22 November 2001
PHYSICS LETTERS B
ELSEVIER
Physics Letters B 521 (2001) 7–14
www.elsevier.com/locate/npe

Charge asymmetry and photon energy spectrum in the decay $B_s \to l^+l^-\gamma$

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Received 17 August 2001; received in revised form 25 September 2001; accepted 5 October 2001
Editor: P.V. Landshoff

Abstract

We consider the structure-dependent amplitude of the decay $B_s \to l^+l^-\gamma$ ($l = e, \mu$) in a model based on the effective Hamiltonian for $b\bar{s} \to l^+l^-$ containing the Wilson coefficients C_7, C_9 and C_{10}. The form factors characterising the matrix elements $\langle\gamma|\bar{s}\gamma_\mu(1 \mp \gamma_5)b|\overline{B}_s\rangle$ and $\langle\gamma|\bar{s}\sigma_{\mu\nu}(1 \mp \gamma_5)b|\overline{B}_s\rangle$ are taken to have the universal form $f_V \approx f_A \approx f_T \approx f_{B_s} M_{B_s} R_s/(3E_\gamma)$ suggested by recent work in QCD, where R_s is a parameter related to the light cone wave function of the B_s meson. Simple expressions are obtained for the charge asymmetry $A(x_\gamma)$ and the photon energy spectrum $d\Gamma/dx_\gamma$ ($x_\gamma = 2E_\gamma/M_{B_s}$). The decay rates are calculated in terms of the decay rate of $B_s \to \gamma\gamma$. The branching ratios are estimated to be $\mathrm{Br}(B_s \to e^+e^-\gamma) = 2.0 \times 10^{-8}$ and $\mathrm{Br}(B_s \to \mu^+\mu^-\gamma) = 1.2 \times 10^{-8}$, somewhat higher than earlier estimates.

1. Introduction

The rare decay $B_s \to l^+l^-\gamma$ is of interest as a probe of the effective Hamiltonian for the transition $b\bar{s} \to l^+l^-$, and as a testing ground for form factors describing the matrix elements $\langle\gamma|\bar{s}\gamma_\mu(1 \mp \gamma_5)b|\overline{B}_s\rangle$ and $\langle\gamma|\bar{s}i\sigma_{\mu\nu}(1 \mp \gamma_5)b|\bar{B}_s\rangle$ [1,2]. The branching ratio for $B_s \to l^+l^-\gamma$ can be sizeable in comparison to the non-radiative process $B_s \to l^+l^-$, since the chiral suppression of the latter is absent in the radiative transition. We will be concerned mainly with the structure-dependent part of the matrix element, since the correction due to bremsstrahlung from the external leptons is small and can be removed by eliminating the end-point region $s_{l^+l^-} \approx M^2_{B_s}$. (For related studies of radiative B decays, we refer to the papers in Ref. [3].)

Our objective is to calculate the decay spectrum of $B_s \to l^+l^-\gamma$ using form factors suggested by recent work in QCD [4]. These form factors have the virtue of possessing a universal behaviour $1/E_\gamma$ for large E_γ, as well as a universal normalization. These features can be tested in measurements of $B^+ \to \mu^+\nu\gamma$ and $B_s \to \gamma\gamma$. We derive simple formulae for the photon energy spectrum $d\Gamma/dx_\gamma$, $x_\gamma = 2E_\gamma/m_{B_s}$, and the charge asymmetry $A(x_\gamma)$, defined as the difference in the probability of events with $E_+ > E_-$ and $E_+ < E_-$, $E_\pm$ being the $l^\pm$ energies. This asymmetry is large over most of the x_γ domain. Predictions are obtained for the branching ratios $\mathrm{Br}(B_s \to e^+e^-\gamma)$ and $\mathrm{Br}(B_s \to \mu^+\mu^-\gamma)$ which are somewhat higher than those estimated in previous literature [1,2].

