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Momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids past a stretching surface with non-uniform heat source/sink

N. Sandeep, C. Sulochana *

Department of Mathematics, Gulbarga University, Gulbarga 585106, Karnataka, India

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KEYWORDS

MHD; Jeffrey nanofluid; Maxwell nanofluid; Oldroyd-B nanofluid; Radiation; Non-uniform heat source/ sink

Abstract In this paper, we proposed a new mathematical model for investigating the momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids over a stretching surface in the presence of transverse magnetic field, non-uniform heat source/sink, thermal radiation and suction effects. The governing boundary layer partial differential equations are transformed into nonlinear ordinary differential equations by using similarity transformation and solved numerically by using Runge–Kutta based shooting technique. The effects of non-dimensional governing parameters on the flow and heat transfer are discussed with the help of graphs. Numerical values of the skin friction coefficient and the local Nusselt number are computed and discussed. We found an excellent agreement of the present results by comparing with the published results. Results indicate that the Jeffrey nanofluid has better heat transfer performance while compared with the Maxwell and Oldroyd-B nanofluids.

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1. Introduction

The flow and heat transfer of non-Newtonian fluids have attained a greatest importance due to their applications in

 $*$ Corresponding author.

E-mail addresses: nsreddy.dr@gmail.com (N. Sandeep), [math.](mailto:math.sulochana@gmail.com) [sulochana@gmail.com](mailto:math.sulochana@gmail.com) (C. Sulochana).

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industry and engineering. Fluids that do not obey Newton's law of motion are called non-Newtonian fluids. Tooth paste, food products, waste fluids, and form oil are some of the good examples of non-Newtonian fluids. The flow and heat transfer characteristics of non-Newtonian fluids are quite different from Newtonian flows. Undoubtedly, the governing equations represent the non-Newtonian fluid flows are highly nonlinear and more complicated compared with the Navier–Stokes equations. Due to this high nonlinearity, closed form solutions for non-Newtonian fluid flows are not possible for the problems with practical interest. Jeffrey, Maxwell and Oldroyd-B nanofluids have their own importance due to their special stress relaxation properties.

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Choi [\[1\]](#page-5-0) was the first person who introduced the concept of nanofluid by immersing the nanometer sized particles into the base fluid. Viscous dissipation and heat transfer effects of a non-Newtonian fluid over a nonlinear stretching surface were discussed by Chen [\[2\].](#page-5-0) Sajid et al. [\[3\]](#page-6-0) have illustrated the stagnation point flow of an Oldroyd-B fluid over a stretching surface in porous medium. Hayat et al. [\[4\]](#page-6-0) have studied the convective heat transfer in two-dimensional boundary layer flow of an Oldroyd-B fluid over a stretching surface in the presence of radiation and found that the temperature profiles of Oldroyd-B fluid increase with the increase in radiation parameter. MHD flow of Jeffrey nanofluid over a stretching surface with Brownian motion and thermophoresis effects was discussed by Abbasi et al. [\[5\]](#page-6-0). In this study they found that Brownian motion and thermophoresis parameters have tendency to enhance the thermal boundary layer thickness. Nonlinear thermal radiation effect on Jeffrey fluid flow in the presence of homogeneous–heterogeneous reactions was discussed by Raju et al. [\[6\]](#page-6-0). Further Raju and Sandeep [\[7\]](#page-6-0) extended this work for bio-convection flow and analysed the heat and mass transfer in non-Newtonian bio-convection flow over a rotating cone/plate.

