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The nonleptonic decays $B_c^+ \to D_s^+ \overline{D^0}$ and $B_c^+ \to D_s^+ D^0$ in a relativistic quark model

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Abstract

In the wake of exploring CP-violation in the decays of B and B_c mesons, we perform the straightforward calculation of their nonleptonic decay rates within a relativistic quark model. We confirm that the decays $B_c \to D_s \overline{D^0}$ and $B_c \to D_s D^0$ are well suited to extract the Cabibbo–Kobayashi–Maskawa angle γ through the amplitude relations because their decay widths are the same order of magnitude. In the b–c sector the decays $B \to DK$ and $B_c \to DD$ lead to squashed triangles which are therefore not so useful to determine the angle γ experimentally. We also determine the rates for other nonleptonic B_c-decays and compare our results with the results of other studies.

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As was pointed out in [1] and [2,3] the decays $B_c^+ \to D_s^+ D^0(\overline{D^0})$ are well suited for an extraction of the CKM angle γ through amplitude relations. These decays are better suited for the extraction of γ than the similar decays of the B_u and B_d mesons because the triangles in latter decays are very squashed. The B_c meson has been observed by the CDF Collaboration [4] in the decay $B_c \to J/\psi l \nu$. One could expect around 5×10^{10} B_c events per year at LHC [5] which gives us hope to use the B_c decay modes for the studying CP-violation.

In the case of the $B_c \to D_s D^0(\overline{D^0})$ decays the relevant amplitude relations can be written in the form [2]

$$\begin{aligned}\sqrt{2}\,A\big(B_c^+ \to D_s^+ D_+^0\big) &= A\big(B_c^+ \to D_s^+ D^0\big) + A\big(B_c^+ \to D_s^+ \overline{D^0}\big),\\ \sqrt{2}\,A\big(B_c^- \to D_s^- D_+^0\big) &= A\big(B_c^- \to D_s^- \overline{D^0}\big) + A\big(B_c^- \to D_s^- D^0\big),\end{aligned} \tag{1}$$

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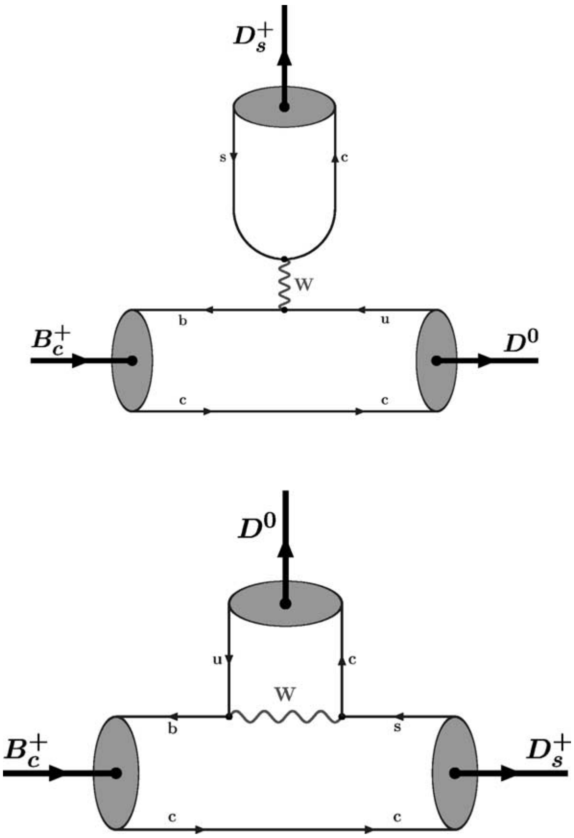

Fig. 1. Diagrams describing the decay $B_c \to D_s D^0$.