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PII: S0370-2693(01)01203-5

2. Matrix element and differential decay rate

The effective Hamiltonian for the interaction $b\bar{s} \to l^+l^-$ has the standard form [5]

$$\mathcal{H}_{\text{eff}} = \frac{\alpha G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^\star \left\{ C_9^{\text{eff}} (\bar{s}\gamma_\mu P_L b)\bar{l}\gamma_\mu l + C_{10}(\bar{s}\gamma_\mu P_L b)\bar{l}\gamma_\mu\gamma_5 l - 2\frac{C_7}{q^2}\bar{s}i\sigma_{\mu\nu}q^\nu(m_b P_R + m_s P_L)b\bar{l}\gamma_\mu l \right\}, \tag{1}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ and q is the sum of the l^+ and l^- momenta. For the purpose of this Letter, we will neglect the small q^2-dependent terms in C_9^{eff}, arising from one-loop contributions of four-quark operators, as well as long-distance effects associated with $c\bar{c}$ resonances. The Wilson coefficients in Eq. (1) will be taken to have the constant values

$$C_7 = -0.315, \qquad C_9 = 4.334, \qquad C_{10} = -4.624. \tag{2}$$

To obtain the amplitude for $B_s \to l^+l^-\gamma$, one requires the matrix elements $\langle\gamma|\bar{s}\gamma_\mu(1 \mp \gamma_5)b|\overline{B}_s\rangle$ and $\langle\gamma|\bar{s}i\sigma_{\mu\nu}(1 \mp \gamma_5)b|\overline{B}_s\rangle$. We parametrise these in the same way as in Ref. [1,2]

$$\begin{aligned}
\langle\gamma(k)|\bar{s}\gamma_\mu b|\overline{B}_s(k+q)\rangle &= e\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\star\nu}q^\rho k^\sigma f_V(q^2)/M_{B_s}, \\
\langle\gamma(k)|\bar{s}\gamma_\mu\gamma_5 b|\overline{B}_s(k+q)\rangle &= -ie\left[\epsilon_\mu^\star k\cdot q - \epsilon^\star\cdot q k_\mu\right] f_A(q^2)/M_{B_s}, \\
\langle\gamma(k)|\bar{s}i\sigma_{\mu\nu}q^\nu b|\overline{B}_s(k+q)\rangle &= -e\,\epsilon_{\mu\nu\rho\sigma}\epsilon^{\star\nu}q^\rho k^\sigma f_T(q^2), \\
\langle\gamma(k)|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\overline{B}_s(k+q)\rangle &= -ie\left[\epsilon_\mu^\star k\cdot q - \epsilon^\star\cdot q k_\mu\right] f_T'(q^2).
\end{aligned} \tag{3}$$

The form factors f_V, f_A, f_T and f_T' are dimensionless, and related to those of Aliev et al. [1] by $f_V = g/M_{B_s}$, $f_A = f/M_{B_s}$, $f_T = -g_1/M_{B_s}^2$, $f_T' = -f_1/M_{B_s}^2$. The matrix element for $\overline{B}_s \to l^+l^-\gamma$ can then be written as (neglecting terms of order m_s/m_b)

$$\begin{aligned}
\mathcal{M}(\overline{B}_s \to l^+l^-\gamma) = \frac{\alpha G_F}{2\sqrt{2}\pi} e V_{tb}V_{ts}^\star \frac{1}{M_{B_s}} \Big[&\epsilon_{\mu\nu\rho\sigma}\epsilon^{\star\nu}q^\rho k^\sigma \left(A_1\bar{l}\gamma^\mu l + A_2\bar{l}\gamma^\mu\gamma_5 l\right) \\
&+ i\left(\epsilon_\mu^\star(k\cdot q) - (\epsilon^\star\cdot q)k_\mu\right)\left(B_1\bar{l}\gamma^\mu l + B_2\bar{l}\gamma^\mu\gamma_5 l\right)\Big],
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
A_1 &= C_9 f_V + 2C_7\frac{M_{B_s}^2}{q^2} f_T, \qquad A_2 = C_{10} f_V, \\
B_1 &= C_9 f_A + 2C_7\frac{M_{B_s}^2}{q^2} f_T', \qquad B_2 = C_{10} f_A.
\end{aligned} \tag{5}$$

