



# “*T*-odd effects” in unpolarized Drell–Yan scattering

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## Abstract

We consider the leading twist “*T*-odd” contributions as the dominant source of the  $\cos 2\phi$  azimuthal asymmetry in unpolarized  $p\bar{p} \rightarrow \mu^- \mu^+ X$  di-lepton production in Drell–Yan scattering. This asymmetry contains information on the distribution of quark transverse spin in an unpolarized proton. In a parton-spectator framework we estimate these asymmetries at 50 GeV center of mass energy. This azimuthal asymmetry is interesting in light of proposed experiments at GSI, where an anti-proton beam is ideal for studying the transversity properties of quarks due to the dominance of *valence* quark effects.

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## 1. Introduction

One of the persistent challenges confronting the QCD parton model is to provide a theoretical basis to understand the experimentally significant azimuthal and transverse spin asymmetries (TSAs) that emerge in exclusive, inclusive and semi-inclusive processes [1–6]. Generally speaking, the spin dependent amplitudes for the scattering will contribute to non-zero transverse single spin asymmetries (TSSAs) if there are imaginary parts of bilinear products of those amplitudes that have overall helicity change. For two-body exclusive reactions, TSSA requires there to be an imaginary part of the product of an helicity non-flip with an helicity flip amplitude. For inclusive reactions, the same conclusion can be reached by taking the amplitudes as two-body helicity amplitudes for the production of a fixed hadron and a state  $|X\rangle$ . Through the generalized optical theorem, TSSAs in inclusive reactions can be related to discontinuities in helicity flip three-body forward scattering amplitudes [7]. In perturbative QCD (PQCD), applicable to the hard scattering region, to obtain an imaginary contribution to quark and/or gluon scattering processes demands introducing

higher order corrections to tree level processes. One approach incorporates the requisite phases through interference of tree level and one-loop contributions in PQCD in an attempt to explain TSSAs [8,9]. On general grounds the helicity conservation property of massless QCD predicts that such contributions are small, going like  $\alpha_s m/Q$ , where  $\alpha_s$  is the strong coupling,  $m$  represents a non-zero quark mass and  $Q$  represents the hard QCD scale. Such contributions have failed to account for the large TSSAs observed in  $\Lambda$  production [1,9]. On the other hand, the twist three quark–quark and quark–gluon correlations described in [10,11] hold promise to describe the phenomena at large  $p_T$ .

By contrast, in soft contributions to hadronic processes, when the transverse momentum of the process  $p_T$  is sensitive to the scale of intrinsic quark momenta, there arises the possibility that there are non-trivial transversity parton distributions that contribute to transverse spin asymmetries. This was realized by Ralston and Soper [12] when they introduced the chiral-odd transversity distribution function [13,14]  $h_1(x)$  which play a role in doubly polarized Drell–Yan processes [15] as well as in TSSAs in semi-inclusive deep inelastic scattering (SIDIS) [20]. In the latter case TSSAs arise from so-called naive *T*-odd correlations (“*T*-odd”) of the form  $i\mathbf{s}_T \cdot (\mathbf{P} \times \mathbf{k}_\perp)$ , with transverse quark spin  $\mathbf{s}_T$ , longitudinal hadron momentum  $\mathbf{P}$ , and intrinsic quark transverse momentum  $\mathbf{k}_\perp$ , implying the existence of “*T*-odd” transverse momentum distribution (TMD) functions

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[16]. The “ $T$ -odd” distributions [17–19] possess both transversality properties and the necessary phases to account for TSSAs and azimuthal asymmetries [20–23] and they exist by virtue of non-zero parton transverse momenta [18,19,24]. They correspond to distributions that would vanish at *tree level* in any  $T$ -conserving model of hadrons and quarks. In this sense they are similar to the decay amplitudes for hadrons that involve single spin asymmetries which are non-zero due to final (and/or initial) state strong interactions. Their existence was established by Sivers to account for the significant SSA in inclusive reactions (e.g.,  $pp^\uparrow \rightarrow \pi X$ ) [17,18], by Collins in SIDIS [20], and by Boer [21] in Drell–Yan scattering.

In contrast to the TSSA generated from the interference of tree-level and one loop correction in PQCD, such effects go like  $\alpha_s(k_\perp)/M$ , where now  $M$  plays the role of the chiral symmetry breaking scale and  $k_\perp$  is characteristic of quark intrinsic motion. This realization was exploited by Brodsky, Hwang and Schmidt [22] which several researchers generalized thereafter [25,26] in parton inspired spectator-models of quark–hadron interactions.