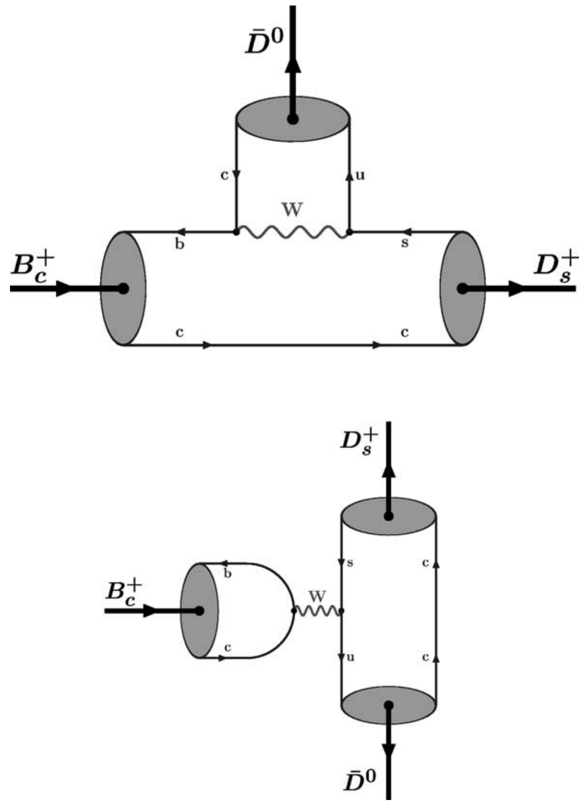

Fig. 2. Diagrams describing the decay $B_c \to D_s \overline{D^0}$.

where $|D_+^0\rangle = (|D^0\rangle + |\overline{D^0}\rangle)/\sqrt{2}$ is a CP-even eigenstate. The diagrams describing the decays $B_c^+ \to D_s^+ D^0$ and $B_c^+ \to D_s^+ \overline{D^0}$ are shown in Figs. 1 and 2, respectively. The color-enhanced amplitude of $B_c^+ \to D_s^+ D^0$ can be seen to be proportional to $V_{ub}^\dagger V_{cs} \approx 0.0029 \exp(i\gamma)$. At the same time the decay amplitude for $B_c^+ \to D_s^+ \overline{D^0}$ is proportional to $V_{bc} V_{us} \approx 0.0088$ but color-suppressed. Simple estimates made in [2] give

$$\left|\frac{A(B_c^+ \to D_s^+ D^0)}{A(B_c^+ \to D_s^+ \overline{D^0})}\right| = \left|\frac{A(B_c^- \to D_s^- \overline{D^0})}{A(B_c^- \to D_s^- D^0)}\right| = \mathcal{O}(1). \tag{2}$$

This implies that all sides of the amplitude triangles suggested in [2,6] have similar lengths as shown in Fig. 3. It allows one to extract the magnitude of the weak CKM-phase γ from the measurement of the $B_c^\pm \to D_s^\pm + (D^0, \overline{D^0}, D_+^0)$ decay widths. The method [6] of the extraction of γ from Eq. (1) is based on the parametrization of the amplitudes as

$$\begin{aligned} &A\big(B_c^+ \to D_s^+ \overline{D^0}\big) = A\big(B_c^- \to D_s^- D^0\big) = |\bar{A}| e^{i\bar{\delta}}, \\ &A\big(B_c^+ \to D_s^+ D^0\big) = |A| e^{i\gamma} e^{i\delta}, \qquad A\big(B_c^- \to D_s^- \overline{D^0}\big) = |A| e^{-i\gamma} e^{i\delta}, \end{aligned} \tag{3}$$