The flow of Jeffrey fluid over an asymmetric rotating channel by using closed form solutions was investigated by Alla and Dahab [\[8\]](#page-6-0) and found that the magnitude of the shear stress is high in symmetric channel compared with the asymmetric channel. Akram et al. [\[9\]](#page-6-0) discussed the consequences of nanofluid flow over an asymmetric channel. The influence of variable viscosity on the non-Newtonian fluid flow over a permeable inclined channel with slip conditions was presented by Khan et al. [\[10\].](#page-6-0) Ellahi et al. [\[11\]](#page-6-0) have studied MHD effects on peristaltic flow of a Jeffrey fluid over a rectangular duct in porous medium. Hayat and Ali [\[12\]](#page-6-0) presented a mathematical model for MHD peristaltic flow in a tube and found that a raise in the value of magnetic field parameter declines the momentum boundary layer thickness. Hayat et al. [\[13\]](#page-6-0) presented a series solution for MHD boundary layer flow of a Maxwell fluid over a permeable stretching sheet with suction and injection effects. Raju et al. [\[14\]](#page-6-0) studied the heat and mass transfer in 3D non-Newtonian nano and Ferro fluids over a bidirectional stretching surface. An analytical solution for axisymmetric flow of a second grade fluid over a stretching surface was discussed by Hayat and Sajid [\[15\].](#page-6-0) Baris and Dokuz [\[16\]](#page-6-0) analysed the stagnation point flow of a second grade fluid past a moving vertical plate. The flow of Jeffrey fluid near the stagnation point over a stretching surface with convective boundary conditions was discussed by Nadeem et al. [\[17\]](#page-6-0).

An unsteady mixed convection flow of a micropolar fluid over a stretching or shrinking sheet with non-uniform heat source/sink was discussed by Sandeep and Sulochana [\[18\]](#page-6-0). Radiation and aligned magnetic field effects on the flow over a stretching surface were studied by Raju et al. [\[19\],](#page-6-0) and they found that radiation parameter have tendency to increase the temperature profiles of the flow. Mehmood et al. [\[20\]](#page-6-0) discussed the stagnation point flow and heat transfer characteristics of micropolar second grade fluid past a stretching surface. Nadeem and Akbar [\[21\]](#page-6-0) depicted the heat and mass transfer in peristaltic motion of a Jeffrey fluid in an annulus. Makinde et al. [\[22\]](#page-6-0) have studied an unsteady flow of non-Newtonian fluid in a saturated porous medium with convective boundary conditions. Hussain et al. [\[23\]](#page-6-0) illustrated a steady twodimensional MHD Jeffrey nanofluid flow over an exponen-

tially stretching sheet in the presence of viscous dissipation and thermal radiation and found a hike in thermal boundary layer with the increase in the viscous dissipation parameter. Influence of viscous dissipation on Couette–Poiseuille flow for pseudo-plastic fluids was studied by Francisca [\[24\]](#page-6-0).

Double diffusive mixed convection flow of a viscoelastic fluid using weak nonlinear stability analysis was discussed by Malashetty [\[25\]](#page-6-0). Makinde and Mhone [\[26\]](#page-6-0) discussed nonlinear flow of an incompressible conducting viscous fluid over converging–diverging channels in the presence of uniform magnetic field. Ibrahim and Makinde [\[27\]](#page-6-0) studied the boundary layer flow and heat transfer of a nanofluid over a stretching surface. Hydro thermal behaviour of nanofluid flow due to external heated plates was studied by Sheikholeslami [\[28\]](#page-6-0). The influence of suction on the flow of nanofluid over a cylinder was discussed by Sheikholeslami [\[29\]](#page-6-0). Sheikholeslami and Rashidi [\[30\]](#page-6-0) illustrated the heat transfer characteristics of ferrofluid flow in the presence of variable magnetic field. Heat transfer enhancement in nanofluid for Lattice Boltzmann simulation was presented by Sheikholeslami et al. [\[31\]](#page-6-0). The effects of magnetic field on nanofluid flow, heat, and mass transfer between two horizontal coaxial cylinders are studied by Sheikholeslami and Abelman [\[32\]](#page-6-0). Magnetohydrodynamic nanofluid flow in a cubic cavity heated from below is discussed by Sheikholeslami and Ellahi [\[33\]](#page-6-0). Recently, the researchers [\[34–40\]](#page-6-0) discussed about the heat and mass transfer in MHD nanofluids in the presence of thermal radiation effect.

In this study we discussed the momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids over a stretching surface in the presence of transverse magnetic field, non-uniform heat source/sink, thermal radiation and suction effects. The governing boundary layer partial differential equations are transformed into nonlinear ordinary differential equations by using self similarity transformation. The ordinary differential equations are solved numerically by using Runge–Kutta based shooting technique. The effects of nondimensional governing parameters on the flow and heat transfer are discussed with the help of graphs. Numerical values of the skin friction coefficient and the local Nusselt number are computed and analysed.

