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Holographic dark energy and cosmic coincidence

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Abstract

In this Letter we demonstrate that any interaction of pressureless dark matter with holographic dark energy, whose infrared cutoff is set by the Hubble scale, implies a constant ratio of the energy densities of both components thus solving the coincidence problem. The equation of state parameter is obtained as a function of the interaction strength. For a variable degree of saturation of the holographic bound the energy density ratio becomes time dependent which is compatible with a transition from decelerated to accelerated expansion.

Nowadays there is a wide consensus among cosmologists that the Universe has entered a phase of accelerated expansion [1]. The debate is now centered on when the acceleration did actually begin, whether it is to last forever or it is just a transient episode and, above all, which is the agent behind it. Whatever the agent, usually called dark energy, it must possess a negative pressure high enough to violate the strong energy condition. A number of dark energy candidates have been put forward, ranging from an incredibly tiny cosmological constant to a variety of exotic fields (scalar, tachyon, k-essence, etc.) with suitably chosen potentials [2]. Most of the candidates, however, suffer from the coincidence problem, namely: *Why are the matter and dark energy densities of precisely the same order today?* [3].

Recently, a new dark energy candidate, based not in any specific field but on the holographic principle, was proposed [4–9]. The latter, first formulated by 't Hooft [10] and Susskind [11], has attracted much attention as a possible short cut to quantum gravity and found interesting applications in cosmology—see, e.g., [12]—and black hole growth [13]. According to this principle, the number of degrees of freedom of physical systems scales with their bounding area rather than with their volume. In this context Cohen et al. reasoned that the dark energy should obey the aforesaid principle and be constrained by the infrared (IR)

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cutoff [14]. In line with this suggestion, Li has argued that the dark energy density should satisfy the bound $\rho_X \leqslant 3M_p^2 c^2/L^2$, where c^2 is a constant and $M_p^2 = (8\pi G)^{-1}$ [7]. He discusses three choices for the length scale L which is supposed to provide an IR cutoff. The first choice is to identify L with the Hubble radius, H^{-1}. Applying arguments from Hsu [6], Li demonstrates that this leads to a wrong equation of state, namely that for dust. The second option is the particle horizon radius. However, this does not work either since it is impossible to obtain an accelerated expansion on this basis. Only the third choice, the identification of L with the radius of the future event horizon gives the desired result, namely a sufficiently negative equation of state to obtain an accelerated universe.

Here, we point out that Li's conclusions rely on the assumption of an independent evolution of the energy densities of dark energy and matter which, in particular, implies a scaling $\rho_M \propto a^{-3}$ of the matter energy density ρ_M with the scale factor $a(t)$. Any interaction between both components will change, however, this dependence. The target of this Letter is to demonstrate that as soon as an interaction is taken into account, the first choice, the identification of L with H^{-1}, can simultaneously drive accelerated expansion and solve the coincidence problem. We believe that models of late acceleration that do not solve the coincidence problem cannot be deemed satisfactory (see, however, [15]).

Let us reconsider the argument Li used to discard the identification of the IR cutoff with Hubble's radius. Setting $L = H^{-1}$ in the above bound and working with the equality (i.e., assuming that the holographic bound is saturated) it becomes $\rho_X = 3c^2 M_P^2 H^2$. Combining the last expression with Friedmann's equation for a spatially flat universe, $3M_P^2 H^2 = \rho_X + \rho_M$, results in $\rho_M = 3(1-c^2)M_P^2 H^2$. Now, the argument runs as follows: the energy density ρ_M varies as H^2, which coincides with the dependence of ρ_X on H. The energy density of cold matter is known to scale as $\rho_M \propto a^{-3}$. This corresponds to an equation of state $p_M \ll \rho_M$, i.e., dust. Consequently, this should be the equation of state for the dark energy as well. Thus, the dark energy behaves as pressureless matter. Obviously, pressureless matter cannot generate accelerated expansion, which seems to rule out the choice $L = H^{-1}$. This is exactly Li's conclusion. What underlies this reasoning is the assumption that ρ_M and ρ_X evolve independently. However if one realizes that the ratio of the energy densities

$$r \equiv \frac{\rho_M}{\rho_X} = \frac{1-c^2}{c^2}, \tag{1}$$

should approach a constant, finite value $r = r_0$ for the coincidence problem to be solved, a different interpretation is possible, which no longer relies on an independent evolution of the components. Given the unknown nature of both dark matter and dark energy there is nothing in principle against their mutual interaction (however, in order not to conflict with "fifth force" experiments [16] we do not consider baryonic matter) to the point that assuming no interaction at all is not less arbitrary than assuming a coupling. In fact, this possibility is receiving growing attention in the literature [17–19] and appears to be compatible not only with SNIa and CMB data [20] but even favored over non-interacting cosmologies [21]. On the other hand, the coupling should not be seen as an entirely phenomenological approach as different Lagrangians have been proposed in support of the coupling—see [22] and references therein.