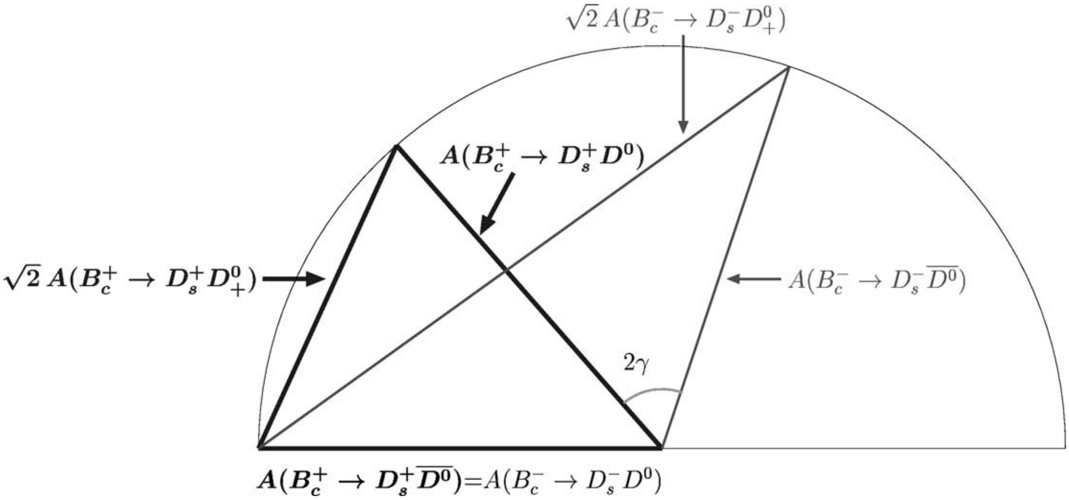

Fig. 3. The amplitude triangles for the decays $B_c^\pm \to D_s^\pm\{D^0, \overline{D^0}, D_+^0\}$.

where δ and $\bar\delta$ are the strong final state interaction phases. Introducing the notation $|A(B_c^+ \to D_s^+ D_+^0)| \equiv |A_+|$ and $|A(B_c^- \to D_s^- D_+^0)| \equiv |A_-|$ Eq. (1) can be rewritten as

$$|A_+|^2 + |A_-|^2 = |A|^2 + |\bar A|^2 + 2|A||\bar A|\cos\gamma\cos(\bar\delta - \delta), \qquad |A_+|^2 - |A_-|^2 = 2|A||\bar A|\sin\gamma\sin(\bar\delta - \delta). \tag{4}$$

The four solutions for $\sin\gamma$ are given by [6]

$$\sin\gamma = \frac{1}{4|A||\bar A|}\left\{\pm\sqrt{Y_{++}Y_{--}} \pm \sqrt{Y_{+-}Y_{-+}}\right\}, \tag{5}$$

where $Y_{\pm+} = [|A| + |\bar A|]^2 - 2|A_\pm|^2$ and $Y_{\pm-} = 2|A_\pm|^2 - [|A| - |\bar A|]^2$. Thus, the measurements of the rates of the six decays in Eq. (1) will determine the magnitude of γ with the four-fold ambiguity in Eq. (5). The way to resolve the ambiguity was discussed in [6].

In contrast to $B_c \to D_s D$, the corresponding ratios for $B \to KD$ and $B_c \to DD$ are [2]

$$\left|\frac{A(B^+ \to K^+ D^0)}{A(B^+ \to K^+ \overline{D^0})}\right| = \left|\frac{A(B^- \to K^- \overline{D^0})}{A(B^- \to K^- D^0)}\right| = \mathcal{O}(0.1), \tag{6}$$

$$\left|\frac{A(B_c^+ \to D^+ D^0)}{A(B_c^+ \to D^+ \overline{D^0})}\right| = \left|\frac{A(B_c^- \to D^- \overline{D^0})}{A(B_c^- \to D^- D^0)}\right| = \mathcal{O}(0.1) \tag{7}$$

which can be seen to lead to squashed triangles which are not very suited to measure γ.

Some estimates of the branching ratios have been obtained before in [7–10] with widely divergent results. We employ here a relativistic quark model [11] to provide an independent evaluation of these branching ratios.