(In the coefficient of C_7, we have approximated $m_b M_{B_s}$ by $M_{B_s}^2$.) The Dalitz plot density in the energy variables $E_\pm$ is

$$\frac{d\Gamma}{dE_+\,dE_-} = \frac{1}{256\pi^3 M_{B_s}} \sum_{\text{spin}} |\mathcal{M}|^2, \tag{6}$$

where [1,2,6]

$$\begin{aligned}
\sum_{\text{spin}} |\mathcal{M}|^2 = &\left|\frac{\alpha G_F}{\sqrt{2}\pi} V_{tb}V_{ts}^\star e\right|^2 \frac{1}{M_{B_s}^2} \\
&\times \Big\{ \left(|A_1|^2 + |B_1|^2\right)\left[q^2\{(p_+\cdot k)^2 + (p_-\cdot k)^2\} + 2m_l^2(q\cdot k)^2\right]
\end{aligned}$$

$$+\left(|A_2|^2+|B_2|^2\right)\left[q^2\left\{(p_+\cdot k)^2+(p_-\cdot k)^2\right\}-2m_l^2(q\cdot k)^2\right]$$
$$+2\,\mathrm{Re}\left(B_1^\star A_2+A_1^\star B_2\right)q^2\left[(p_+\cdot k)^2-(p_-\cdot k)^2\right]\Big\}. \tag{7}$$

It is convenient to introduce dimensionless variables

$$x_\gamma=2E_\gamma/M_{B_s},\qquad x_\pm=2E_\pm/M_{B_s},\qquad \Delta=x_+-x_-,\qquad r=m_l^2/M_{B_s}^2 \tag{8}$$

in terms of which $q^2=M_{B_s}^2(1-x_\gamma)$. Taking x_γ and Δ as the two coordinates of the Dalitz plot, phase space is defined by

$$|\Delta|\leqslant vx_\gamma,\quad v=\sqrt{1-4m_l^2/q^2}=\sqrt{1-4r/(1-x_\gamma)},$$
$$0\leqslant x_\gamma\leqslant 1-4r. \tag{9}$$

In terms of x_γ and Δ, the differential decay width takes the form

$$\frac{d\Gamma}{dx_\gamma\,d\Delta}=\mathcal{N}\bigg[\left(|A_1|^2+|B_1|^2\right)\left\{\frac{(1-x_\gamma)(x_\gamma^2+\Delta^2)}{8}+\frac{1}{2}rx_\gamma^2\right\}$$
$$+\left(|A_2|^2+|B_2|^2\right)\left\{\frac{(1-x_\gamma)(x_\gamma^2+\Delta^2)}{8}-\frac{1}{2}rx_\gamma^2\right\}+2\,\mathrm{Re}\left(B_1^\star A_2+A_1^\star B_2\right)(1-x_\gamma)\frac{1}{4}x_\gamma\Delta\bigg], \tag{10}$$

where $\mathcal{N}=[\alpha^2G_F^2/(256\pi^4)]|V_{tb}V_{ts}^\star|^2M_{B_s}^5$. The last term is linear in Δ and produces an asymmetry between the l^+ and l^- energy spectra.

We will derive from Eq. (10) two distributions of interest:

(i) The charge asymmetry $A(x_\gamma)$ defined as

$$A(x_\gamma)=\frac{\left(\int_0^{vx_\gamma}\frac{d\Gamma}{dx_\gamma\,d\Delta}-\int_{-vx_\gamma}^{0}\frac{d\Gamma}{dx_\gamma\,d\Delta}\right)d\Delta}{\int_{-vx_\gamma}^{+vx_\gamma}\frac{d\Gamma}{dx_\gamma\,d\Delta}\,d\Delta}$$
$$=\frac{3}{4}v(1-x_\gamma)\frac{2\,\mathrm{Re}(B_1^\star A_2+A_1^\star B_2)}{\left\{(|A_1|^2+|B_1|^2)(1-x_\gamma+2r)+(|A_2|^2+|B_2|^2)(1-x_\gamma-4r)\right\}}. \tag{11}$$

(ii) The photon energy spectrum

$$\frac{d\Gamma}{dx_\gamma}=\frac{\alpha^3G_F^2}{768\pi^4}\left|V_{tb}V_{ts}^\star\right|^2M_{B_s}^5vx_\gamma^3\left[\left(|A_1|^2+|B_1|^2\right)(1-x+2r)+\left(|A_2|^2+|B_2|^2\right)(1-x_\gamma-4r)\right]. \tag{12}$$

To proceed further, we must introduce a model for the form factors which appear in the functions $A_{1,2}$ and $B_{1,2}$ defined in Eq. (5).