2. Mathematical analysis

Consider a steady incompressible, laminar, hydromagnetic flow of Jeffrey, Maxwell and Oldroyd-B nanofluids over a stretching sheet. A transverse magnetic field of strength B_0 is

Figure 1 Physical model of the problem.

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applied to the flow as displayed in [Fig.1](#page-1-0). A non-uniform heat source/sink is taken into account. It is assumed that the surface is heated by a hot fluid at the reference temperature T_r . Magnetic Reynolds number is assumed to be very small so that the induced magnetic field is neglected. As per the aforesaid assumptions the governing boundary layer equations are given as follows $[5,13]$:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}
$$

$$
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\lambda_1 \left(u^2 \frac{\partial^2 u}{\partial x^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} + v^2 \frac{\partial^2 u}{\partial y^2} \right) + \frac{v_f}{(1 + \lambda_2)} \left(\frac{\partial^2 u}{\partial y^2} + \lambda_3 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} \right) \right)
$$
 (2)
- $\frac{\partial B_0^2}{\rho_f} u$,

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_f} \frac{\partial q_r}{\partial y} + \frac{1}{(\rho c_p)_f} q''',\tag{3}
$$

The boundary conditions of the flow are as follows:

$$
u = u_w(x) = cx, v = V_w, -k\frac{\partial T}{\partial y} = h(T_r - T), \text{ at } y = 0,
$$

$$
u \to 0, T \to T_\infty, \text{ as } y \to \infty,
$$
 (4)

where u and v are the velocity components along x and y directions respectively, λ_1 is the relaxation time, λ_2 is the ratio of the relaxation to retardation times, λ_3 is the retardation time, ρ_f is the density of the fluid, σ is the electrical conductivity, T is the temperature of the fluid, T_r is the reference temperature, k is the thermal conductivity, h is the heat transfer coefficient, α is the thermal diffusivity of the nanofluid, $(\rho c_p)_f$ is the specific heat capacitance, q_r is the radiative heat flux, $q^{\prime\prime\prime}$ is the nonuniform heat source/sink and V_w is the suction velocity.

The problem is studied based on the following conditions:

- (i) If $\lambda_1 = 0$, $\lambda_2 \neq 0$, $\lambda_3 \neq 0$ then the problem represents Jeffrey nanofluid model.
- (ii) If $\lambda_1 \neq 0$, $\lambda_2 = 0$, $\lambda_3 \neq 0$ then the problem represents Oldroyd-B nanofluid model.
- (iii) If $\lambda_1 \neq 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ then the problem represents Maxwell nanofluid model.

The radiative heat flux q_r under Rosseland approximation has the form [\[44\]](#page-7-0)

$$
q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y},\tag{5}
$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding $T⁴$ using Taylor series and neglecting higher order terms yields [\[41\]](#page-6-0)

$$
T^4 \cong 4T_{\infty}^3 T - 3T_{\infty}^4,\tag{6}
$$

The space and temperature dependent heat generation/ absorption (non-uniform heat source/sink) $q^{\prime\prime\prime}$ are defined as $[18]$

$$
q''' = \left(\frac{ku_w(x)}{xv_f}\right) (A^*(T_w - T_\infty)f' + B^*(T - T_\infty)),\tag{7}
$$

where A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption. The positive and negative values of A^* and B^* represent heat generation and absorption respectively. To convert the governing equations into set of nonlinear ordinary differential equations we now introduce the following similarity transformation

$$
u = cxf'(\eta), v = -(v_f c)^{0.5} f(\eta), \eta = (c/v_f)^{0.5} y, \theta(\eta)
$$

= $(T - T_{\infty})/(T_r - T_{\infty}),$ (8)

 \sim

Using Eqs. (5) – (8) , Eqs. (2) and (3) can be reduced into the following nonlinear ordinary differential equations.