This model is based on an effective interaction Lagrangian which describes the coupling between hadrons and their constituent quarks. For example, the coupling of the meson H to its constituent quarks q_1 and $\bar q_2$ is given by the Lagrangian

$$\mathcal{L}_{\rm int}(x) = g_H H(x)\int dx_1 \int dx_2\, F_H(x, x_1, x_2)\bar q(x_1)\Gamma_H \lambda_H q(x_2). \tag{8}$$

Here, λ_H and Γ_H are Gell-Mann and Dirac matrices which entail the flavor and spin quantum numbers of the meson field $H(x)$. The shape of the vertex function F_H can in principle be found from the Bethe–Salpeter equation as was done, e.g., in [12]. However, we choose a phenomenological approach where the vertex functions are modelled by a simple form. The function F_H must be invariant under the translation $F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)$ and should decrease quite rapidly in the Euclidean momentum space.

In our previous papers [13] we omit a possible dependence of the vertex functions on external momenta under calculation of the Feynman diagrams. This implies a dependence on how loop momenta are routed through the diagram at hand. In our last paper [14] and in the present calculation we employ a particular form of the vertex function given by

$$F_H(x, x_1, x_2) = \delta\left(x - \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}\right)\Phi_H\big((x_1 - x_2)^2\big), \tag{9}$$

where m_1 and m_2 are the constituent quark masses. The vertex function $F_H(x, x_1, x_2)$ evidently satisfies the above translational invariance condition. We are able to make calculations explicitly without any assumptions concerning the choice of loop momenta.

The coupling constants g_H in Eq. (8) are determined by the so-called *compositeness condition* proposed in [15] and extensively used in [16]. The compositeness condition means that the renormalization constant of the meson field is set equal to zero

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2}\widetilde{\Pi}'_H\big(m_H^2\big) = 0, \tag{10}$$

where $\widetilde{\Pi}'_H$ is the derivative of the meson mass operator. In physical terms the compositeness condition means that the meson is composed of a quark and antiquark system. For the pseudoscalar and vector mesons treated in this Letter one has

$$\widetilde{\Pi}'_P\big(p^2\big) = \frac{1}{2p^2}p^\alpha\frac{d}{p^\alpha}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_P^2\big(-k^2\big)\,\mathrm{tr}\Big[\gamma^5 S_1(\not k + w_{21}\not p)\gamma^5 S_2(\not k - w_{12}\not p)\Big],$$

$$\widetilde{\Pi}'_V\big(p^2\big) = \frac{1}{3}\bigg[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\bigg]\frac{1}{2p^2}p^\alpha\frac{d}{p^\alpha}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_V^2\big(-k^2\big)\,\mathrm{tr}\big[\gamma^\nu S_1(\not k + w_{21}\not p)\gamma^\mu S_2(\not k - w_{12}\not p)\big],$$

where $w_{ij} = m_j/(m_i + m_j)$.

The leptonic decay constant f_P is calculated from

$$\frac{3g_P}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_P\big(-k^2\big)\,\mathrm{tr}\big[O^\mu S_1(\not k + w_{21}\not p)\gamma^5 S_2(\not k - w_{12}\not p)\big] = f_P\, p^\mu,$$

$$\frac{3g_V}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_V\big(-k^2\big)\,\mathrm{tr}\big[O^\mu S_1(\not k + w_{21}\not p)\gamma\epsilon_V S_2(\not k - w_{12}\not p)\big] = \frac{1}{m_V}f_V\epsilon_V^\mu.$$

