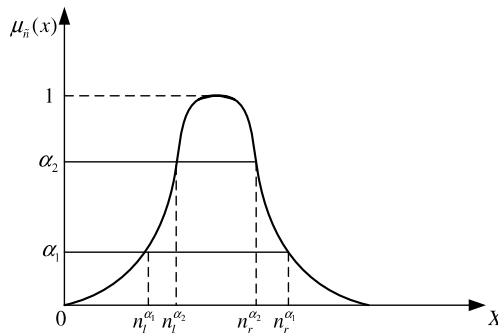

Fig. 3. The fuzzy number $\tilde{n}$ and α-cuts.

Definition 3. A linguistic variable is a variable whose values are linguistic terms [5,12]. Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and/or imprecision qualitative of a decision maker's assessments [5,29,31].

3. New technique for group decision-making

In a real-life environment, decision processes often teem with vagueness, imprecise, indefinite, and subjective data or vague information. Fuzzy set theory might provide the flexibility to represent the imprecise/vague information resulting from the lack of knowledge/information [5,9,15,31]. To efficiently resolve these problems, this study will provide interesting results on group decision-making and MCDM with the help of fuzzy set theory. A new fuzzy MCDM technique, based on the concepts of combining grey relations and pairwise comparisons, is proposed to solve fuzzy MCDM problems.

In this paper, the ratings of qualitative criteria are considered as linguistic terms, and those linguistic terms are characterized by positive L–R fuzzy numbers, as shown in Table 1. Moreover, the imprecise assessment values of quantitative criteria are also expressed by L–R fuzzy numbers. As an evaluator will always perceive the weighting with the judge's own subjective evaluation, an exact or precise weighting for a specified criterion can therefore not be given [9]. In general, the importance weight of each criterion can be obtained by either direct assignment or indirectly using pairwise comparisons [8,13]. Here, the fuzzy AHP method is suggested. The main advantages of the fuzzy AHP method are the relative ease with which it handles multiple criteria [16,20] and, as it is more difficult for the decision maker to provide deterministic preferences, perception-based judgment intervals can be used instead. Furthermore, the preferences in AHP are essentially judgments of human beings based on perception (this is especially true for intangibles), and the fuzzy approach allows a more accurate description of the decision-making process [3,9,26]. Buckley [3] considered a fuzzy positive reciprocal matrix $\widetilde{A} = [\tilde{a}_{jk}]$, and used a geometric mean technique to define the fuzzy geometric mean of each row $\tilde{e}_j$ and fuzzy weight $\tilde{w}_j$ corresponding to each criterion. The calculation formulae are as follows:

$$\tilde{e}_j = (\tilde{a}_{j1}(\cdot)\tilde{a}_{j2}(\cdot)\cdots(\cdot)\tilde{a}_{jn})^{1/n}, \quad \tilde{w}_j = \tilde{e}_j(\cdot)(\tilde{e}_1(+)\tilde{e}_2(+)\cdots(+)\tilde{e}_n)^{-1}; \tag{1}$$

where $\tilde{w}_j$ can be indicated by a triangular fuzzy number $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$. In addition, w_{j1}, w_{j2} and w_{j3} are the lower, middle and upper bounds of the available area for the evaluation data.

The triangular fuzzy number can transform into a L–R triangular fuzzy number after we obtain the triangular fuzzy number of the weight $\tilde{w}_j$. In this paper, the importance weight will be expressed by the L–R triangular fuzzy number $\tilde{w}_j = (w_j^m, w_j^\alpha, w_j^\beta)_{LR}$. Assuming that a

Table 1
Linguistic terms for the fuzzy ratings (example)

Very poor (VP)	$(0, 0, 1)_{LR}$
Poor (P)	$(1, 1, 2)_{LR}$
Medium poor (MP)	$(3, 2, 2)_{LR}$
Fair (F)	$(5, 2, 2)_{LR}$
Medium good (MG)	$(7, 2, 2)_{LR}$
Good (G)	$(9, 2, 1)_{LR}$
Very good (VG)	$(10, 1, 0)_{LR}$

decision group has K judges, the weight of each criterion can be calculated by Eq. (1), and the rating of alternatives with respect to each criterion can be calculated as follows:

$$\tilde{x}_{ij} = \frac{1}{K}\left[\tilde{x}_{ij}^1(+)\tilde{x}_{ij}^2(+)\cdots(+)\tilde{x}_{ij}^K\right] = \frac{1}{K}\sum_{t=1}^{K}\tilde{x}_{ij}^t, \tag{2}$$

where $\tilde{x}_{ij}^t$ is the fuzzy rating assigned by the t-th judge.