The possible existence of the “ $T$ -odd” quark distribution functions are important in light of the observation some 35 years ago by Drell and Yan [27] that high energy hadron scattering processes that produce large invariant mass lepton pairs, can be probes of quark–antiquark distribution functions. Indeed,  $p\bar{p}$  scattering is a preferred reaction to study the role that “ $T$ -odd” quark distribution functions play in the transverse spin structure of the proton through spin and azimuthal asymmetries in QCD [21]. That is the direction we pursue herein.

## 2. Drell–Yan and “ $T$ -odd” correlations

At the parton level the Drell–Yan cross section receives contributions from quark–antiquark annihilation into the heavy photon. In unpolarized Drell–Yan scattering early cross section data as a function of the transverse momentum of the muon pair indicated deviations from the Bjorken scaling prediction [28, 29]. The implication was that the collinear approximation was insufficient to describe the data [30,31]. Transverse momentum of a parton arises due to hard Bremsstrahlung of gluons, which is calculable from PQCD when the momentum transfers are large [32]. On the other hand, quark confinement implies that quarks have soft or intrinsic transverse momenta  $k_\perp$ . This latter effect is significant at low transverse momentum where the di-muon pair’s transverse momentum is much less than its invariant mass ( $m_{\mu\mu} \equiv Q$ )  $q_T \ll Q$ .  $q_T$  dependence has been incorporated into the factorized Drell–Yan model [12,33]<sup>1</sup> by extending the parton probability distribution to be a function of  $k_\perp$  [16]

$$\int d^2\mathbf{k}_\perp \mathcal{P}(k_\perp, x) = f(x). \quad (1)$$

If the parton distributions within the incoming hadrons have transverse momentum dependence, there will be a continuum

of values of their  $k_\perp$  for which a time-like photon of fixed 4-momentum will be formed. Ignoring or summing over spin (and the lepton pair orientation), the  $k_\perp$  dependent distribution functions appear in the differential cross section,

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy d^2\mathbf{q}_T} &= \frac{4\pi\alpha^2}{3Q^4} \sum_a e_a^2 \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp \\ &\times \delta^{(2)}(\mathbf{k}_\perp + \mathbf{p}_\perp - \mathbf{q}_T) \\ &\times f_{a/A}(x, k_\perp) \bar{f}_{\bar{a}/B}(\bar{x}, p_\perp), \end{aligned}$$

where  $f_{a/A}(x, k_\perp)$  is a distribution function for a quark  $a$  to be found in hadron  $A$  with transverse momentum  $k_\perp$  and longitudinal momentum fraction  $x$  and  $\bar{f}$  is the corresponding antiquark distribution in hadron  $B$ . In case the transverse momentum  $q_T$  of the muon pair is not negligible compared to the invariant mass  $Q$ , the beam and target are not collinear in their center of mass frame. Thus the cross section can depend on the lepton pair orientation. The angular dependence can be expressed as

$$\begin{aligned} \frac{dN}{d\Omega} &= \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi \right. \\ &\left. + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right), \quad (2) \end{aligned}$$

where  $\frac{dN}{d\Omega} \equiv \frac{d\sigma}{dQ^2 dy d^2\mathbf{q}_T d\Omega} / \frac{d\sigma}{dQ^2 dy d^2\mathbf{q}_T}$ . The solid angle  $\Omega$  refers to the lepton pair orientation in the pair rest frame relative to the boost direction and the incoming hadrons’ plane [33] and  $y = 1/2 \ln(x/\bar{x})$  rapidity.  $\lambda, \mu, \nu$  are functions that depend on  $s, x, Q^2, q_T$ , the square of the center of mass energy, the quark’s fraction of the hadron’s longitudinal momentum, the invariant mass squared of the produced lepton pair, and the transverse momentum of the di-muon pair.<sup>2</sup> All of these asymmetry functions have parton model contributions. Taking into account NLO [32] and NNLO [46] the QCD improved parton model predicts  $1 - \lambda - 2\nu = 0$ , the so-called Lam–Tung relation [47]. However, experimental measurements of  $\pi p \rightarrow \mu^+ \mu^- X$  discovered unexpectedly large values of these asymmetries [2, 3] compared to parton-model expectations resulting in a serious violation of this relation.

An early theoretical explanation for non-trivial azimuthal  $\cos 2\phi$  dependence based on the Drell–Yan model, Eq. (2), was given by Collins and Soper [33] where the “ $T$ -even” contribution to the asymmetry

$$v_4 = \frac{\frac{1}{Q^2} \sum_a e_a^2 \mathcal{F}[w_4 f_1(x, k_\perp) \bar{f}_1(\bar{x}, p_\perp)]}{\sum_a e_a^2 \mathcal{F}(f_1(x, k_\perp) \bar{f}_1(\bar{x}, p_\perp))}, \quad (3)$$

where  $w_4 = 2(\hat{\mathbf{h}} \cdot (\mathbf{k}_\perp - \mathbf{p}_\perp))^2 - (\mathbf{k}_\perp - \mathbf{p}_\perp)^2$  and,  $\hat{\mathbf{h}} = \mathbf{q}_T / Q_T$  and  $\mathcal{F}$  is the convolution integral

$$\begin{aligned} \mathcal{F}[f \bar{f}] &= \int d^2k_\perp d^2p_\perp \delta^{(2)}(\mathbf{k}_\perp + \mathbf{p}_\perp - \mathbf{q}_T) \\ &\times f_{a/A}(x, k_\perp) \bar{f}_{\bar{a}/B}(\bar{x}, p_\perp). \end{aligned}$$