3. Model for form factors

First of all, we note that the form factors f_T and f_T' defined in Eq. (3) are necessarily equal, by virtue of the identity

$$\sigma_{\mu\nu}=\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5. \tag{13}$$

This was pointed out by Korchemsky et al. [4]. We, therefore, have to deal with three independent form factors f_V, f_A and f_T. These have been computed in Ref. [4] using perturbative QCD methods combined with heavy quark effective theory. For the vector and axial vector form factors of the radiative decay $B^+ \to l^+\nu\gamma$, and their tensor counterpart, defined as in Eq. (3), these authors obtain the remarkable result

$$f_V(E_\gamma) = f_A(E_\gamma) = f_T(E_\gamma) = \frac{f_B m_B}{2E_\gamma}\left(Q_u R - \frac{Q_b}{m_b}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{E^2_\gamma}\right), \tag{14}$$

where R is a parameter related to the light-cone wave-function of the B meson, with an order of magnitude $R^{-1} \sim \bar{\Lambda} = M_B - m_b$, where the binding energy $\bar{\Lambda}$ is estimated to be between 0.3 and 0.4 GeV. Applying the same reasoning to the form factors for $\overline{B}_s \to l^+l^-\gamma$, we conclude that

$$f_V(E_\gamma) = f_A(E_\gamma) = f_T(E_\gamma) = \frac{f_{B_S} M_{B_s}}{2E_\gamma}\left(-Q_s R_s + \frac{Q_b}{m_b}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{E^2_\gamma}\right). \tag{15}$$

In what follows, we will neglect the term Q_b/m_b, and approximate the form factors by

$$f_{V,A,T}(E_\gamma) \approx \frac{f_{B_s} M_{B_s}}{2E_\gamma}\frac{1}{3\bar{\Lambda}_s} = \frac{1}{3}\frac{f_B}{\bar{\Lambda}_s}\frac{1}{x_\gamma}, \tag{16}$$

where $\bar{\Lambda}_s = M_{B_s} - m_b$ will be taken to have the nominal value 0.5 GeV. Several of our results will depend only on the universal form $f_{V,A,T}(E_\gamma) \sim 1/E_\gamma$, independent of the normalization. As pointed out in [4], a check of the behaviour $f_{V,A} \sim 1/E_\gamma$ in the case of $B^+ \to \mu^+\nu\gamma$ is afforded by the photon energy spectrum, which is predicted to be

$$\frac{d\Gamma}{dx_\gamma} \sim \left[f^2_V(E_\gamma) + f^2_A(E_\gamma)\right]x^3_\gamma(1-x_\gamma) \sim x_\gamma(1-x_\gamma). \tag{17}$$

In the case of the reaction $B_s \to l^+l^-\gamma$, the normalization of the tensor form factor $f_T(E_\gamma)$ at $E_\gamma = M_B/2$ (i.e., $x_\gamma = 1$) can be checked by appeal to the decay rate of $B_s \to \gamma\gamma$. To see this connection, we note that the matrix element of $B_s \to \gamma(k,\epsilon) + \gamma(k',\epsilon')$ can be obtained from that of $B_s \to l^+l^-\gamma$ by putting $C_9 = C_{10} = 0$, and replacing the factor $(ef_T C_7/q^2)(\bar{l}\gamma_\mu l)$ by $f_T(x_\gamma = 1)\epsilon^{\star\prime}_\mu$. This yields the matrix element

$$\mathcal{M}\left(\overline{B}_s \to \gamma(\epsilon,k)\gamma(\epsilon',k')\right) = -i\frac{G_F e^2}{\sqrt{2}\pi^2}\left(V_{tb}V^\star_{ts}\right)\left[A^+ F_{\mu\nu}F^{\mu\nu'} + iA^- F_{\mu\nu}\widetilde{F}^{\mu\nu'}\right]$$