It is assumed that an evaluation problem contains m possible alternatives and n criteria with which alternative performances are measured. As stated above, a fuzzy MCDM problem concerning group decision-making can be expressed concisely in matrix format as follows:

$$\widetilde{\mathbf{D}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix} = [\tilde{x}_{ij}]_{m\times n}, \tag{3}$$

$$\tilde{w} = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_2 & \cdots & \tilde{w}_n \end{bmatrix},$$

where $\tilde{x}_{ij}$, $\forall i,j$ is the fuzzy rating of possible alternative A_i, $i = 1, 2, \ldots, m$, with respect to criterion C_j, and $\tilde{w}_j$ is the fuzzy weight of criterion C_j, $j = 1, 2, \ldots, n$. These values can be indicated by a L–R triangular fuzzy number, $\tilde{w}_j = (w_j^m, w_j^\alpha, w_j^\beta)_{\text{LR}}$ and $\tilde{x}_{ij} = (m_{ij}, \alpha_{ij}, \beta_{ij})_{\text{LR}}$.

In order to ensure compatibility between evaluation of objective/quantitative criteria and linguistic ratings of subjective criteria, a normalization method is utilized to transform the various criteria scales into a comparable scale. To avoid complex calculation of the normalization models, we used the linear scale transformation proposed by Hsu and Chen [6,8,11] to transform the various criteria scales into a comparable scale. Through this method, we obtain the normalized fuzzy decision matrix, denoted by $\widetilde{\mathbf{R}}$,

$$\widetilde{\mathbf{R}} = [\tilde{r}_{ij}]_{m\times n},$$

$$\tilde{r}_{ij} = \left(\frac{m_{ij}}{c_j^*}, \frac{\alpha_{ij}}{c_j^*}, \frac{\beta_{ij}}{c_j^*}\right)_{\text{LR}}, \quad j \in B,$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{m_{ij}}, \frac{\beta_{ij}a_j^-}{m_{ij}(m_{ij}+\beta_{ij})}, \frac{\alpha_{ij}a_j^-}{m_{ij}(m_{ij}-\alpha_{ij})}\right)_{\text{LR}}, \quad j \in C, \tag{4}$$

$$c_j^* = \max_i(m_{ij}+\beta_{ij}) \quad \text{if } j \in B,$$

$$a_j^- = \min_i(m_{ij}-\alpha_{ij}) \quad \text{if } j \in C,$$

where B is benefit criteria set, C is cost criteria set. The normalization method mentioned above is used to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0,1].

After performance normalization of various criteria scales, we can define the referential sequence (positive ideal solution) A^* by (5) as follows:

$$A^* = [\tilde{r}_1^*, \tilde{r}_2^*, \ldots, \tilde{r}_n^*],$$

where

$$\tilde{r}_{0j}^* = (1, 0, 0)_{\text{LR}}, \quad j = 1, 2, \ldots, n. \tag{5}$$

From (5), the Hamming distance between comparative sequence (possible alternative) A_i and the referential sequence (positive ideal solution) A^* can be calculated respectively by

$$\tilde{d}_{ij} = (\tilde{r}_{0j}^* - \tilde{r}_{ij}); \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \tag{6}$$

Therefore, we can obtain the distance matrix $\widetilde{\mathbf{H}}$, after which each distance is obtained, denoted by

$$\widetilde{\mathbf{H}} = [\tilde{d}_{ij}]_{m \times n}, \tag{7}$$

where $\tilde{d}_{ij}$ is an L–R triangular fuzzy number denoted by $\tilde{d}_{ij} = (dm_{ij}, d\alpha_{ij}, d\beta_{ij})_{LR}$.