The transition form factors $P(p_1) \to P(p_2)(V(p_2))$ are calculated from the Feynman integrals

$$\begin{aligned}&\frac{3g_P g_{P'}}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_P\big(-(k+w_{13}p_1)^2\big)\widetilde{\Phi}_{P'}\big(-(k+w_{23}p_2)^2\big)\,\mathrm{tr}\big[S_2(\not k + \not p_2)O^\mu S_1(\not k + \not p_1)\gamma^5 S_3(\not k)\gamma^5\big]\\ &\quad= F_+\big(q^2\big)P^\mu + F_-\big(q^2\big)q^\mu,\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{3g_P g_V}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\widetilde{\Phi}_P\big(-(k+w_{13}p_1)^2\big)\widetilde{\Phi}_V\big(-(k+w_{23}p_2)^2\big)\,\mathrm{tr}\big[S_2(\not k + \not p_2)O^\mu S_1(\not k + \not p_1)\gamma^5 S_3(\not k)\gamma\cdot\epsilon_V\big]\\ &\quad= \frac{(\epsilon_V)_\nu}{m_P + m_V}\big\{-g^{\mu\nu}PqA_0\big(q^2\big) + P^\mu P^\nu A_+\big(q^2\big) + q^\mu P^\nu A_-\big(q^2\big) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V\big(q^2\big)\big\},\end{aligned} \tag{12}$$

where $O^\mu = \gamma^\mu(1-\gamma^5)$. We use the local quark propagators $S_i(\not k) = 1/(m_i - \not k)$ where m_i is the constituent quark mass. As discussed in [13,14], we assume that $m_H < m_1 + m_2$ in order to avoid the appearance of imaginary parts in the physical amplitudes. This holds true for the light pseudoscalar mesons but is no longer true for the

Table 1
Form factors for $B_c^+ \to D^0(D^{*0})$ and $B_c^+ \to D_s^+(D_s^{*+})$ transitions. Form factors are *approximated* by the form $F(q^2) = F(0)/(1 - a\hat{s} + b\hat{s}^2)$ with $\hat{s} = q^2/m_{B_c}^2$

	$B_c^+ \to D^0(D^{*0})$			$B_c^+ \to D_s^+(D_s^{*+})$		
	$F(0)$	a	b	$F(0)$	a	b
F_+	0.189	2.47	1.62	0.194	2.47	1.61
F_-	−0.194	2.43	1.54	−0.183	2.43	1.53
A_0	0.284	1.30	0.15	0.312	1.40	0.16
A_+	0.158	2.15	1.15	0.168	2.21	1.19
A_-	−0.328	2.40	1.51	−0.329	2.41	1.51
V	0.296	2.40	1.49	0.298	2.41	1.49

light vector mesons. We shall therefore employ identical masses for the pseudoscalar mesons and the vector mesons in our matrix element calculations but use physical masses in the phase space calculation. This is quite a reliable approximation for the heavy mesons, e.g., D^* and B^* whose masses are almost the same as the D and B, respectively. However, for the light mesons this approximation is not so good since the $K^*(892)$ has a mass much larger than the $K(494)$. For this reason we exclude the light vector mesons from our considerations. The fit values for the constituent quark masses are taken from our papers [13,14] and are given in Eq. (13):

$$\begin{array}{ccccc} m_u & m_s & m_c & m_b & \\ \hline 0.235 & 0.333 & 1.67 & 5.06 & \text{GeV} \end{array} \tag{13}$$

We employ a Gaussian for the vertex function $\widetilde{\Phi}_H(k_E^2) = \exp(-k_E^2/\Lambda_H^2)$ and determine the size parameters Λ_H^2 by a fit to the experimental data, when available, or to lattice simulations for the leptonic decay constants. The numerical values for Λ_H are

$$\begin{array}{cccccccc} \Lambda_\pi & \Lambda_K & \Lambda_D & \Lambda_{D_s} & \Lambda_B & \Lambda_{B_s} & \Lambda_{B_c} & \\ \hline 1.00 & 1.60 & 1.70 & 1.70 & 2.00 & 2.00 & 2.05 & \text{GeV} \end{array} \tag{14}$$