with

$$A^+ = -A^- = \frac{1}{4}M_{B_s} f_T(x_\gamma = 1)C_7. \tag{18}$$

The result for $A^\pm$ coincides with that obtained in Refs. [7–9] when $f_T(x_\gamma = 1) = -\frac{Q_d f_B}{\bar{\Lambda}_s} = \frac{1}{3}\frac{f_B}{\bar{\Lambda}_s}$. (In Refs. [8,9], the role of the parameter Λ_s is played by the constituent quark mass m_s.) Thus the decay width of $B_s \to \gamma\gamma$,

$$\Gamma(B_s \to \gamma\gamma) = \frac{M^3_{B_s}}{16\pi}\left|\frac{G_F e^2}{\sqrt{2}\pi^2}V_{tb}V^\star_{ts}\right|^2\left(|A_+|^2 + |A_-|^2\right) \tag{19}$$

serves as a test of the normalization factor $f_T(x_\gamma = 1)$.

We remark, parenthetically, that the calculation of $B_s \to \gamma\gamma$, based on an effective interaction for $b \to s\gamma\gamma$, produces the amplitudes A^+ and A^- given in Eq. (18) in the limit of retaining only the 'reducible' diagrams related to the transition $b \to s\gamma$. Inclusion of 'irreducible' contributions like $b\bar{s} \to c\bar{c} \to \gamma\gamma$ introduces a correction term in A_- causing the ratio $|A_+/A_-|$ to deviate from unity. Estimates in Ref. [7,8] yield values for this ratio between 0.75 and 0.9. The branching ratio $\mathrm{Br}(B_s \to \gamma\gamma)$ is estimated at 5×10^{-7}, with an uncertainty of about 50%.

Having specified our model for the form factors $f_V(x_\gamma)$, $f_A(x_\gamma)$ and $f_T(x_\gamma)$, we proceed to present results for the spectrum and branching ratio of $B_s \to l^+l^-\gamma$ [10]. We use $M_{B_s} = 5.3$ GeV, $f_{B_s} = 200$ MeV and, where necessary, $\bar{\Lambda}_s = 0.5$ GeV in the normalization of the form factors in Eq. (16).

4. Results

4.1. Charge asymmetry

With the assumption of universal form factors $f_V = f_A = f_T \sim \frac{1}{x_\gamma}$, the asymmetry $A(x_\gamma)$ in Eq. (11) assumes the simple form

$$A(x_\gamma) = \frac{3}{4}v\frac{2C_{10}\big(C_9 + 2C_7\frac{1}{1-x_\gamma}\big)(1-x_\gamma)}{\big(C_9 + 2C_7\frac{1}{1-x_\gamma}\big)^2(1-x_\gamma+2r) + C_{10}^2(1-x_\gamma-4r)}. \tag{20}$$

This is plotted in Fig. 1, and is clearly large and negative over most of the x_γ domain, changing sign at $x_\gamma = 1 + \frac{2C_7}{C_9}$. (A negative asymmetry corresponds to l^- being more energetic, on average, than l^+ in the decay $\overline{B}_s(=b\bar{s}) \to l^+l^-\gamma$.) The average charge asymmetry is

$$\langle A\rangle = \frac{3}{4}\frac{\int_0^{1-4r} dx_\gamma\, v^2 x_\gamma(1-x_\gamma)2C_{10}\big(C_9 + 2C_7\frac{1}{1-x_\gamma}\big)}{\int_0^{1-4r} dx_\gamma\, v x_\gamma\big[(1-x_\gamma+2r)\big(C_9 + 2C_7\frac{1}{1-x_\gamma}\big)^2 + (1-x_\gamma-4r)C_{10}^2\big]} \tag{21}$$

and has the numerical value $\langle A\rangle_e = -0.28$, $\langle A\rangle_\mu = -0.47$ for the modes $l = e, \mu$, the difference arising essentially from the end-point region $x_\gamma \approx 1-4r$.