The ideal solution is taken to be the referential sequence and each of the alternatives to be the comparative sequence in order to obtain the fuzzy grey relation coefficient (FGRC) of each alternative ideal value $\gamma(\tilde{r}_{0j}^*, \tilde{r}_{ij})$.

$$\gamma(\tilde{r}_{0j}^*, \tilde{r}_{ij}) = \frac{\min_i \min_j \widetilde{\Delta}_{ij} + \zeta \max_i \max_j \widetilde{\Delta}_{ij}}{\widetilde{\Delta}_{ij} + \zeta \max_i \max_j \widetilde{\Delta}_{ij}}, \tag{8}$$

where $\widetilde{\Delta}_{ij} = \tilde{d}_{ij} = (\tilde{r}_{0j}^* - \tilde{r}_{ij})$, and ζ is the resolving coefficient $\zeta \in [0, 1]$. $\gamma(\tilde{r}_{0j}^*, \tilde{r}_{ij})$ is an L–R triangular fuzzy number denoted by $\gamma(\tilde{r}_{0j}^*, \tilde{r}_{ij}) = (\gamma m_{ij}, \gamma \alpha_{ij}, \gamma \beta_{ij})_{LR}$.

To determine the grade of fuzzy grey relation (FGRG) of each alternative to a ideal solution, the calculation equations are as follows:

$$\widetilde{\Gamma}_{0,i} = \sum_{j=1}^{m} \tilde{w}_j(\cdot) r(\tilde{r}_{0j}^*, \tilde{r}_{ij}). \tag{9}$$

where $\widetilde{\Gamma}_{0,i}$ is the final fuzzy evaluation value (FGRG) of alternative A_i denoted by $\widetilde{\Gamma}_{0,i} = (\tau m_{ij}, \tau \alpha_{ij}, \tau \beta_{ij})_{LR}$.

Finally, after calculating the FGRG of each alternative, the pairwise comparison of the preference relationships between the alternatives can be established, as stated in the following section.

To obtain the preference relationships between the alternatives and to avoid an immediately defuzzified process, this paper uses the concepts of preference relation [8] and ranking procedure proposed by Hsu and Chen [13]. First, we must calculate fuzzy preference relationships between the alternatives. The preference relationship is based on a concept of the fuzzy difference between two fuzzy numbers. Let the FGRG $\widetilde{\Gamma}_{0,i}$ and $\widetilde{\Gamma}_{0,j}$ be L–R triangular fuzzy number. The fuzzy difference between $\widetilde{\Gamma}_{0,i}$ and $\widetilde{\Gamma}_{0,j}$ is also a L–R triangular fuzzy numbers and can be calculated as

$$\widetilde{Z}_{ij} = \widetilde{\Gamma}_{0,i}(-)\widetilde{\Gamma}_{0,j}, \tag{10}$$

$$\widetilde{Z}_{ij}^\alpha = [z_{ijl}^\alpha, z_{ijr}^\alpha], \tag{11}$$

where

$$\Gamma_{0,i}^\alpha = [\Gamma_{0,il}^\alpha, \Gamma_{0,ir}^\alpha], \quad \Gamma_{0,j}^\alpha = [\Gamma_{0,jl}^\alpha, \Gamma_{0,jr}^\alpha], \quad z_{ijl}^\alpha = \Gamma_{0,il}^\alpha - \Gamma_{0,jr}^\alpha, \quad z_{ijr}^\alpha = \Gamma_{0,ir}^\alpha - \Gamma_{0,jl}^\alpha,$$
$$\Gamma_{0,il}^\alpha = \tau m_i^\alpha - \tau \alpha_i^\alpha, \quad \Gamma_{0,ir}^\alpha = \tau m_i^\alpha + \tau \beta_i^\alpha, \quad \Gamma_{0,jl}^\alpha = \tau m_j^\alpha - \tau \alpha_j^\alpha, \quad \Gamma_{0,jr}^\alpha = \tau m_j^\alpha + \tau \beta_j^\alpha,$$
$$\alpha \in [0, 1].$$

If $z_{ijl}^\alpha > 0$, then alternative A_i is absolutely preferred to alternative A_j. That is to say, alternative A_i is greater than alternative A_j absolutely. If $z_{ijl}^\alpha < 0$, then alternative A_i is not absolutely preferred to alternative A_j. If $z_{ijr}^\alpha > 0$ and $z_{ijl}^\alpha < 0$ with some α values, then e_{ij} is a judgment value of the fuzzy preference relation between alternative A_i and alternative A_j. The e_{ij} is defined as

$$e_{ij} = \frac{S_1}{S}, \quad S > 0, \tag{12}$$

where

$$S = S_1 + S_2,$$
$$S_1 = \int_{x>0} \mu_{\tilde{z}_{ij}}(x)\, dx, \quad S_2 = \int_{x<0} \mu_{\tilde{z}_{ij}}(x)\, dx.$$

e_{ij} is the preference degree value (PG) of alternative A_i over alternative A_j, and it is also a judgment value. $\mu_{\tilde{z}_{ij}}(x)$ is the membership function of $\widetilde{\varGamma}_{0,i}(-)\widetilde{\varGamma}_{0,j}$.