<sup>1</sup> New work on the factorization theorem for Drell–Yan can be found in [45].

<sup>2</sup> We are working in the Collins–Soper frame where  $q_T$  retains its meaning.

However this contribution as well as attempts to account for the violation in terms of higher twist effects [34,35] have been unsuccessful.

Once transverse momentum dependence of parton distributions enters the picture of scattering processes a larger set of transverse momentum distribution (TMD) [24] become relevant, particularly for azimuthal and spin asymmetries. Among such functions are the possible leading twist “ $T$ -odd” quark distribution [17] and fragmentation functions [20].

More recently, Boer [21] proposed that there is a dominant leading twist contribution to  $\nu$  coming from the “ $T$ -odd” transversity distributions  $h_1^\perp(x, k_\perp)$  for both hadrons, which dominates in the kinematic range,  $q_T \ll Q$ . The  $\cos 2\phi$  azimuthal asymmetry in unpolarized  $p\bar{p} \rightarrow \mu^+\mu^-X$  involves the convolution of the leading twist “ $T$ -odd” function,  $h_1^\perp$  [21,36–38]

$$v_2 = \frac{\sum_a e_a^2 \mathcal{F}[w_2 h_1^\perp(x, k_\perp) \bar{h}_1^\perp(\bar{x}, p_\perp) / (M_1 M_2)]}{\sum_a e_a^2 \mathcal{F}[f_1(x, k_\perp) \bar{f}_1(\bar{x}, p_\perp)]}, \quad (4)$$

where  $w_2 = (2\hat{h} \cdot \mathbf{k}_\perp \cdot \hat{h} \cdot \mathbf{p}_\perp - \mathbf{p}_\perp \cdot \mathbf{k}_\perp)$  is the weight in the convolution integral,  $\mathcal{F}$  [21]. A simple model for these distributions, inspired by Collins’ ansatz for the transversity fragmentation function, led to  $q_T$  dependent  $\nu$  which could be fit to the low values of the  $\pi p$  data. Further work along those lines [36–38] incorporated a more realistic model for the “ $T$ -odd” functions, as first developed in SIDIS [22] for the functions  $f_{1T}^\perp(x, k_\perp)$  which were related to the  $h_1^\perp(x, k_\perp)$  in Ref. [26]. The results were presented for  $p\bar{p}$  scattering.<sup>3</sup> This azimuthal asymmetry is interesting in light of proposed experiments at Darmstadt GSI [39], where an anti-proton beam is ideal for studying the transversity property of quarks due to the dominance of *valence* quark effects [40]. Herein we extend our calculations for “ $T$ -odd” contributions to the unpolarized Drell–Yan scattering first reported in [37]. We perform a detailed analysis displaying  $q_T$  and, for the first time,  $x$ , and  $Q$  (or  $m_{\mu\mu}$ ) dependence of this effect. In addition we compare the double “ $T$ -odd” contribution to the conventional sub-leading twist “ $T$ -even” contribution [33] from Eq. (3).

### 3. “ $T$ -odd” transversity distribution

The leading twist “ $T$ -odd” distributions functions emerge from the color gauge invariant factorized hadronic tensor [20, 21]. In a non-singular gauge (e.g., Feynman gauge) for  $p\bar{p}$  scattering the generalization [41] of the Drell–Yan model [12,33] is

$$W(Q, P_A, P_B) = \int d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp \delta^2(\mathbf{q}_\perp - \mathbf{k}_\perp - \mathbf{p}_\perp) \times \text{Tr}(\gamma_\mu \Phi^{[-]}(x, \mathbf{k}_\perp) \gamma_\nu \bar{\Phi}^{[-]}(\bar{x}, \mathbf{p}_\perp)). \quad (5)$$

<sup>3</sup> Very recently instanton induced effects have been investigated [48].