$$
f''' + \gamma_2 (f''^2 - ff'''') - M(1 + \lambda_2)f' - (1 + \lambda_2)f'^2 - ff'' + \gamma_1 (f^2 f''' - 2ff' f'')]= 0,
$$
(9)

$$
\left(1+\frac{4}{3}R\right)\theta'' + Pr\ f\theta' + A^*f' + B^*\theta = 0,\tag{10}
$$

with the transformed boundary conditions

$$
f = S, f' = 1, \quad \theta' = -Bi(1 - \theta(0)), \quad \text{at } \eta = 0,
$$

$$
f' \to 0, \quad \theta \to 0, \quad \text{as } \eta \to \infty,
$$
 (11)

where $M = \sigma B_0^2 / \rho_f c$ is the magnetic field parameter, $\gamma_1 = \lambda_1 c$ is the Deborah number with respect to relevation time, $\gamma_1 = \lambda_1 c$ is the Deborah number with respect to relaxation time, $\gamma_2 = \lambda_3 c$ is the Deborah number with respect to retardation time, λ_2 is the ratio of the relaxation to retardation times, $Pr = v_f / \alpha$ is the Prandtl number, A^* , B^* are the non-uniform heat source/sink parameters, $R = 4\sigma T_{\infty}^3 / k k^*$ is the radiation parameters $R = \frac{4\sigma T_{\infty}^3}{k^*} / k k^*$ eter, $Bi = (h/k)\sqrt{v_f/c}$ is the Biot number and $S = V_w/(v_f c)^{0.5}$ is the suction parameter.

For engineering interest the shear stress coefficient or friction factor (C_f) and local Nusselt number (Nu_x) are given by

$$
Re_x^{0.5} C_f = \frac{1 + \gamma_1}{1 + \gamma_2} f''(0),
$$
\n(12)

$$
Re_x^{-0.5} Nu_x = -\left(1 + \frac{4}{3}R\right) \theta'(0), \tag{13}
$$

where $Re_x = \frac{u_{w}x}{v_f}$ is the local Reynolds number.

3. Numerical procedure

The nonlinear ordinary differential Eqs. (9) and (10) subjected to the boundary conditions (11) are solved numerically using Runge–Kutta based shooting technique [\[18\].](#page-6-0) We assumed the unspecified initial conditions for unknown values, and the transformed first order differential equations are integrated numerically as an initial valued problem to a given terminal point. We have checked the accuracy of the assumed missing initial conditions by comparing with the calculated value. The calculations are carried out by the program using MATLAB.

4. Results and discussion

The effects of dimensionless governing parameters namely magnetic field parameter (M) , Radiation parameter (R) , nonuniform heat source/sink parameters (A^*, B^*) , Biot number $B(i)$ and Prandtl number (Pr) on the flow and heat transfer of Jeffrey, Maxwell and Oldroyd-B nanofluids are discussed

Figure 2 Velocity field for different values of magnetic field parameter M.

Figure 3 Temperature field for different values of magnetic field parameter M.

and presented through graphs. Also, the influence of nondimensional governing parameters on the skin friction coefficient and local Nusselt number is computed and presented through tables. For numerical computations we considered $Bi = 0.8$, $Pr = 6.8$, $M = R = 1$, $A^* = B^* = 0.1$, $\lambda_2 = \gamma_1$ $\gamma_2 = \gamma_2 = 0.5$. These values are kept as common in entire study except the varied values as shown in the respective figures and tables.

Figs. 2 and 3 depict the effect of magnetic field parameter on the velocity and temperature profiles of the flow. It is evident that rise in the magnetic field parameter decreases the velocity profiles and increases the temperature profiles of the flow. Generally, an increase in the magnetic field parameter develops the opposite force to the flow, is called the Lorentz force. This force has tendency to reduce the momentum boundary layer and enhance the thermal boundary layer thickness. It is interesting to mention here that the Jeffrey nanofluid

Figure 4 Temperature field for different values of non-uniform heat source/sink parameter A^* .

Figure 5 Temperature field for different values of non-uniform heat source/sink parameter B^* .

Figure 6 Velocity field for different values of suction parameter S.

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Figure 7 Temperature field for different values of suction parameter S.

Figure 8 Temperature field for different values of radiation parameter R.

Figure 9 Temperature field for different values of Biot number Bi.

Figure 10 Temperature field for different values of Prandtl number Pr.

is highly influenced by magnetic field parameter compared with other two nanofluids.

[Figs. 4 and 5](#page-3-0) depict the effect of non-uniform heat source/ sink parameter on the temperature profiles of the flow. It is clear from the figures that an increase in the values of A^* , B^* increases the temperature profiles of the flow. It is also observed that the influence of A^* is high on Jeffery nanofluid. Maxwell nanofluid is highly influenced by B^* . The positive values of non-uniform heat source/sink parameters act like heat generators. Generating the heat causes to release the heat energy to the flow and hence develops the temperature profiles of the flow. This agrees with the general physical behaviour of heat source parameter.