In Fig. 4, we can see the shaded area of S_1 and the other area of S_2, and the shaded area (S_1) is greater than the other area (S_2). That is to say, alternative A_i is preferred to alternative A_j in the most favourable situation. The e_{ij} can indicate the degree of preference of alternative A_i over alternative A_j. Furthermore, we can see that $e_{ij} + e_{ji} = 1$ according to the definition of e_{ij} from Eq. (12). Therefore, if $e_{ij} > 0.5$, then alternative A_i is preferred to alternative A_j. If $e_{ij} < 0.5$, then alternative A_j is preferred to alternative A_i. If $e_{ij} = 0.5$, then there is no difference between the two alternatives. Finally, we can construct a fuzzy preference relation matrix $\mathbf{E}$ to draw support from fuzzy preferences e_{ij} and e_{ji}.

$$\mathbf{E} = [e_{ij}]_{m \times n}. \tag{13}$$

The fuzzy preference relation matrix represents the PG value of each pair of alternatives. According to fuzzy preference relation matrix $\mathbf{E}$, the fuzzy strict preference relation matrix $\mathbf{E}^s$ is defined as [13]

$$\mathbf{E}^s = [e_{ij}^s]_{m \times n}, \tag{14}$$

where

$$e_{ij}^s = \begin{cases} e_{ij} - e_{ji}, & \text{when } e_{ij} \geqslant e_{ji}, \\ 0, & \text{otherwise.} \end{cases}$$
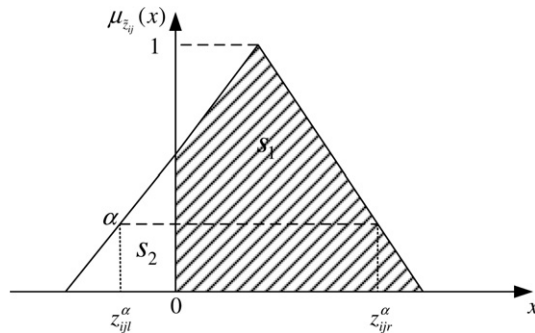
Fig. 4. The illustration of calculating e_{ij}.

The e_{ij}^s value is the strict dominance degree. If alternative A_i is preferred to alternative A_j, then we use the fuzzy strict preference relation matrix $\mathbf{E}^s$, and the non-dominated degree of each alternative A_i can be defined as

$$\mu^{\text{ND}}(A_i) = \min_{\substack{j \in \Omega \\ j \neq i}} \{1 - e_{ji}^s\} = 1 - \max_{\substack{j \in \Omega \\ j \neq i}} e_{ji}^s, \tag{15}$$

where $\Omega = \{A_1, A_2, \ldots, A_m\}$ is a set of alternative.

A large value of $\mu^{\text{ND}}(A_i)$ indicates that alternative A_i has a higher non-dominated degree than the other alternatives. Therefore, we can rank a set of alternatives by the $\mu^{\text{ND}}(A_i)$ values, and obtain preference degrees and ranking orders for each alternative.

4. Numerical example

In this section, a hypothetical location selection problem of international logistics (IL) was designed to demonstrate the computational process of the proposed method.

First, we assume that an international logistics company desires to select a suitable location for establishing a new "distribution" and/or "export processing" center. The hierarchical structure of this decision problem is shown in Fig. 5. The evaluation is done by a committee of five judges $D_1, D_2, \ldots, D_5$. After preliminary screening, three possible alternatives A_1, A_2 and A_3 remain for further evaluation. The international logistics company considers six criteria to select the most suitable possible alternatives. The six estimation criteria are considered as follows:

(1) cost criterion: investment cost (C_1),
(2) benefit criteria: expansion possibility (C_2), availability of acquirement material (C_3), closeness to international port and/or international airport (C_4), human resources (C_5), square measure of area (C_6).