We have used the technique outlined in our previous papers [13,14] for the numerical evaluation of the Feynman integrals in Eqs. (11) and (12). The results of our numerical calculations are well represented by the parametrization

$$F(s) = \frac{F(0)}{1 - a\hat{s} + b\hat{s}^2} \tag{15}$$

with $\hat{s} = q^2/m_{B_c}^2$. Using such a parametrization facilitates further integrations. The values of $F(0)$, a and b are listed in Table 1. The calculated values of the leptonic decay constants are given in Eq. (16). They agree with the available experimental data and the results of the lattice simulations

$$\begin{array}{cccccccc} f_{K^+} & f_{D^0} & f_{D^{*0}} & f_{D_s} & f_{D_s^*} & f_B & f_{B_c} & \\ \hline 0.161 & 0.215 & 0.348 & 0.222 & 0.329 & 0.180 & 0.398 & \text{GeV} \end{array} \tag{16}$$

The relevant effective Hamiltonian for the decays $B_c \to D_s \overline{D^0}$ and $B_c \to D_s D^0$ is written as

$$\begin{aligned} H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \big\{ & C_1(\mu)\big(V_{cs}V_{ub}^\dagger \cdot (\bar{b}u)_{V-A}(\bar{c}s)_{V-A} + V_{us}V_{cb}^\dagger \cdot (\bar{b}c)_{V-A}(\bar{u}s)_{V-A}\big) \\ & + C_2(\mu)\big(V_{cs}V_{ub}^\dagger \cdot (\bar{b}s)_{V-A}(\bar{c}u)_{V-A} + V_{us}V_{cb}^\dagger \cdot (\bar{b}s)_{V-A}(\bar{u}c)_{V-A}\big)\big\}, \end{aligned} \tag{17}$$

where $V - A$ refers to $O^\mu = \gamma^\mu(1 - \gamma^5)$. We use the numerical values of the Wilson coefficients at the renormalization scale $\mu = m_{b,\text{pole}}$ given by $C_1 = 1.107$ and $C_2 = -0.248$ as given in [17]. Note that we interchange the labeling $1 \leftrightarrow 2$ of the coefficients to be consistent with the papers [7–10].

Straightforward calculation of the matrix elements of the decays $B_c \to D_s \overline{D^0}(D_s D^0)$ by using the effective Hamiltonian in Eq. (17) reproduces the result of the factorization method. We have

$$A\big(B_c^+ \to D_s^+ D^0\big) = \frac{G_F}{\sqrt{2}} V_{ub}^\dagger V_{cs} \big\{a_1\big[f_+^{B_c D}\big(m_{D_s}^2\big)\big(m_{B_c}^2 - m_{D^0}^2\big) + f_-^{B_c D}\big(m_{D_s}^2\big)m_{D_s}^2\big] f_{D_s} + a_2\big[f_+^{B_c D_s}\big(m_{D^0}^2\big)\big(m_{B_c}^2 - m_{D_s}^2\big) + f_-^{B_c D_s}\big(m_{D^0}^2\big)m_{D^0}^2\big] f_{D^0}\big\}, \tag{18}$$

$$A\big(B_c^+ \to D_s^+ \overline{D^0}\big) = \frac{G_F}{\sqrt{2}} V_{bc}^\dagger V_{us} a_2\big[f_+^{B_c D_s}\big(m_{D^0}^2\big)\big(m_{B_c}^2 - m_{D_s}^2\big) + f_-^{B_c D_s}\big(m_{D^0}^2\big)m_{D^0}^2\big] f_{D^0} + \text{annihilation channel}, \tag{19}$$

where $a_1 = C_1 + \xi C_2$ and $a_2 = C_2 + \xi C_1$ with $\xi = 1/N_c$. As usual we put the QCD color factor $\xi = 0$ according to $1/N_c$-expansion. Also we drop the annihilation processes from the consideration. Note that the calculation of the matrix elements of the nonleptonic decays involving the vector D-mesons in the final states proceed in a similar way. We extend our analysis to the semileptonic and nonleptonic decays of B-meson.