As a consequence of their mutual interaction neither component conserves separately,

$$\dot\rho_M + 3H\rho_M = Q, \qquad \dot\rho_X + 3H(1+w)\rho_X = -Q, \tag{2}$$

though the total energy density, $\rho = \rho_M + \rho_X$, does. Here Q denotes the interaction term, and w the equation of state parameter of the dark energy. Without loss of generality we shall describe the interaction as a decay process with $Q = \Gamma\rho_X$ where Γ is an arbitrary (generally variable) decay rate. Then we may write

$$\dot\rho_M + 3H\rho_M = \Gamma\rho_X \tag{3}$$

and

$$\dot\rho_X + 3H(1+w)\rho_X = -\Gamma\rho_X. \tag{4}$$

Consequently, the evolution of r is governed by

$$\dot r = 3Hr\left[w + \frac{1+r}{r}\frac{\Gamma}{3H}\right]. \tag{5}$$

In the non-interacting case ($\Gamma = 0$) and for a constant equation of state parameter w this ratio scales as $r \propto a^{3w}$. If we now assume $\rho_X = 3c^2 M_P^2 H^2$, this definition implies

$$\dot{\rho}_X = -9c^2 M_P^2 H^3 \left[1 + \frac{w}{1+r}\right], \tag{6}$$

where we have employed Einstein's equation $\dot{H} = -\frac{3}{2}H^2[1 + \frac{w}{1+r}]$. Inserting (6) in the left-hand side of the balance equation (4) yields a relation between the equation of state parameter w and the interaction rate Γ, namely,

$$w = -\left(1 + \frac{1}{r}\right)\frac{\Gamma}{3H}. \tag{7}$$

The interaction parameter $\frac{\Gamma}{3H}$ together with the ratio r determine the equation of state. In the absence of interaction, i.e., for $\Gamma = 0$, we have $w = 0$, i.e., Li's result is recovered as a special case. For the choice $\rho_X = 3c^2 M_P^2 H^2$ an interaction is the only way to have an equation of state different from that for dust. Any decay of the dark energy component ($\Gamma > 0$) into pressureless matter is necessarily accompanied by an equation of state $w < 0$. The existence of an interaction has another interesting consequence. Using the expression (7) for Γ in (5) provides us with $\dot{r} = 0$, i.e., $r = r_0 =$ const. Therefore, if the dark energy is given by $\rho_X = 3c^2 M_P^2 H^2$ and if an interaction with a pressureless component is admitted, the ratio $r = \rho_M/\rho_X$ is necessarily constant, irrespective of the specific structure of the interaction. Under this condition we have [cf. (1)]

$$c^2 = \frac{1}{1+r_0}. \tag{8}$$

At variance with [7,9], the fact that c^2 is lower than unity does not prompt any conflict with thermodynamics. For the case of a constant interaction parameter $\frac{\Gamma}{3H} \equiv \nu =$ const, it follows that

$$\rho, \rho_M, \rho_X \propto a^{-3m} \quad \left(m = 1 + \frac{w}{1+r_0} = 1 - \frac{\nu}{r}\right), \tag{9}$$

while the scale factor obeys $a \propto t^n$ with $n = 2/(3m)$. Consequently, the condition for accelerated expansion is $w/(1+r_0) < -1/3$, i.e., $\nu > r_0/3$.

Accordingly, the expression for the holographic dark energy with the identification $L = H^{-1}$ fits well into the interacting dark energy concept. The Hubble radius is not only the most obvious but also the simplest choice. It is not only compatible with a constant ratio between the energy densities but requires it. In a sense, the holographic dark energy with $L = H^{-1}$ together with the observational fact of an accelerated expansion almost calls for an interacting model. Note that the interaction is essential to simultaneously solve the coincidence problem and have late acceleration. There is no non-interacting limit, since in the absence of interaction, i.e., $Q = \Gamma = 0$, there is no acceleration.