The proposed method is currently applied to solve this problem. The computational procedure is summarized as follows:

Step 1. (1) Take the judges' subjective judgments and use Eq. (1) to calculate the fuzzy weights of all criteria.

Fig. 5. The hierarchical structure.

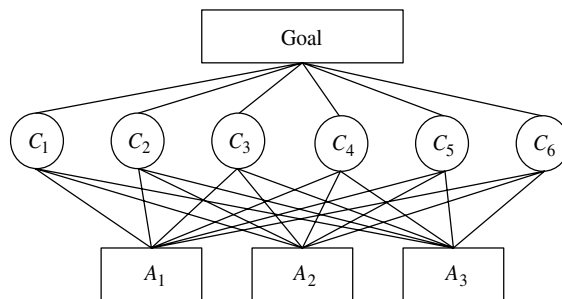(2) Judges use the linguistic terms (shown in Table 1) to evaluate the rating of alternatives versus each criterion. The results are shown in Table 2.

(3) By using Eqs. (2) and (3), the decision matrix of fuzzy ratings of comparative sequences (possible alternatives) and the weights of all criteria can be obtained. The fuzzy decision matrix $\widetilde{\mathbf{D}}$ and the fuzzy weight matrix $\tilde{w}$ are shown in Table 3.

Step 2. Construct the fuzzy normalized decision matrix by using Eq. (4). The result is shown in Table 4.

Step 3. Determine referential sequence (positive ideal solution) by using Eq. (5).

$$A^* = [(1,0,0)_{\text{LR}}, (1,0,0)_{\text{LR}}, (1,0,0)_{\text{LR}}, (1,0,0)_{\text{LR}}, (1,0,0)_{\text{LR}}, (1,0,0)_{\text{LR}}].$$

Step 4. Calculate the Hamming distance between comparative sequence (each alternative) and referential sequence (positive ideal solution) by Eq. (6). Then, construct the distance matrix as shown in Table 5.

Table 2
The ratings of the three candidates by judges under all criteria

Criteria	Candidates	Judges				
		D_1	D_2	D_3	D_4	D_5
C_1	A_1			6 million		
	A_2			5 million		
	A_3			5.5 million		
C_2	A_1	VG	VG	G	VG	VG
	A_2	G	G	MG	G	G
	A_3	G	G	G	G	G
C_3	A_1	G	MG	G	MG	MG
	A_2	G	VG	MG	G	VG
	A_3	G	G	F	G	G
C_4	A_1	P	MP	P	P	P
	A_2	G	MG	G	G	VG
	A_3	VG	G	G	G	VG
C_5	A_1	VG	G	G	VG	G
	A_2	VG	G	G	G	G
	A_3	G	G	G	G	MG
C_6	A_1			4 ha		
	A_2			3 ha		
	A_3			7 ha		

Table 3
The fuzzy weight of criteria and fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
Weight	$(0.232, 0.068, 0.14)_{\text{LR}}$	$(0.113, 0.033, 0.046)_{\text{LR}}$	$(0.136, 0.04, 0.055)_{\text{LR}}$	$(0.188, 0.058, 0.086)_{\text{LR}}$	$(0.139, 0.044, 0.063)_{\text{LR}}$	$(0.192, 0.055, 0.081)_{\text{LR}}$
A_1	$(6,0,0)_{\text{LR}}$	$(9.8, 1.2, 02)_{\text{LR}}$	$(7.8, 2, 1.6)_{\text{LR}}$	$(1.4, 1.2, 2)_{\text{LR}}$	$(9.4, 1.6, 0.6)_{\text{LR}}$	$(4,0,0)_{\text{LR}}$
A_2	$(5,0,0)_{\text{LR}}$	$(8.6, 2, 1.2)_{\text{LR}}$	$(9, 1.6, 0.8)_{\text{LR}}$	$(8, 0.2, 1.8)_{\text{LR}}$	$(9.2, 1.8, 0.8)_{\text{LR}}$	$(3,0,0)_{\text{LR}}$
A_3	$(5.5, 0, 0)_{\text{LR}}$	$(9, 2, 1)_{\text{LR}}$	$(8.2, 2, 1.2)_{\text{LR}}$	$(9.4, 1.6, 0.6)_{\text{LR}}$	$(8.6, 2, 1.2)_{\text{LR}}$	$(7,0,0)_{\text{LR}}$