[Figs. 6 and 7](#page-3-0) illustrate the effect of suction parameter on the velocity and the temperature profiles of the flow. It is observed that with the increase in the suction parameter we noticed depreciation in the velocity and the temperature profiles of the flow. It is also observed that the Oldroyd-B fluid is highly influenced by suction parameter compared with the Maxwell and Oldroyd-B nanofluids. This agrees with the general physical behaviour of the suction parameter.

Figs. 8 and 9 show the effect of radiation parameter and Biot number on the temperature profiles of the flow. It is evident that an increase in the values of the radiation parameter and the Biot number enhances the temperature profiles of the flow. An increase in the radiation parameter releases the heat energy to the flow, and this helps to develop the thermal boundary layer thickness. In the other hand an increase in the

value of Biot number enhances the heat transfer coefficient, and these causes to enhance the temperature profiles of the flow. The effect of Prandtl number on the temperature profiles of the flow is displayed in [Fig. 10](#page-4-0). It is clear that an increase in the Prandtl number declines the temperature profiles of the flow. This is due to the fact that with the increase in the Prandtl number causes to increase the viscosity of the fluid, which causes to enhance the wall friction along with the thermal conductivity. Due to this reason we have seen fall in the temperature profiles of the flow.

[Table 1](#page-4-0) shows the comparison of the present results with the existed results of Ferdows et al. [\[42\]](#page-6-0) and Rashidi et al. [\[43\].](#page-7-0) Under some special conditions, present results have an excellent agreement with the existed results. This shows the validity of the present results along with the accuracy of the numerical technique we used in this study. Table 2 shows the variation in local Nusselt number at different nondimensional governing parameters. It is evident that an increase in the magnetic field parameter, non-uniform heat source or sink parameter and thermal radiation parameter depreciates the heat transfer rate. But for higher values of Biot number, Prandtl number and suction parameter we noticed a hike in the local Nusselt number. Table 3 displays the variation in the skin friction coefficient at different non-dimensional governing parameters. It is observed that a raise in the values of magnetic field parameter and suction parameter depreciates the friction factor. But an increase in the radiation parameter and Biot number does not shown significant variation in the skin friction coefficient.

5. Conclusions

This study presents a numerical solution to analyze the momentum and heat transfer behaviour of Jeffrey, Maxwell and Oldroyd-B nanofluids over a stretching surface in the presence of transverse magnetic field, non-uniform heat source/ sink, thermal radiation and suction effects. The governing boundary layer partial differential equations are transformed into nonlinear ordinary differential equations by using self similarity transformation and solved numerically. The conclusions of the present study are made as follows:

- Biot number and suction parameter have tendency to enhance the heat transfer rate.
- Jeffrey nanofluid has better heat transfer performance while compared with the Maxwell and Oldroyd-B nanofluids.
- Rise in the value of the magnetic field parameter reduces the friction factor.
- Increasing values of the non-uniform heat source or sink parameters enhances the thermal boundary layer thickness.
- Higher values of radiation parameter increase the temperature profiles of the flow.

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Dr. N. Sandeep obtained his M.Sc., and Ph.D. degrees from Sri Venkateswara University, Tirupati, M.Phil degree from Madurai Kamaraj University, Madurai. He published more than 30 papers in reputed scientific journals and presented papers in national and international conferences. At present he is serving as a Post Doctoral Fellow under UGC Dr.D.S.Kothari Fellowship Scheme at Department of Mathematics, Gulbarga University, Gulbarga. His area of interest is Fluid Mechanics.

Prof. C. Sulochana obtained her M.Sc., and M.Phil., degrees from Gulbarga University, Gulbarga, Karnataka and Ph.D degree from Sri Krishna Devaraya University, Anantapur. She produced 6 Ph.D and 7 M.Phil, degrees. She published more than 35 papers in reputed scientific journals and presented papers in national and international conferences. She handled various prestigious positions at Gulbarga University. At present she is serving as a professor at Department of Mathematics, Gulbarga University, Gulbarga. Her area of interest is Fluid Mechanics.