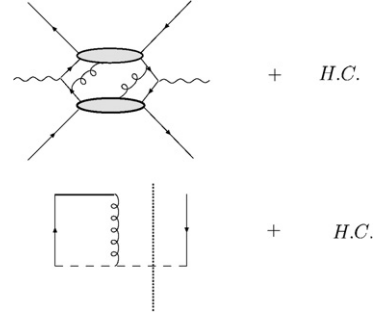


Fig. 1. *Above*: Feynman diagram representing final state interactions giving rise to “ $T$ -odd” contribution to Drell–Yan scattering. *Below*: Quark–target scattering amplitude depicting the “ $T$ -odd” contribution to the quark distribution function in the eikonal approximation.

$\Phi^{[-]}(x, k_\perp)$  is the gauge invariant quark–quark correlations function

$$\Phi^{[-]}(x, \mathbf{k}_\perp) = \sum_X \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ik \cdot \xi} \langle P | \bar{\psi}(0) \mathcal{G}_{[0, -\infty]}^- | X \rangle \times \langle X | \mathcal{G}_{[-\infty, \xi]}^- \psi(\xi) | P \rangle |_{\xi^+ = 0},$$

with a gauge link  $\mathcal{G}_{[-\infty, \xi]}^-$ , running to  $-\infty$  (“past pointing”) along the “minus” light-cone direction and  $\bar{\Phi}^{[-]}(x, p_\perp)$  is the antiquark correlator with gauge link running to  $-\infty$  along the “plus” light-cone direction.  $h_1^\perp$  is projected from the Dirac trace, that is

$$\begin{aligned} \Phi^{[i\sigma^{\perp+}\gamma_5]}(x, \mathbf{k}_\perp) &= \text{Tr}(i\sigma^{\perp+}\gamma_5 \Phi^{[-]}(x, \mathbf{k}_\perp)) \\ &= \frac{2\varepsilon_T^{ij} k_{\perp j}}{M} h_1^\perp(x, k_\perp) \dots, \end{aligned}$$

where  $\varepsilon_T^{ij} = \varepsilon_{-+ij}$ .

In our work on SIDIS [42–44] we used a parton model within the quark–diquark spectator framework. Here we extend that treatment to Drell Yan  $p\bar{p} \rightarrow \mu\mu^+X$  scattering. We expand the gauge link operator to leading order in the strong coupling  $g$ . Here we have an  $A^+$  gluon collinear to the proton running along the minus light-cone direction [41] resulting in a “ $T$ -odd” TMD represented in Fig. 1 and given by,

$$\begin{aligned} \Phi^{[-]}(x, \mathbf{p}_\perp) &= \sum_X \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) | X \rangle \\ &\times \langle X | \left( -ig \int_{-\infty}^{\xi^-} d\eta^- A^+(\eta) \right) \psi(\xi) | P \rangle |_{\xi^+ = 0} \\ &+ \text{H.C.} \end{aligned} \quad (6)$$

Noting that parton intrinsic transverse momentum yields a natural regularization for the moments of these distributions, we incorporate a Gaussian form factor into our model. The non-perturbative vertex functions entering the correlation functions  $\Phi^{[-]}(x, \mathbf{k}_\perp)$  [26,42], in this spectator framework are

$$\langle X_{sdq} | \psi(0) | P \rangle = \left( \frac{i}{\not{k} - m} \right) \gamma(k_\perp^2) U(P, S), \quad (7)$$

where  $\Upsilon(k_{\perp}^2) = \mathcal{N}e^{-bk_{\perp}^2}$  and  $b \equiv 1/\langle k_{\perp}^2 \rangle$ .  $\mathcal{N}$  is the scalar diquark normalization [42],  $k$  is the momentum of the quark in the target proton,  $k_{\perp}$  and  $\langle k_{\perp}^2 \rangle$  are the intrinsic and average intrinsic transverse momentum respectively, and  $U(P, S)$  is the nucleon spinor. The quark–diquark–gluon vertex function (in momentum space) is

$$\begin{aligned} \langle X_{sdq} | \frac{A^+(\ell)}{\ell^+ + i\epsilon} \psi(k - \ell) | P \rangle \\ = \frac{1}{(\ell^2 + i\epsilon)(\ell^+ + i\epsilon)} \\ \times \frac{ie_2(2(P - k) - \ell)^+}{(P - k - \ell)^2 - \mu^2 + i\epsilon} \frac{i}{\not{k} - \not{\ell} - m + i\epsilon} \\ \times \Upsilon(k_{\perp}^2) U(P, S), \end{aligned} \quad (8)$$

where  $\ell$  is the loop momentum. Inserting Eqs. (7) and (8) into (6), performing the loop integration, and finally projecting the unpolarized piece from  $\Phi^{[i\sigma^{\perp+}\gamma_5]}$  results in the leading twist, “ $T$ -odd”, scalar diquark contribution to the Boer Mulders function (which is equal to the Sivers function [26]),