Table 4
The fuzzy normalized decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(0.833,0,0)_{LR}$	$(0.98,0.12,0.02)_{LR}$	$(0.796,0.204,0.163)_{LR}$	$(0.14,0.12,0.2)_{LR}$	$(0.94,0.16,0.06)_{LR}$	$(0.571,0,0)_{LR}$
A_2	$(1,0,0)_{LR}$	$(0.86,0.2,0.12)_{LR}$	$(0.918,0.163,0.082)_{LR}$	$(0.8,0.02,0.18)_{LR}$	$(0.92,0.18,0.08)_{LR}$	$(0.429,0,0)_{LR}$
A_3	$(0.909,0,0)_{LR}$	$(0.9,0.2,0.1)_{LR}$	$(0.837,0.204,0.122)_{LR}$	$(0.94,0.16,0.06)_{LR}$	$(0.86,0.2,0.12)_{LR}$	$(1,0,0)_{LR}$

Table 5
The fuzzy distance matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(0.167,0,0)_{LR}$	$(0.02,0.02,0.12)_{LR}$	$(0.204,0.163,0.204)_{LR}$	$(0.86,0.2,0.12)_{LR}$	$(0.06,0.06,0.16)_{LR}$	$(0.429,0,0)_{LR}$
A_2	$(0,0,0)_{LR}$	$(0.14,0.12,0.2)_{LR}$	$(0.082,0.082,0.163)_{LR}$	$(0.2,0.18,0.02)_{LR}$	$(0.08,0.08,0.18)_{LR}$	$(0.571,0,0)_{LR}$
A_3	$(0.091,0,0)_{LR}$	$(0.1,0.1,0.2)_{LR}$	$(0.163,0.122,0.204)_{LR}$	$(0.06,0.06,0.16)_{LR}$	$(0.14,0.12,0.2)_{LR}$	$(0,0,0)_{LR}$

Step 5. (1) Calculate the FGRC values of all alternatives by using Eq. (8). The results are shown in Table 6. Then, calculate the FGRG values of all alternatives by using Eq. (9). The results are

$$\widetilde{\Gamma}_{0,1} = (0.849, 0.62, 0.733)_{LR};$$
$$\widetilde{\Gamma}_{0,2} = (0.882, 0.645, 0.782)_{LR};$$
$$\widetilde{\Gamma}_{0,3} = (0.918, 0.703, 0.811)_{LR}.$$

(2) By using Eqs. (10) and (11), the difference between two FGRG values can be calculated as

$$\widetilde{\Gamma}_{0,1} - \widetilde{\Gamma}_{0,2} = (-0.033, 1.401, 1.378)_{LR};$$
$$\widetilde{\Gamma}_{0,1} - \widetilde{\Gamma}_{0,3} = (-0.069, 1.43, 1.436)_{LR};$$
$$\widetilde{\Gamma}_{0,2} - \widetilde{\Gamma}_{0,3} = (-0.036, 1.456, 1.485)_{LR}.$$

Step 6. Construct the fuzzy preference relation matrix by using Eqs. (12) and (13). The result is

$$\mathbf{E} = \begin{bmatrix} 0.5 & 0.472 & 0.454 \\ 0.528 & 0.5 & 0.481 \\ 0.546 & 0.519 & 0.5 \end{bmatrix}.$$

Step 7. Calculate the fuzzy strict preference relation matrix by using Eq. (14). The result is

Table 6
The FGRC matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$(0.838, 0.293, 0.28)_{LR}$	$(0.977, 0.494, 0.381)_{LR}$	$(0.808, 0.434, 0.389)_{LR}$	$(0.5, 0.186, 0.186)_{LR}$	$(0.935, 0.502, 0.395)_{LR}$	$(0.667, 0.217, 0.197)_{LR}$
A_2	$(1, 0.372, 0.372)_{LR}$	$(0.86, 0.475, 0.395)_{LR}$	$(0.913, 0.487, 0.401)_{LR}$	$(0.811, 0.296, 0.404)_{LR}$	$(0.915, 0.505, 0.4)_{LR}$	$(0.601, 0.19, 0.268)_{LR}$
A_3	$(0.904, 0.324, 0.316)_{LR}$	$(0.896, 0.507, 0.405)_{LR}$	$(0.84, 0.462, 0.382)_{LR}$	$(0.935, 0.502, 0.345)_{LR}$	$(0.86, 0.475, 0.395)_{LR}$	$(1, 0.372, 0.372)_{LR}$

$$\mathbf{E}^s = \begin{bmatrix} 0 & 0 & 0 \\ 0.055 & 0 & 0 \\ 0.092 & 0.038 & 0 \end{bmatrix}.$$

Step 8. Set $K = 0$.