Table 2
Comparison of some branching ratios of the B-meson decays with the available experimental data

	This work	PDG [18]
$B^+ \to \overline{D^0} e^+ \nu$	0.024	0.0215 ± 0.0022
$B^+ \to \overline{D^{*0}} e^+ \nu$	0.056	0.053 ± 0.008
$B^+ \to K^+ \overline{D^0}$	2.8×10^{-4}	$(2.9 \pm 0.8) \times 10^{-4}$
$B^+ \to D_s^+ \overline{D^0}$	0.013	0.013 ± 0.004
$B^+ \to D_s^+ \overline{D^{*0}}$	0.008	0.012 ± 0.005
$B^+ \to D_s^{*+} \overline{D^0}$	0.019	0.009 ± 0.004
$B^+ \to D_s^{*+} \overline{D^{*0}}$	0.046	0.027 ± 0.010

Table 3
Exclusive nonleptonic decay widths of the B and B_c mesons in 10^{-15} GeV

$B^+ \to K^+ \overline{D^0}$	$(0.364 a_1 + 0.286 a_2)^2$	$B^+ \to K^+ D^0$	$0.00915 a_2^2$
$B^+ \to K^+ \overline{D^{*0}}$	$(0.342 a_1 + 0.442 a_2)^2$	$B^+ \to K^+ D^{*0}$	$0.0219 a_2^2$
$B^+ \to D_s^+ \overline{D^0}$	$4.367 a_1^2$		
$B^+ \to D_s^+ \overline{D^{*0}}$	$2.707 a_1^2$		
$B^+ \to D_s^{*+} \overline{D^0}$	$6.300 a_1^2$		
$B^+ \to D_s^{*+} \overline{D^{*0}}$	$14.84 a_1^2$		
$B_c^+ \to D^+ D^0$	$(0.0147 a_1 + 0.0146 a_2)^2$	$B_c^+ \to D^+ \overline{D^0}$	$0.753 a_2^2$
$B_c^+ \to D^+ D^{*0}$	$(0.0107 a_1 + 0.0234 a_2)^2$	$B_c^+ \to D^+ \overline{D^{*0}}$	$1.925 a_2^2$
$B_c^+ \to D^{*+} D^0$	$(0.0233 a_1 + 0.0106 a_2)^2$	$B_c^+ \to D^{*+} \overline{D^0}$	$0.399 a_2^2$
$B_c^+ \to D^{*+} D^{*0}$	$(0.0235 a_1 + 0.0235 a_2)^2$	$B_c^+ \to D^{*+} \overline{D^{*0}}$	$1.95 a_2^2$
$B_c^+ \to D_s^+ D^0$	$(0.0689 a_1 + 0.0672 a_2)^2$	$B_c^+ \to D_s^+ \overline{D^0}$	$0.0405 a_2^2$
$B_c^+ \to D_s^+ D^{*0}$	$(0.0503 a_1 + 0.106 a_2)^2$	$B_c^+ \to D_s^+ \overline{D^{*0}}$	$0.101 a_2^2$
$B_c^+ \to D_s^{*+} D^0$	$(0.101 a_1 + 0.0498 a_2)^2$	$B_c^+ \to D_s^{*+} \overline{D^0}$	$0.0222 a_2^2$
$B_c^+ \to D_s^{*+} D^{*0}$	$(0.104 a_1 + 0.110 a_2)^2$	$B_c^+ \to D_s^{*+} \overline{D^{*0}}$	$0.109 a_2^2$

Table 4
Branching ratios of some nonleptonic decay widths of the B and B_c mesons calculated for $a_1 = 1.107$ and $a_2 = -0.248$

