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Holographic dark energy in Brans–Dicke theory

Narayan Banerjee [a], Diego Pavón [b,*]

[a] *Relativity and Cosmology Research Centre, Department of Physics, Jadavpur University, Calcutta 700032, India*
[b] *Departamento de Física, Facultad de Ciencias, Universidad Autónoma de Barcelona, 08193 Bellaterra (Barcelona), Spain*

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Abstract

In this Letter it is shown that when the holographic dark energy is combined with the Brans–Dicke field equations the transition from decelerated to accelerated expansion of the Universe can be more easily accounted for than when resort to the Einstein field equations is made. Likewise, the coincidence problem of late cosmic acceleration gets more readily softened.

1. Introduction

Arguably, the finding that the Universe is currently accelerating its expansion constitutes the most intriguing discovery in observational cosmology of recent years [1,2]. The long-lived Einstein–de Sitter cosmological model is no longer fit to explain the present state of affairs, and must be replaced by some other model compatible with a transition from decelerated to accelerated expansion. Very often, to achieve this transition, a novel energy component (dubbed "dark energy"), that violates the strong energy condition and clusters only at the largest accessible scales, is invoked. But, aside from these two features, nothing is known for certain about the nature of dark energy, which has become a matter of intense debate [3]. By far the simplest dark energy candidate is the cosmological constant, Λ. However, albeit it fits reasonably well into the cosmological data it faces two serious drawbacks on the theoretical side. In the first place, its quantum field value comes about 123 orders of magnitude larger than that observed. Secondly, it gives rise to the *coincidence problem*: "Why are the vacuum and dust energy densities of precisely the same order today?" Bear in mind that the energy density of dust red-shifts with expansion as a^{-3}, where a denotes the scale factor of the Friedmann–Robertson–Walker (FRW) metric. This is why a number of candidates of various degrees of plausibility have been proposed over the last few years with no clear winner in sight. Here we focus on a dark energy candidate grounded on sound thermodynamic considerations that is receiving growing attention in the literature, namely, the "holographic dark energy". It arises from the holographic principle (which as formulated by 't Hooft and Susskind [4] says that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume) and the realization that it should be constrained by infrared cutoff [5]. On these basis, Li [6] suggested the following constraint on its energy density $\rho_X \leqslant 3M_P^2 c^2/L^2$, the equality sign holding only when the holographic bound is saturated. In this expression M_P stands for the reduced Planck mass, c^2 is a dimensionless constant and L denotes the infrared cutoff radius. The latter is not specified at all by the holographic principle and different options have been tried with different degrees of success, namely, the particle horizon [7], the future event horizon [8], and the Hubble horizon [9–11].

Scalar-tensor theories of gravity have been widely applied in cosmology (see Faraoni's monograph [12] for an authorized review and Ref. [13] for a recent work) and very recently also in connection to holographic energy [14]. The aim of this Letter is to build a cosmological model of late acceleration based on the Brans–Dicke theory of gravity [15] and on the assumption that the (pressureless) dark matter and holographic dark energy do not conserve separately but interact with each other in a manner

* Corresponding author.
E-mail addresses: narayan@juphys.ernet.in (N. Banerjee), diego.pavon@uab.es (D. Pavón).

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to be specified below. At this point the interaction (coupling) may look purely phenomenological but different Lagrangians have been proposed in support of it—see [16] and references therein. On the other hand, in the absence of a symmetry that forbids the interaction there is nothing, in principle, against it. Further, the interacting dark mater–dark energy (the latter in the form of a quintessence scalar field and the former as fermions whose mass depends on the scalar field) has been investigated at one quantum loop with the result that the coupling leaves the dark energy potential stable if the former is of exponential type but it renders it unstable otherwise [17]. So, microphysics seems to allow enough room for the coupling; however, this point is not fully settled and should be further investigated. The difficulty lies, among other things, in that the very nature of both dark energy and dark matter remains unknown whence the detailed form of the coupling cannot be elucidated at this stage.

As infrared cutoff we shall choose the Hubble horizon—i.e., $L = H^{-1}$—as it seems more natural. As it turns out, the transition from decelerated to accelerated expansion is more readily effected and the coincidence problem gets substantially alleviated, both for spatially flat and curved FRW spaces. Our work differs from that of Ref. [14] in many important respects, notably in that we take the Hubble length as infrared cutoff not the future event horizon, and that the author of [14] assumes that the dark energy does not interact with matter. On the other hand, while we focus on alleviating the coincidence problem and providing a natural transition from decelerated to accelerated expansion, the latter aims to obtain the conditions the model must fulfill to prevent that the equation of state of the dark energy crosses the phantom divide.

This Letter is outlined as follows. Section 2 presents the model. Section 3 extends it to the non-flat case. Section 4 examines the implications of a varying M_P. Lastly, Section 5 summarizes our conclusions and provides some final remarks.