Step 9. Calculate the non-dominated degree of each alternative: alternative A_3 has the highest non-dominated degree and set $r(A_3) = 1$.

$$\mu^{\mathrm{ND}}(A_1) = 0.908;$$
$$\mu^{\mathrm{ND}}(A_2) = 0.962;$$
$$\mu^{\mathrm{ND}}(A_3) = 1.$$

Step 10. Delete alternative A_3 from the fuzzy strict preference relation matrix.

Step 11. After deleting alternative A_2, the new fuzzy strict preference relation matrix is

$$\mathbf{E}^s = \begin{bmatrix} 0 & 0 \\ 0.055 & 0 \end{bmatrix}.$$

This paper postulated a resolving coefficient value $\zeta = 1$ while the non-dominated degrees of each alternative A_1, A_2 and A_3 were 0.908, 0.962 and 1, respectively. Therefore, the ranking order of the three alternatives is $A_3 \succ A_2 \succ A_1$. We can see that the best location to establish a new "distribution" and/or "export processing" center is alternative A_3, as alternative A_3 has the highest non-dominated degree. If we used various resolving coefficient values to evaluate the problem, the results obtained, as shown in Table 7, are satisfactory. We can then find the variation of the non-dominated degree value for various resolving coefficient values. This paper found that the ranking orders of the three alternatives are the same despite changes to the values of the resolving coefficient (as shown in Table 7 and Fig. 6), so we can confirm that the results obtained from the ranking orders using the proposed method are reliable, and these results can help decision-makers to identify the best alternative. In addition, the proposed method found that the gap between the non-dominated degree values of the alternatives became larger when the values of the resolving coefficient decreased from 1 to 0.1. Though the gap between the non-dominated degree values of the alternatives can easily be found using various resolving coefficient values, these results are not conspicuous owing to the closeness of the performances of the alternatives. The proposed method can distinguish the differences among the alternatives more easily, and the above analysis shows that the proposed method has produced satisfactory results. Furthermore, the proposed method not only allows judges to determine the ranking order of alternatives but can also indicate the preference degree of alternatives in pairwise comparisons. That is, this method can easily obtain a best alternative from the

Table 7
The non-dominated degree value based on each resolving coefficient (ζ)

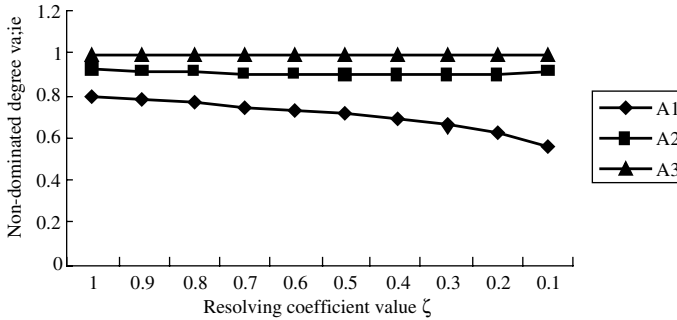| ζ	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
A_1	0.793	0.781	0.767	0.751	0.733	0.712	0.688	0.658	0.621	0.564
A_2	0.923	0.920	0.915	0.911	0.906	0.901	0.897	0.894	0.896	0.919
A_3	1	1	1	1	1	1	1	1	1	1

Fig. 6. The variation analysis of non-dominated degree value for each alternative.

candidates, and it is more suitable and effective than other methods to deal with various types of vagueness/imprecision or subjectiveness in a fuzzy environment.