$$h_1^{\perp}(x, k_{\perp}) = \mathcal{N}\alpha_s M \frac{(1-x)(m+xM)}{\Lambda(k_{\perp}^2)k_{\perp}^2} \mathcal{R}(k_{\perp}^2; x), \quad (9)$$

where

$$\mathcal{R}(k_{\perp}^2; x) = \exp^{-2b(k_{\perp}^2 - \Lambda(0))} (\Gamma(0, 2b\Lambda(0)) - \Gamma(0, 2b\Lambda(k_{\perp}^2)))$$

is the regularization function,  $\Lambda(k_{\perp}^2) = k_{\perp}^2 + (1-x)m^2 + x\mu^2 - x(1-x)M^2$ , and  $M$ ,  $m$ , and  $\mu$  are the nucleon, quark, and spectator diquark masses, respectively. The normalization factor  $\mathcal{N}$ , and  $b = 1/\langle k_{\perp}^2 \rangle$  are determined with respect to the unpolarized  $u$ -quark distribution  $f_1^{(u)}(x, k_{\perp})$

$$f_1(x, k_{\perp}) = \frac{\mathcal{N}(1-x)((m+xM)^2 + k_{\perp}^2)}{\Lambda^2(k_{\perp}^2)} \mathcal{R}_f(k_{\perp}^2),$$

where  $\mathcal{R}_f(k_{\perp}^2) = e^{-bk_{\perp}^2}$ . Taking the first moment yields

$$\begin{aligned} f(x) = \frac{g^2}{(2\pi)^2} \frac{b^2}{\pi^2} (1-x) \left\{ \frac{(m+xM)^2 - \Lambda(0)}{\Lambda(0)} \right. \\ \left. - [2b((m+xM)^2 - \Lambda(0)) - 1] \right. \\ \left. \times e^{2b\Lambda(0)} \Gamma(0, 2b\Lambda(0)) \right\}, \end{aligned} \quad (10)$$

which multiplied by  $x$  with  $\langle k_{\perp}^2 \rangle = (0.4)^2 (\text{GeV}/c)^2$ , is in good agreement [42] with the valence distribution of Ref. [49]. The quark–gluon coupling  $g \equiv e_1$  and  $e_2$  the gluon–scalar diquark together yield  $\alpha_s = C_F e_1 e_2 / 4\pi$ , where  $\alpha_s = 0.4$  and  $C_F = 4/3$ .

#### 4. Drell–Yan kinematics

Before we perform the convolution integral in Eq. (4), the Drell–Yan kinematics demand some special attention. Since the incoming partons have non-vanishing transverse momentum  $k_{\perp}$  and we are considering the kinematic higher twist corrections  $\nu_4$  relative to  $\nu_2$ , consistency demands that we take into account  $q_T/Q$  corrections in the two body kinematics. They enter the

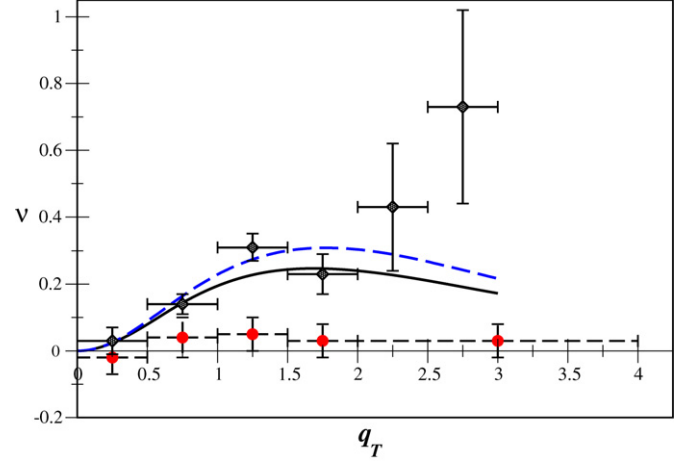


Fig. 2.  $\nu$  plotted as a function of  $q_T$  for  $s = 50 \text{ GeV}^2$ ,  $x$  in the range 0.2–1.0, and  $Q$  ranging from 3–6 GeV/ $c$ . Solid line leading twist contribution  $\nu_2$ , dashed line leading and sub-leading twist ( $\nu_2 + \nu_4$ ). Data: Diamonds are for E615  $\pi^- + p$  at 252 GeV/ $c$ . Circles are for E866  $p + d$  at 800 GeV/ $c$ . Horizontal bars refer to bin size, vertical error bars refer to statistical uncertainties.