2. The spatially flat FRW case

For a spatially flat FRW universe filled with dust (dark matter) and dark energy, the Brans–Dicke field equations take the form

$$3\frac{\dot{a}^2}{a^2} = \frac{1}{\phi M_P^2}(\rho_M + \rho_X) + \frac{1}{2}\omega\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{a}\dot{\phi}}{a\phi}, \tag{1}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{\phi M_P^2}p_X - \frac{1}{2}\omega\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\ddot{\phi}}{\phi}, \tag{2}$$

where ω stands for the Brans–Dicke parameter [15].

As stated above, both components—the pressureless dark matter and the holographic dark energy—are assumed to interact with each other; thus, one may grow at the expense of the other. The conservation equations for them read

$$\dot{\rho}_M + 3H\rho_M = Q,$$
$$\dot{\rho}_X + 3H(1+w)\rho_X = -Q, \tag{3}$$

where $w \equiv p_X/\rho_X$ denotes the equation of state parameter for the dark energy, and Q stands for the interaction term. Following [10,11] we shall assume for the latter the ansatz $Q = \Gamma\rho_X$ with $\Gamma > 0$ being the interaction rate which, in general, can vary with time.

The wave equation for the Brans–Dicke scalar field, $(2\omega + 3)[\ddot{\phi} + 3(\dot{a}/a)\dot{\phi}] = T$—where T denotes the trace of the stress-energy tensor of dark matter and dark energy—is not an independent expression as it follows from the Bianchi identities alongside Eqs. (1)–(3). This wave equation is not altered by the interaction (3) since although the matter and dark energy components do not conserve separately the overall fluid—matter plus dark energy—does.

Taking up Li's expression, with the equality sign, for the holographic dark energy [6] and $L = H^{-1}$, we write

$$\rho_X = 3c^2 M_P^2 H^2. \tag{4}$$

At this point our system of equations is not closed and we still have freedom to choose one. We shall assume that Brans–Dicke field can be described as a power law of the scale factor, $\phi \propto a^n$. In principle there is no compelling reason for this choice. However, we shall see in due course that for small $|n|$ it leads to consistent results. Thus, a partial justification will be seen a posteriori.

By combining Eq. (4) with the above expression for ϕ and the field equations (1) and (2), we get

$$\dot{\rho_X} = -\frac{6c^2M_P^2H^3}{2+n} \times \left[1 + \frac{w}{(1+r)}\left(3(1+n) - \frac{n^2\omega}{2}\right) - \left(n^2\omega + n^2 - n\right)\right], \tag{5}$$

where $r \equiv \rho_M/\rho_X$. Inserting this into the second equation of (3), we obtain an expression for the equation of state parameter of the dark energy,

$$w = (1+r)\frac{(3n^2\omega + 2n^2 - 5n) + (n+2)\frac{\Gamma}{H}}{3[n - (n+2)r] - n^2\omega}. \tag{6}$$

It is important to note that if n is zero, the Brans–Dicke scalar field ϕ becomes trivial, and the last two equations reduce to their respective expressions in general relativity [10]. Eq. (6) clearly shows that w can be negative. This requirement only puts some bounds on the values of n and ω. A case of particular interest is that when $|n|$ is small whereas ω is high so that the product $n^2\omega$ results of order unity. This is interesting because local astronomical experiments set a very high lower bound on ω [18,19]; in particular, the Cassini experiment [20] implies that $\omega > 10^4$. Likewise, a slow fractional variation of ϕ will lead to a small fractional variation of G, consistent with observations. In this case w takes a simpler form,

$$w \simeq -\frac{(1+r)}{6r + n^2\omega}\left(3n^2\omega + 2\frac{\Gamma}{H}\right). \tag{7}$$

It is clearly negative-definite and decreases with expansion whenever Γ/H augments with time.

Now, as the dynamics of the scale factor is governed not only by the dark matter and the holographic dark energy, but also by the Brans–Dicke field, the signature of the deceleration parameter, $q = -\ddot{a}/(aH^2)$, has to be examined carefully. If we divide Eq. (2) by H^2, combine the resulting expression with Eq. (4) and the relationship $\phi \propto a^n$, we get

$$q = \frac{3wc^2}{(2+n)\phi} + \frac{n^2\omega}{2(2+n)} + \frac{n^2+n+1}{2+n}. \tag{8}$$

So, with a negative w, q can obviously be negative if

$$\left|\frac{3wc^2}{(2+n)\phi}\right| > \left|\frac{n^2+n+1}{2+n}\right| + \left|\frac{n^2\omega}{2(2+n)}\right|.$$