5. Discussion

As mentioned above, this method can be applied to entire/individual evaluators according to their own preferences to select their ideal alternative. Through a numerical example, it is shown that the proposed method can be utilized to tackle fuzzy MCDM problems in a fuzzy environment very well. In addition, the results are satisfactory. According to the results of the analysis, one can see the gap between the ideal alternative and each alternative, and the best alternative from all alternatives can be found. Moreover, in order that the FGRG values of the evaluation do not damage the true values, this study avoids an immediately defuzzified process. The concepts of preference relation and ranking procedure [13] are also used in this paper. Therefore, the proposed method can find the gap between the ideal alternative and each alternative, and the preference relation between two alternatives in addition to the ranking order of alternatives. In order to interpret the difference between FGRG values of defuzzified and non-dominated degree values of the preference relation, this study uses defuzzification of the best nonfuzzy performance (BNP) value [33] to the FGRG value. In addition, this paper used the discrimination index [30] to examine the proposed method. This index is calculated based on the final ranking values of the integrated performance produced by a method of evaluation of decision problems, and the decision maker will have more confidence in making decisions based on the ranking values produced by this method. The discrimination index DI_k is defined as

$$DI_k = \frac{\sum_{i=1}^{k} \dfrac{P_i - P_{i+1}}{\max\left(\frac{1}{k}, (P_i - P_{i+1})\right)}}{k}, \quad k \in \{1, 2, \ldots, n-1\}, \tag{16}$$

where k is the number of alternatives that a decision maker wants to select from. The DI_k is measured by the average ratio of the difference between each pair of adjacent ranking values "P_i and P_{i+1}" and an ideal difference. The ideal difference is determined by the value of k.

Table 8 shows the discrimination index of the ranking values produced by the proposed method (including: FGRG values and non-dominated degree values) and the FSAW [5]

Table 8
Comparison of discrimination index between ranking values of FGRG values, non-dominated degree values, and fuzzy MCDM

Method	$\zeta = 0.1$ $A_3 \succ A_2 \succ A_1$		$\zeta = 0.3$ $A_3 \succ A_2 \succ A_1$		$\zeta = 0.5$ $A_3 \succ A_2 \succ A_1$		$\zeta = 0.7$ $A_3 \succ A_2 \succ A_1$		$\zeta = 0.9$ $A_3 \succ A_2 \succ A_1$		$\zeta = 1$ $A_3 \succ A_2 \succ A_1$	
	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$
FGRG values of proposed method	0.03881	0.24060	0.06792	0.22491	0.0669	0.19863	0.06178	0.17570	0.05639	0.15690	0.05388	0.14884
Non-dominated degree values of proposed method	0.08054	0.43641	0.10617	0.34169	0.09895	0.28788	0.08928	0.24897	0.08045	0.21935	0.07652	0.20704

	$A_3 > A_2 > A_1$	
	$k = 1$	$k = 2$
FSAW [5]	0.08875	0.23576

evaluation method. The non-dominated degree value of the proposed method has higher discriminating performance, as compared with the FGRG value of the proposed and fuzzy SAW methods. Lastly, this study found that the non-dominated degree value can give unsatisfactory results. As the proposed method can produce satisfactory results in terms of rationality and discriminatory ability, it is an appropriate and good means for tackling fuzzy MCDM problems. Through the discrimination index, this study would have more confidence in the proposed method if the alternative numbers were much larger. Furthermore, this study found the ranking orders of the evaluation alternatives are the same despite changing the values of the resolving coefficient, but the gap of the non-dominated degree values or FGRG values among the alternatives became larger when the values of the resolving coefficient decreased from 1 to 0.1. Through the resolving coefficient, the proposed method can distinguish the differences between the alternatives more easily, thus this method is better than the other methods. However, the proposed method and the other methods calculate the performances of the alternatives to be close when the gaps among the alternatives are not conspicuous.

6. Conclusion

In this paper, an effective fuzzy multicriteria analysis method based on incorporated grey relations and pairwise comparisons is presented to solve fuzzy MCDM problems. The proposed method can simultaneously obtain the gap between the ideal alternative and each alternative, the preference relation between two alternatives, and the ranking order of alternatives. A numerical example regarding IL location selection has been conducted to examine the applicability of the proposed method. Furthermore, a comparative study is used to examine the rationality and discriminatory ability of the results of the proposed method. This paper finds that the performances among the alternatives are close or similar when the gaps between the alternatives are not conspicuous. This issue is little discussed in the literature. In addition, the proposed method is constructed under the condition of independent criteria in this study. A subsequent method should extend the proposed method to consider independent, dependent and/or interdependent criteria.

Although the proposed method presented in this paper is illustrated by a location selection problem, it could also be applied to problems such as information project selection, material selection, and many other areas of management decision problems or strategy selection problems.

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