constraints among  $x$  and  $\bar{x}$ , the fractional longitudinal momenta of the quark and antiquark,

$$\begin{aligned} x\bar{x} = (Q^2 + q_T^2)/s \equiv \tilde{\tau}, \quad \frac{x - \bar{x}}{2} \equiv \eta = x_F/2 \quad \text{and} \\ x = \eta + \sqrt{\eta^2 + \tilde{\tau}^2}, \quad \bar{x} = -\eta + \sqrt{\eta^2 + \tilde{\tau}^2}, \end{aligned} \quad (11)$$

where  $x_F$  is Feynman- $x$ . Due to the constraint on  $x\bar{x}$  the allowed range of  $x$  is restricted for each  $Q$  value, from  $x_{\min} = (Q^2 + q_T^2)/s$  to 1. Furthermore, evaluating the convolutions of  $h_1^{\perp}\bar{h}_1^{\perp}$  and  $f_1\bar{f}_1$  for a sampling of  $x$  will not treat the  $\bar{x}$  and the corresponding antiparticle structure functions symmetrically. So it is more appropriate to use the symmetrical variable,  $x_F$ . However, the allowed range of  $x_F$  depends on  $Q$  (from  $-1/2(1 - (Q^2 + q_T^2)/s)$  to  $+1/2(1 - (Q^2 + q_T^2)/s)$ ). That is, the variables  $x_F$  and  $Q$  are not orthogonal. Since we aim to present partially integrated values of  $\nu$ , approximating experimentalists’ measurements, it is advantageous to work with orthogonal variables. We choose the variable

$$\zeta = \frac{1}{2} \frac{x_F}{(1 - (Q^2 + q_T^2)/s)}, \quad (12)$$

with range from  $-\frac{1}{2}$  to  $+\frac{1}{2}$ , independent of  $q$  and  $q_T$ . Values of the asymmetry fill the rectangular space of variables,  $\zeta$ ,  $Q$ ,  $q_T$ . We have been careful with this choice because our model predictions have considerable structure in all 3 variables. Hence the meaning of a graph of  $\nu(Q)$  or  $\nu(x)$  has particular significance when comparing to experimental data.

A crucial point in selecting these variables involves how experimenters determine various asymmetries and angular dependences, in order to maximize statistics when extracting possibly small effects like  $\nu$ . Events appear distributed over allowed regions (modified by experimental acceptances) of all three variables along with the  $\mu$  pair angular variables, of course. To obtain the dependence on one variable, large bins are defined and event numbers averaged over those bins. How are those results to be compared with theoretical predictions [50]? The two



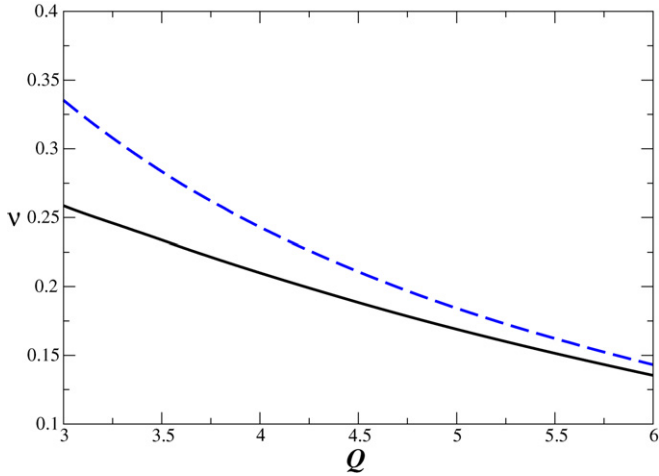


Fig. 3.  $\nu$  plotted as a function of  $Q$  for  $s = 50 \text{ GeV}^2$ ,  $x$  in the range  $0.2-1.0$ , and  $q_T$  ranging from  $1-2 \text{ GeV}/c$ . Solid line leading twist contribution  $\nu_2$ , dashed line leading and sub-leading twist ( $\nu_2 + \nu_4$ ).

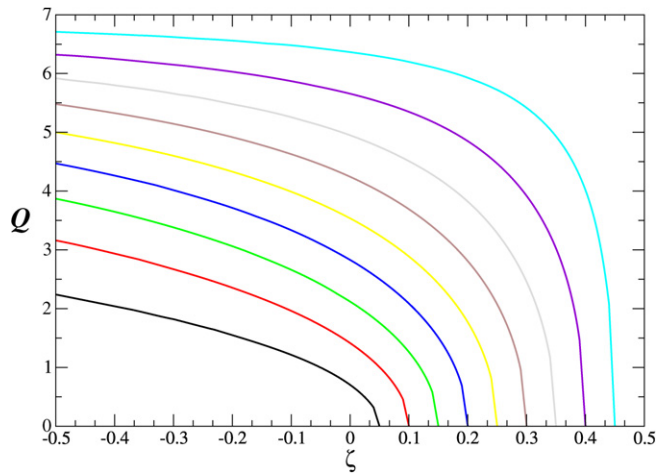


Fig. 4. Contours of constant  $x$  (for fixed  $q_T$ ) as a function of  $\zeta$  and  $Q$ .

experiments for which relevant data have been published have different ranges of variables [2,3]. The binning procedures are not easily compared. To be most general and adaptable for future experimental comparisons we have determined the value of  $\nu$  as a function of  $\zeta$ ,  $Q$ ,  $q_T$ . We then integrate over pairs of those variables for particular ranges of the variables. At  $s = 50 \text{ GeV}^2$  we take  $q_T \leq 2 \text{ GeV}/c$  and  $3 \text{ GeV}/c \leq Q \leq 6 \text{ GeV}/c$ , while  $\zeta$  always varies from  $-1/2$  to  $1/2$ .