4.2. Photon energy spectrum

With the form factors of Eq. (16), the photon energy spectrum simplifies to

$$\frac{d\Gamma}{dx_\gamma} = \frac{1}{3}\mathcal{N} v x_\gamma\left\{(1-x_\gamma+2r)\left(C_9 + 2C_7\frac{1}{1-x_\gamma}\right)^2 + (1-x_\gamma-4r)C_{10}^2\right\}, \tag{22}$$

where the constant factor $\mathcal{N}$ is defined after Eq. (10). It is expedient to write this distribution in terms of the decay rate of $\overline{B}_s \to \gamma\gamma$. We then obtain the prediction

$$\frac{d\Gamma(\overline{B}_s \to l^+l^-\gamma)/dx_\gamma}{\Gamma(\overline{B}_s \to \gamma\gamma)} = \left\{\frac{2\alpha}{3\pi}\frac{x_\gamma^3}{(1-x_\gamma)^2}v(1-x_\gamma+2r)\right\}\left(\frac{1}{x_\gamma}\right)^2\left[\left\{\eta_9(1-x_\gamma)+1\right\}^2 + \left\{\eta_{10}(1-x_\gamma)\right\}^2\frac{1-x_\gamma-4r}{1-x_\gamma+2r}\right]. \tag{23}$$

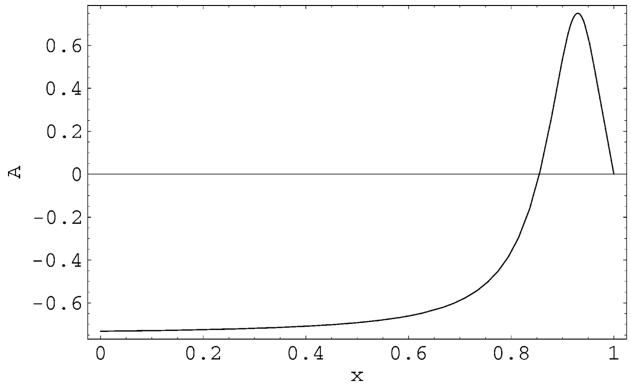

Fig. 1. Asymmetry versus x_γ.

The first factor (in curly brackets { }) is the QED result expected if the decay $\overline{B}_s \to l^+l^-\gamma$ is interpreted as a Dalitz pair reaction $\overline{B}_s \to \gamma\gamma^\star \to \gamma l^+l^-$, without form factors. The factor $(1/x_\gamma)^2$ results from the universal behaviour $f_{V,A,T} \sim 1/x_\gamma$ given in Eq. (10), while the last factor is the electroweak effect associated with the coefficients $\eta_9 = C_9/(2C_7)$ and $\eta_{10} = C_{10}/(2C_7)$. This distribution is plotted in Figs. 2 and 3, where the QED result is shown for comparison.

4.3. Rates and branching ratios

From the photon spectrum given in Eq. (23), we derive the 'conversion ratios'

$$R_l = \frac{\int_0^{1-4r} \frac{d\Gamma}{dx_\gamma}(B_s \to l^+l^-\gamma)}{\Gamma(B_s \to \gamma\gamma)}. \tag{24}$$

The numerical values are $R_e = 4.0\%$ and $R_\mu = 2.3\%$. These are to be contrasted with the QED result given by

$$R_l(\text{QED}) = \frac{2\alpha}{3\pi}\left[\left(1 - 18r^2 + 8r^3\right)\ln\frac{1+\sqrt{1-4r}}{1-\sqrt{1-4r}} + \sqrt{1-4r}\left(-\frac{7}{2} + 13r + 4r^2\right)\right] \tag{25}$$

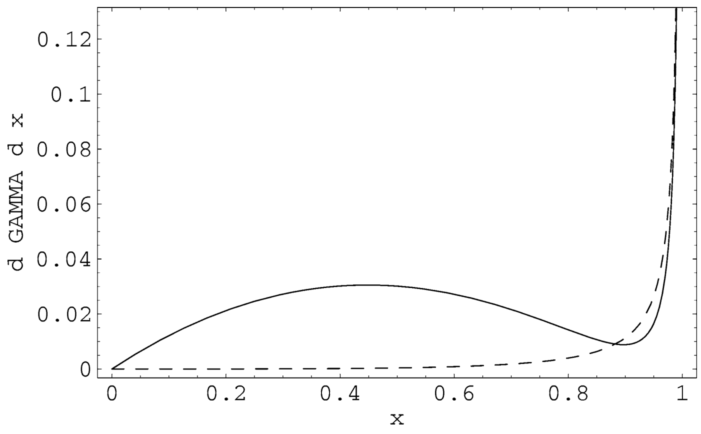

Fig. 2. Photon energy distribution for $\overline{B}_s \to e^+e^-\gamma$, normalized to $\overline{B}_s \to \gamma\gamma$. (Dashed line is the QED result.)