It is interesting to note that although q does not contain Γ or r explicitly, it actually depends on these two via the expression for w. If ϕ decreases with a, i.e., if $n < 0$, then the absolute value of the first term in Eq. (8) will increase and q might also have a signature flip from a positive to a negative value. With $|n| \ll 1$ and $(n^2\omega) \sim \mathcal{O}(1)$, this equation reads as

$$q \simeq \frac{3wc^2}{2\phi} + \frac{n^2\omega}{4} + \frac{1}{2}. \tag{9}$$

This implies a clear improvement with respect to the model of Ref. [10]. There, no acceleration can be achieved in the absence of interaction. Here, even in the non-interacting limit ($\Gamma = 0$) we can have $q < 0$—see Eqs. (7) and (8). Besides, to stage a transition from deceleration to acceleration in [10] it was necessary that the quantity c^2, entering the expression for the holographic dark energy density (Eq. (4)), should be slightly dynamical. While, that assumption seems to us very reasonable as there is no reason why the holographic bound should be already saturated at present, in the model considered in this Letter it is not necessary at all, though a slowly varying c^2 may also help the transition.

Our holographic interacting model shares with the model of Ref. [10] the advantage of considerably alleviating the coincidence problem. It is alleviated in the sense that the ratio between the energy densities of matter and dark energy, r, can vary more slowly in this model than in the conventional ΛCDM model, where $|\dot{r}/r|_0 = 3H_0$ (here and throughout a zero subscript indicates present time). Indeed, by virtue of the conservation equations (3) the evolution for the aforesaid ratio is seen to obey

$$\dot{r} = 3Hr\left[w + \frac{1+r}{r}\frac{\Gamma}{3H}\right]. \tag{10}$$

Therefore, keeping in mind that at present $w \simeq -1$ [2,21], a "soft" coincidence can be achieved if $0 < (\Gamma/3H)_0 < (1 + r_0)/r_0$. This is consistent with the requirement that $\Gamma > 0$. It should be noted that the field equations (1) and (2) do not enter the derivation of (10). Thus, the latter remains the same as that in general relativity—see Eq. (5) of Ref. [10].

3. Curved FRW cases

As is widely believed, inflation practically washes out the effect of curvature in the early stages of cosmic evolution. However, it does not necessarily imply that the curvature has to be wholly neglected at present. Indeed, aside from the sake of generality, there are sound reasons to include it: (i) Inflation drives the k/a^2 ratio close to zero but it cannot set it to zero if $k \neq 0$ initially. (ii) The closeness to perfect flatness depends on the number of e-folds and we can only speculate about the latter. (iii) After inflation the absolute value of the k/a^2 term in the field equations may increase with respect to the matter density term, thereby the former should not be ignored when studying the late Universe. (iv) Observationally there is room for a small but non-negligible spatial curvature [2,21].

After incorporating the curvature term, the field equations (1) and (2) generalize to

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \frac{1}{\phi M_P^2}(\rho_M + \rho_X) + \frac{1}{2}\omega\frac{\dot{\phi}^2}{\phi^2} - 3\frac{\dot{a}\dot{\phi}}{a\phi}, \tag{11}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -\frac{1}{\phi M_P^2}p_X - \frac{1}{2}\omega\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\ddot{\phi}}{\phi}, \tag{12}$$

respectively. The equation of state parameter of dark energy—Eq. (6)—now reads

$$w = (1+r)\frac{(3n^2\omega + 2n^2 - 5n) + (n+2)\frac{\Gamma}{H} + 10\Omega_k}{3[n(1-r) - 2r] - n^2\omega - 6\Omega_k}, \tag{13}$$

where $\Omega_k \equiv -k/(a^2H^2)$.

The deceleration parameter follows from Eq. (12) and takes the form

$$q = \frac{3wc^2}{(2+n)\phi} + \frac{n^2\omega}{2(2+n)} + \frac{n^2+n+1}{2+n} - \frac{\Omega_k}{(2+n)}. \tag{14}$$

Thus, the spatial curvature does not seriously modify the qualitative picture of the previous section but it may however affect the time of onset of the acceleration. For an open universe ($\Omega_k > 0$) the acceleration sets in earlier whereas in a closed universe ($\Omega_k < 0$) the accelerated phase is delayed. A more careful look at the equation for w reveals that in the second case, i.e., for a closed universe, a further condition has to be satisfied. For example, in the small $|n|$ limit, $3n^2\omega + \frac{2\Gamma}{H} + 10\Omega_k > 0$, together with $6r + n^2\omega + 6\Omega_k > 0$, or the reversed direction of the inequality for both. For an open universe, however, there is no further restriction on the onset of acceleration.