Obviously, a change of r_0 demands a corresponding change of c^2. Within the framework discussed so far, a dynamical evolution of the energy density ratio is impossible. As a way out it has been suggested again to replace the Hubble scale by the future event horizon [23]. Here we shall follow a different strategy to admit a dynamical energy density ratio. Motivated by the relation (8) in the stationary case $r = r_0 =$ const, we retain the expression $\rho_X = 3c^2 M_P^2 H^2$ for the dark energy but allow the so far constant parameter c^2 to vary, i.e., $c^2 = c^2(t)$. Since the precise value of c^2 is unknown, some time dependence of this parameter cannot be excluded. Then this definition of ρ_X implies

$$\dot{\rho}_X = -9c^2 M_P^2 H^3 \left[1 + \frac{w}{1+r}\right] + \frac{(c^2)^{\cdot}}{c^2}\rho_X, \tag{10}$$

which generalizes Eq. (6). Using now the expression (10) for $\dot{\rho}_X$ on the left-hand side of the balance equation (4), leads to

$$\frac{(c^2)^{\cdot}}{c^2} = -3H\frac{r}{1+r}\left[w + \frac{1+r}{r}\frac{\Gamma}{3H}\right]. \tag{11}$$

A vanishing left-hand side, i.e., $c^2 =$ const, consistently reproduces (7). Comparing the right-hand sides of Eqs. (11) and (5) yields $(c^2)^{\cdot}/c^2 = -\dot{r}/(1+r)$, whose solution is

$$c^2(1+r) = 1. \tag{12}$$

The constant has been chosen to have the correct behavior (8) for the limit $r = r_0 =$ const. We conclude that if the dark energy is given by $\rho_X = 3c^2 M_P^2 H^2$ and c^2 is allowed to be time dependent, this time dependence must necessarily preserve the quantity $c^2(1+r)$. The time dependence of c^2 thus fixes the dynamics of r (and vice versa). Since r is expected to decrease in the course of cosmic expansion, $\dot{r} < 0$, this is accompanied by an increase in c^2, i.e., $(c^2)^{\cdot} > 0$.

Solving (11) for the equation of state parameter w we find

$$w = -\left(1 + \frac{1}{r}\right)\left[\frac{\Gamma}{3H} + \frac{(c^2)^{\cdot}}{3Hc^2}\right]. \tag{13}$$

For $(c^2)^{\cdot} = 0$ one recovers expression (7). It is obvious, that both a decreasing r and an increasing c^2 in (13) tend to make w more negative compared with $w = -(1 + \frac{1}{r})\frac{\Gamma}{3H}$ from (7). A variation of the c^2 parameter can be responsible for a change in the equation of state parameter w. Such a change to (more) negative values is required for the transition from decelerated to accelerated expansion. For a specific dynamic model assumptions about the interaction have to be introduced. This may be done, e.g., along the lines of [18, 19]. However, as is well known, the holographic energy must fulfill the dominant energy condition [24] whereby it is not compatible with a phantom equation of state ($w < -1$). This automatically sets a constraint on Γ and c^2.

It is noteworthy that in allowing c^2 to vary, contrary to what one may think, the infrared cutoff does not necessarily change. This may be seen as follows. The holographic bound can be written as $\rho_X \leqslant 3c^2 M_p^2/L^2$ with $L = H^{-1}$. Now, Li and Huang [7–9]—as well as ourselves—assume that the holographic bound is saturated (i.e., the equality sign is assumed in the above expression). Since the saturation of the bound is not at all compelling, and the "constant" $c^2(t)$ increases with expansion (as r decreases) up to reaching the constant value $(1 + r_0)^{-1}$, the expression $\rho_X = 3c^2(t) M_p^2 H^2$, in reality, does not imply a modification of the infrared cutoff, which is still $L = H^{-1}$. What happens is that, as $c^2(t)$ grows, the bound gets progressively saturated up to full saturation when, asymptotically, c^2 becomes a constant. In other words, the infrared cutoff always remains $L = H^{-1}$, what changes is the degree of saturation of the holographic bound.

In this Letter we have shown that *any* interaction of a dark energy component with density $\rho_X = 3c^2 M_P^2 H^2$ (and $c^2 = \text{const}$) with a pressureless dark matter component necessarily implies a constant ratio of the energy densities of both components. The equation of state parameter w is determined by the interaction strength. A time evolution of the energy density ratio is uniquely related to a time variation of the c^2 parameter. Under this condition a decreasing ratio ρ_M/ρ_X sends w to lower values.

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