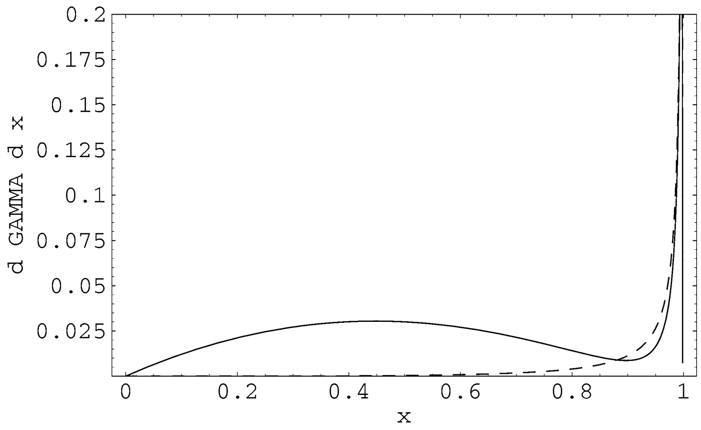

Fig. 3. Photon energy distribution for $\overline{B}_s \to \mu^+\mu^-\gamma$, normalized to $\overline{B}_s \to \gamma\gamma$. (Dashed line is the QED result.)

Table 1
Average charge asymmetry, Conversion ratio and Branching ratio for the decays $\overline{B}_s \to e^+e^-\gamma$ and $\overline{B}_s \to \mu^+\mu^-\gamma$. (Last column assumes $\mathrm{Br}(\overline{B}_s \to \gamma\gamma) = 5 \times 10^{-7}$)

Decay	Average charge asymmetry $\langle A \rangle$	Conversion ratio $\frac{\Gamma(\overline{B}_s \to l^+l^-\gamma)}{\Gamma(\overline{B}_s \to \gamma\gamma)}$	Branching ratio $\frac{\Gamma(\overline{B}_s \to l^+l^-\gamma)}{\Gamma(\overline{B}_s \to \mathrm{all})}$
$\overline{B}_s \to e^+e^-\gamma$	−0.28	4.0%	2.0×10^{-8}
$\overline{B}_s \to \mu^+\mu^-\gamma$	−0.47	2.3%	1.2×10^{-8}

which yields $R_e(\mathrm{QED}) = 2.3\%$, $R_\mu(\mathrm{QED}) = 0.67\%$. The absolute branching ratios of $\overline{B}_s \to l^+l^-\gamma$, obtained by taking $\mathrm{Br}(B_s \to \gamma\gamma) = 5 \times 10^{-7}$ [7,8] are $\mathrm{Br}(\overline{B}_s \to e^+e^-\gamma) = 2.0 \times 10^{-8}$, $\mathrm{Br}(\overline{B}_s \to \mu^+\mu^-\gamma) = 1.2 \times 10^{-8}$. Our results for the average charge asymmetry $\langle A \rangle_l$, the conversion ratios R_l and the branching ratios are summarized in Table 1.

5. Comments

(i) The branching ratios calculated by us are somewhat higher than those obtained in previous work [1,2], which used a different parametrization of the form factors f_V, f_A, f_T, $f_{T'}$ based on QCD sum rules [1] and light-front models [2]. In particular, these parametrizations do not satisfy the relation $f_T = f_T'$ which, as noted in [4], follows from the identity $\sigma_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5$.