4. A model with a varying M_P

It is well known that as a result of the non-minimal coupling between the scalar field and the Ricci scalar, in Brans–Dicke theory, the gravity "constant" G is no longer a constant. The relation between G and ϕ is given by $G = G_N/\phi$ with G_N denoting its Newtonian (constant) value. The reduced Planck mass is defined as

$$M_P^2 = \frac{1}{8\pi G}. \tag{15}$$

In the previous section it was tacitly assumed that M_P was a constant given by $M_P^2 = 1/(8\pi G_N)$. One may be tempted to check what happens in this case if we stick to the relation (15). Here, the behavior of a spatially flat FRW model ($k = 0$) is considered. As Eq. (15) yields that $M_P^2\phi$ is a constant, the equation for the deceleration parameter becomes independent of ϕ, i.e., Eq. (8) now reads

$$q = \frac{3wc^2}{n+2} + \frac{n^2\omega}{2(n+2)} + \frac{n^2+n+1}{n+2}, \tag{16}$$

whereas the expression for the equation of state parameter becomes

$$w = (1+r)\frac{3(n^2\omega + n^2 - n) + (n+2)\frac{\Gamma}{H}}{3[n(1-r) - 2r] - n^2\omega}. \tag{17}$$

So, there is hardly any change in the qualitative behavior of the scenario. In fact, when $|n| \ll 1$ it can be checked that Eq. (17) reduces to (7). The time behavior of q, more specifically the possibility of a signature flip, has now to be taken care of by the variation of w.

5. Discussion

As the accelerated expansion of the Universe seems to be a comparatively recent episode and must have taken over from the more sedate decelerated expansion in the matter dominated era itself, it is good to have a signature flip in the deceleration parameter q from a positive to a negative value in this era. This holographic dark energy model in Brans–Dicke theory serves this purpose even with a constant c^2 whereas the same kind of a dark energy yields an ever-accelerated expansion in general relativity [10].

It is important to note that Brans–Dicke theory, either by itself or together with some form of dark energy, indeed provides a model to the accelerated expansion. However, these are all with a very small value of ω [22]. But it is quite well known now that local astronomical experiments severely restrict this parameter to very high values [20]. One important feature of the present model is that with a small value of $|n|$, ω can indeed have a high value.

The fractional rate of change of G, $|\dot{G}/G| = |nH|$, which follows from our simplifying assumption that $\phi \propto a^n$ with a low value of $|n|$, is quite consistent with the observational requirement that $|\dot{G}/G|_0$ must be lower than the current value of Hubble's expansion rate [23]—for a long list of experiments, see [24]. (In retrospect, this could serve as a motivation for the assumption.)

There is another important point. In this model, the interaction between the dark energy and the dark matter is certainly crucial. It is interesting to note that while in a similar general relativity model, [10], there is no non-interacting limit which yields an acceleration, in Brans–Dicke theory indeed such a possibility does exist. Eqs. (7) and (8) readily show that q can have negative values even for vanishing Γ.

As is well known, holographic energy is not compatible with phantom energy [25] thereby we must impose $w \geqslant -1$. This, combined with Eq. (7), yields

$$r \geqslant \frac{2(n^2\omega + \frac{\Gamma}{H})}{6 - 3n^2\omega - \frac{2\Gamma}{H}}.$$

This constraint is certainly satisfied provided both $n^2\omega$ and Γ/H do not exceed order one.

Clearly, it should be highly desirable to be in position to determine Γ from a fundamental theory but, as hinted in the introduction, we are far from it. Nevertheless, observational astrophysics might soon be able to set reliable bounds on the present value of Γ. In this connection, Tetradis et al. [26] have devised ways based on the rates of direct detection of dark matter particle that live in the Galaxy halo to establish limits on κ, the ratio between the strength of the dark matter–dark energy interaction and the strength of Newtonian gravity. In particular, they found that the average and maximum energy of the dark matter particles varies as $1+\kappa^2$. However, before their method may be applied unambiguous direct detection of the particles is mandatory.

Associated to a never-ending accelerated era there is a future event horizon, the existence of which means a serious handicap for string theories. This is why it has been speculated that the current phase of accelerated expansion must be followed by a fresh decelerated era and cosmological models featuring such transition back to a decelerated expansion has been proposed—see, e.g., [27]. Inspection of the three pair of expressions for w and q reveals that, in each case, our model can achieve such a transition provided the interaction rate, Γ, evolves to negative values. Clearly, such a possibility looks contrived. However, we should not wonder since the proposal of reverting to a decelerated era just for the purpose of getting rid of the event horizon appears unnatural, especially because no observational data suggests it. Nevertheless, this possibility cannot be discarded right away whereby we should keep an open mind. In any case, we would like to remark that holographic dark energy models that identify the infrared cutoff L with the future event horizon cannot account for such a transition.

It is also noteworthy that if at present $-1 < w < -1/3$ and the interaction rate is not high compared to H_0, i.e., $(\Gamma/3H)_0$ is small, then it is quite possible to have $|\dot{r}/r|_0 < H_0$. Put in another way, the model significantly alleviates the coincidence problem.

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