For  $\nu(q_T)$  and  $\nu(Q)$  the resulting values are shown in Figs. 2 and 3. There is a corresponding  $\nu(\zeta)$  shown in Fig. 5. To connect with the  $x$  or  $x_F$  dependence we have to take the  $q_T$  and  $q$  dependence into account. For each  $q_T$  the fixed  $x$  or  $x_F$  values form contours in the  $\zeta, q$  plane. So an integral of  $\nu$  over  $q$  for a fixed  $x_F$  follows the relevant contour in  $\zeta, Q$  and has a limit on  $\bar{\tau} = (Q^2 + q_T^2)/s$  of  $(1 - \frac{x_F}{2\zeta})$ . This limits the range of the  $Q$  integral until the limiting value of the range at  $Q = 6 \text{ GeV}/c$  for  $s = 50 \text{ GeV}^2$ . Similarly, for each  $q_T$  the fixed  $x$  values form asymmetrical contours in the  $\zeta, Q$  plane as shown in Fig. 4. The limit on  $\bar{\tau}$  for a fixed  $x$  will be  $(\frac{2\zeta - x}{2\zeta x - 1})$ . The resulting values of  $\nu(x)$  are shown in Fig. 6.

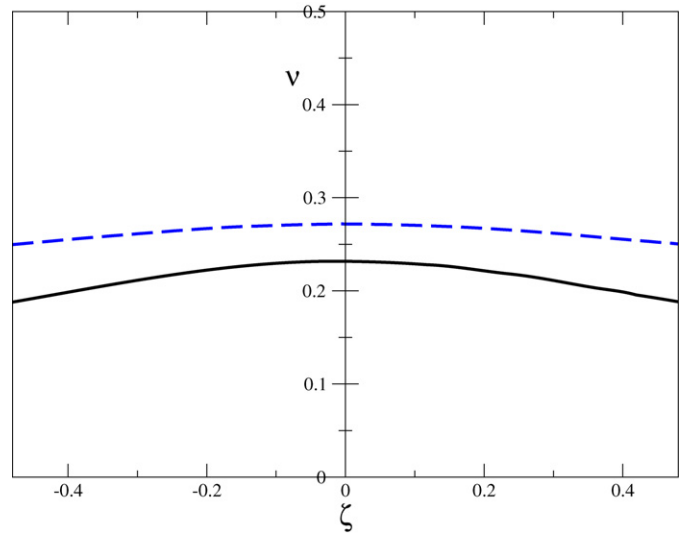


Fig. 5.  $\nu$  plotted as a function of  $\zeta$  for  $s = 50 \text{ GeV}^2$ ,  $q_T$  ranging from  $1-2 \text{ GeV}/c$  and  $Q$  from  $3-6 \text{ GeV}/c$ . Solid line leading twist contribution  $\nu_2$ , dashed line leading and sub-leading twist ( $\nu_2 + \nu_4$ ).

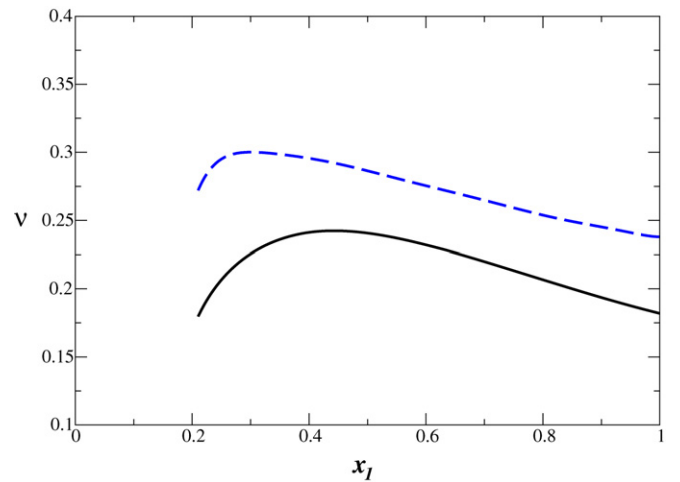


Fig. 6.  $\nu$  plotted as a function of  $x$  for  $s = 50 \text{ GeV}^2$ ,  $q_T$  ranging from  $1-2 \text{ GeV}/c$  and  $Q$  from  $3-6 \text{ GeV}/c$ . Solid line leading twist contribution  $\nu_2$ , dashed line leading and sub-leading twist ( $\nu_2 + \nu_4$ ).