(ii) Our predictions for the charge asymmetry $\langle A \rangle$ and the conversion ratio $\Gamma(\overline{B}_s \to l^+l^-\gamma)/\Gamma(\overline{B}_s \to \gamma\gamma)$ are independent of the parameter $\bar{\Lambda}_s$ which appears in the form factor in Eq. (16). The branching ratios in Table 1 assume $\mathrm{Br}(\overline{B}_s \to \gamma\gamma) = 5 \times 10^{-7}$, and can be rescaled when data on this channel are available.

(iii) A full analysis of the decay $\overline{B}_s \to l^+l^-\gamma$ requires inclusion of the bremsstrahlung amplitude corresponding to photon emission from the leptons in $B_s \to l^+l^-$. This contribution is proportional to $f_{B_s} m_l$ and affects the photon energy spectrum in the small x_γ region. We have calculated the corrected spectrum for $B_s \to l^+l^-\gamma$, following the procedure in [11], and the result is shown in Fig. 4 for the case $l = \mu$. As anticipated, the correction is limited to small x_γ, and can be removed by a cut at small photon energies.

(iv) The QCD form factors in Eq. (16) are valid up to corrections of order $(\Lambda_{\mathrm{QCD}}/E_\gamma)^2$. In the small x_γ region, arguments based on heavy hadron chiral perturbation theory suggest form factors dominated by the $B^\star$ pole

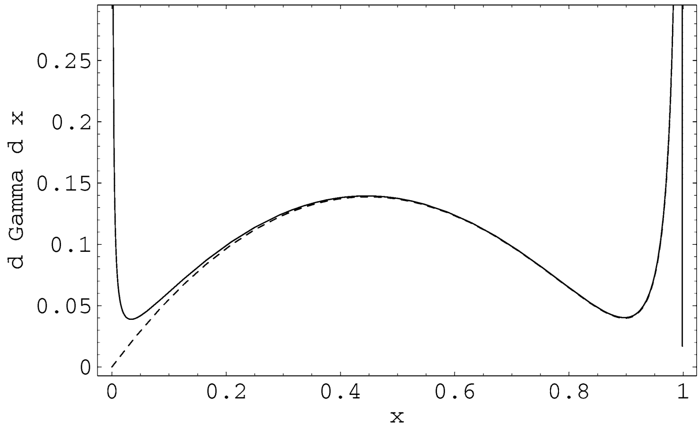

Fig. 4. Photon energy spectrum in $\overline{B}_s \to \mu^+\mu^-\gamma$, with bremsstrahlung (solid line) and without bremsstrahlung (dashed line).

with the appropriate quantum numbers, for example,

$$f_V(x_\gamma) \sim \frac{1}{M_{B_s}^2(1-x_\gamma) - M_{B_s^\star}^2}. \tag{26}$$

Defining $M_{B_s^\star} - M_{B_s} = \Delta M$, this form factor has the behaviour $f_V(x_\gamma) \sim \frac{1}{x_\gamma+\delta}$, with $\delta \approx 2\Delta M/M_{B_s} \approx 0.02$. We have investigated the effect of replacing the QCD form factor of Eq. (16) by a different universal form $f_{V,A,T}(x_\gamma) = f_{B_s}/(3\bar{\Lambda}_s(x_\gamma+\delta))$, and found only minor changes in the numbers given in Table 1. In general, one must expect some distortion in the spectrum at low x_γ, compared to that shown in Figs. 1–4.

(v) We will examine separately the predictions for $A(x_\gamma)$ and $d\Gamma/dx_\gamma$ in the reaction $B_s \to \tau^+\tau^-\gamma$, in which the bremsstrahlung part of the matrix element plays a significant role [11]. We will consider also refinements due to the q^2-dependent term in C_9^{eff}, and the effects of $c\bar{c}$ resonances.

In view of their clear signature, non-negligible branching ratios and interesting dynamics, the decays $B_s \to l^+l^-\gamma$ could form an attractive domain of study at future hadron colliders.

Acknowledgements

We are indebted to Dr. Gudrun Hiller for drawing our attention to a sign error in the first version of this Letter. We thank Dr. Thorsten Feldmann for a helpful discussion. One of us (Y.D.) acknowledges a Doctoral stipend under the Graduiertenförderungs-gesetz of the state of Nordrhein–Westphalen.

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