$B^+ \to K^+ \overline{D^0}$	2.76×10^{-4}	$B^+ \to K^+ D^0$	1.41×10^{-6}
$B^+ \to K^+ \overline{D^{*0}}$	1.82×10^{-4}	$B^+ \to K^+ D^{*0}$	3.38×10^{-6}
$B_c^+ \to D^+ D^0$	1.11×10^{-7}	$B_c^+ \to D^+ \overline{D^0}$	3.24×10^{-5}
$B_c^+ \to D^+ D^{*0}$	0.25×10^{-7}	$B_c^+ \to D^+ \overline{D^{*0}}$	8.28×10^{-5}
$B_c^+ \to D^{*+} D^0$	3.76×10^{-7}	$B_c^+ \to D^{*+} \overline{D^0}$	1.71×10^{-5}
$B_c^+ \to D^{*+} D^{*0}$	2.84×10^{-7}	$B_c^+ \to D^{*+} \overline{D^{*0}}$	8.38×10^{-5}
$B_c^+ \to D_s^+ D^0$	2.48×10^{-6}	$B_c^+ \to D_s^+ \overline{D^0}$	1.74×10^{-6}
$B_c^+ \to D_s^+ D^{*0}$	0.60×10^{-6}	$B_c^+ \to D_s^+ \overline{D^{*0}}$	4.34×10^{-6}
$B_c^+ \to D_s^{*+} D^0$	6.88×10^{-6}	$B_c^+ \to D_s^{*+} \overline{D^0}$	0.95×10^{-6}
$B_c^+ \to D_s^{*+} D^{*0}$	5.41×10^{-6}	$B_c^+ \to D_s^{*+} \overline{D^{*0}}$	4.69×10^{-6}

Table 5
Exclusive nonleptonic decay widths of the B_c meson in units of 10^{-15} GeV. Comparison with other studies

Process	This work	[7]	[8]	[9]	[10]	[5]
$B_c^+ \to D_s^+ \overline{D^0}$	$0.0405\,a_2^2$	$0.0340\,a_2^2$	$0.168\,a_2^2$	$0.01\,a_2^2$	$0.0415\,a_2^2$	$0.176\,a_2^2$
$B_c^+ \to D_s^+ \overline{D^{*0}}$	$0.101\,a_2^2$	$0.0354\,a_2^2$	$0.143\,a_2^2$	$0.009\,a_2^2$	$0.0495\,a_2^2$	$0.260\,a_2^2$
$B_c^+ \to D_s^{*+} \overline{D^0}$	$0.0222\,a_2^2$	$0.0334\,a_2^2$	$0.0658\,a_2^2$	$0.087\,a_2^2$	$0.0201\,a_2^2$	$0.166\,a_2^2$
$B_c^+ \to D_s^{*+} \overline{D^{*0}}$	$0.109\,a_2^2$	$0.0564\,a_2^2$	$0.128\,a_2^2$	$0.15\,a_2^2$	$0.0597\,a_2^2$	$0.951\,a_2^2$

For numerical evaluation we have used the set of the parameters: $m_{B^+} = 5.279$ GeV, $\tau_{B^+} = 1.655$ ps, $m_{B_c} = 6.4$ GeV, $\tau_{B_c} = 0.46$ ps, $a_1|_{\xi=0} = 1.107$, $a_2|_{\xi=0} = -0.248$ and

$$\begin{array}{cccccc} |V_{ud}| & |V_{us}| & |V_{ub}| & |V_{cd}| & |V_{cs}| & |V_{bc}| \\ \hline 0.98 & 0.22 & 0.003 & 0.22 & 0.98 & 0.040 \end{array} \tag{20}$$

First, to illustrate the quality of our calculations, we list some branching ratios of the B-meson decays in Table 2 and compare them with the experimental data. The exclusive nonleptonic decay widths of the B and B_c mesons for arbitrary values of a_1 and a_2 are listed in Table 3 whereas their branching ratios for $a_1 = 1.107$ and $a_2 = -0.248$ are given in Table 4. One can see that as it was expected the magnitudes of the branching ratios of the decays $B_c \to D_s D^0$ and $B_c \to D_s \overline{D^0}$ are very close to each other. It gives us hope that they can be measured in the forthcoming experiments. Finally, in Table 5 we compare our results with the results of other studies. One can see that there are quite large differences between the predictions of the different models.

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