The predictions shown in Figs. 2, 3, 5, and 6 are specifically for  $\bar{p} + p$  scattering. While there are no data for this reaction—a proposed facility at GSI [39] would provide this. As we have noted, there are data on the Drell–Yan asymmetries for  $\pi + p$  [2,3] and recently,  $p + d$  [52]. The analog of the  $T$ -odd contribution to the  $\nu$  asymmetry for the  $\pi$  induced process involves a transversely polarized valence antiquark with transverse momentum,  $\bar{h}_1^{\perp(\pi)}(x, k_\perp)$ . For the sake of illustration, we include data points from E615 [2] in Fig. 2. They fall roughly near our predictions. This suggests that the analogous  $T$ -odd structure function for  $\pi$  is comparable in magnitude to  $h_1^\perp$  for the proton (see also [21,38]). By contrast, the E866 deuteron target data [52] show measured values of  $\nu$  between 0 and 0.1 over the same range of  $q_T$ , as indicated in Fig. 2. One of the pair of structure functions in the convolution will involve sea anti-quarks, the  $\bar{h}_1^{\perp(\text{sea})}$  for  $N \rightarrow \bar{u}$  or  $\bar{d}$ . We have not provided a model

for this subprocess, but it is theoretically suppressed by another factor of  $\alpha_s$  in our approach (as well as possible kinematic factors). A consideration of this data implies that this sea structure function must be at most roughly  $\frac{1}{3}$  of the magnitude of our predicted valence quark structure function. In making these comparisons with the data we are only providing suggestions about magnitudes of contributions, given that different structure functions are involved and differing kinematic regimes are explored.

## 5. $T$ -even contribution

Long before the realization that there is a leading twist 2 contribution to the Drell–Yan azimuthal asymmetry, Collins and Soper [33] proposed that the spin independent, transverse momentum dependent distributions  $f_1$  and  $\tilde{f}_1$  could contribute via Eq. (3). It is important to compare this *kinematic* twist 4 contribution to the leading twist contribution Eq. (4) shown above. We combined both convolutions to determine the magnitude of the shift. The additional contribution for  $s = 50 \text{ GeV}^2$  to each of the partially integrated functions  $\nu$  is shown in Figs. 2, 3, 5, 6 as slightly higher curves (dashed lines). The additional contribution is around 5–7%. For higher  $s$  values the effect is even smaller, as expected [37].

## 6. Conclusion

A perusal of the figures shows that the  $\cos 2\phi$  azimuthal asymmetry  $\nu$  is not small at center of mass energies of  $50 \text{ GeV}^2$ . We estimated the leading twist 2 and twist 4 contributions [37]. In Fig. 2, the “ $T$ -odd” portion (solid line) contributes about 25–30% with an additional 5% from the sub-leading “ $T$ -even” piece (dashed line). The distinction between the leading order “ $T$ -odd” and sub-leading order “ $T$ -even” contributions diminish at center of mass energy of  $s = 500 \text{ GeV}^2$  [37]. Even in Fig. 6,  $\nu$  versus  $x$  at  $s = 50 \text{ GeV}^2$ , where  $q_T$  ranges from 1–2  $\text{GeV}/c$ , the higher twist contribution is a rather small addition.

Thus, aside from the competing “ $T$ -even” effect, the experimental observation of a strong  $x$ -dependence would indicate the presence of “ $T$ -odd” structures in *unpolarized* Drell–Yan scattering, implying that novel transversity properties of the nucleon can be accessed *without invoking beam or target polarization*.

It should be noted that at order  $\alpha_s$ , a complete analysis for the full range of  $q_T$  would entail including gluon bremsstrahlung contributions [32]. Furthermore, collinear Sudakov corrections have not been accounted for here [51]. A thorough explication of Drell–Yan dynamics would require more care with regions in which divergent contributions become important to address. For this study, however, we have considered the implications of our model, unencumbered by subtleties at the edges of the phase space on which we concentrate.

We also conclude that “ $T$ -odd” correlations of intrinsic transverse quark momentum and transverse spin of quarks are intimately connected with studies of the  $\cos 2\phi$  azimuthal asymmetries in  $p\bar{p}$ -Drell–Yan scattering. Due to the dominance

of valence quark effects we estimate that the proposed proton anti-proton experiments at GSI [39] provide an excellent opportunity to study the role that “ $T$ -odd” correlations play in characterizing intrinsic transverse spin effects within the proton.

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