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Experimental determination of the effective strong coupling constant

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Abstract

We present a first attempt to experimentally extract an effective strong coupling constant that we define to be a low Q^2 extension of a previous definition by S. Brodsky et al. following an initial work of G. Grunberg. Using Jefferson Lab data and sum rules, we establish its Q^2-behavior over the complete Q^2-range. The result is compared to effective coupling constants inferred from different processes and to calculations based on Schwinger–Dyson equations, hadron spectroscopy or lattice QCD. Although the connection between the experimentally extracted effective coupling constants and the calculations is not established it is interesting to note that their behaviors are similar.
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At experimentally accessible distances, the strong force remains the only interaction that resists satisfactory understanding. Quantum Chromodynamics (QCD), the gauge theory of the strong force, is well known at short distances ($\lesssim 10^{-16}$ m) where it is solvable perturbatively. QCD, however, is not perturbatively calculable at larger distances, typically the scale of the nucleon radius. Recent precision data on moments of nucleon structure functions [1–4] reveal a smooth transition from small to large scales, while in contrast, a feature of perturbative QCD (pQCD) is that at Λ_{QCD}, the running strong coupling constant α_s becomes infinite. An approach using an effective coupling constant could reconcile these two seemingly paradoxical aspects of strong interaction.

In lepton scattering a scale at which the target structure is probed is given by the inverse of Q^2, the square of the four-momentum transfered to the target. One way to extract α_s at large Q^2 is to fit the Q^2-dependence of the moments of structure functions. Among all moments, the Bjorken sum rule [5] is a convenient relation for such an extraction [6]. Furthermore, as will be discussed, the Bjorken sum may offer unique advantages to define an effective coupling at low Q^2.

In the limit where the energy transfer ν and Q^2 are infinite, while $x \equiv Q^2/(2M\nu)$ remains finite (M is the nucleon mass), the Bjorken sum rule reads:

$$\Gamma_1^{p-n} \equiv \Gamma_1^p - \Gamma_1^n \equiv \int_0^1 dx \left(g_1^p(x) - g_1^n(x) \right) = \frac{1}{6} g_A. \qquad (1)$$

g_1^p and g_1^n are spin structure functions for the proton and neutron. The axial charge of the nucleon, g_A, is known from neutron β-decay. For finite Q^2 much greater than Λ_{QCD}^2, the Q^2-dependence of the Bjorken sum rule is given by a double series in Q^{2-t} ($t = 2, 4, \dots$ being the twist) and in $(\alpha_s/\pi)^n$. The $(\alpha_s/\pi)^n$ series is given by pQCD evolution equations. At leading twist ($t = 2$) and 3rd order in α_s, in the $\overline{\text{MS}}$ scheme and dimensional regularization, we have [7]:

$$\Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.21 \left(\frac{\alpha_s}{\pi} \right)^3 \right]. \qquad (2)$$

The validity of the sum rule is verified at $Q^2 = 5$ GeV^2 to better than 10% [8].

The extraction of the Bjorken integral using data from the Thomas Jefferson National Accelerator Facility (JLab) in the Q^2-range of 0.17–1.10 GeV^2 has been reported recently [9]. The use of Eq. (2) as an ansatz for definition of an effective

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running coupling constant at low Q^2 has ambiguities and difficulties as well as some practical advantages. The advantages are the following: firstly, it was pointed out in Ref. [10] that the extraction of α_s does not depend strongly on the low-x extrapolation. Secondly, this flavor non-singlet contribution does not mix quark and gluon operators when evolved. Hence the pQCD evolution is known to higher order. Thirdly, the JLab data are at constant Q^2. This avoids a possible ambiguity encountered in previous experiments, namely that in order to combine neutron and proton data, structure functions must be evolved to a common Q_0^2, which needs α_s itself as input. This is especially important in our case since we cannot anticipate the value of α_s at such low Q^2, due to the breakdown of pQCD. Hence, no evolution of the data to Q_0^2 is possible. Difficulties within the pQCD approach are: firstly, at low Q^2 higher twist effects become important and are not well known [9]. Secondly, the pQCD expansion loses its meaning as Q approaches Λ_{QCD} where, as a consequence of renormalization and regularization, α_s^{pQCD} itself is singular. It is necessary to use an appropriate theoretical framework to circumvent these difficulties. Such frameworks have been developed, see for example [11] and [12]. We use the method of "effective charges" of Grunberg [13], where the non-perturbative terms and higher order perturbative processes are absorbed in the definition of the coupling constant. In our case, it obeys the following definition:

$$\Gamma_1^{p-n} \equiv \frac{1}{6} g_A \left[1 - \frac{\alpha_{s,g_1}}{\pi} \right]. \tag{3}$$

Eq. (3) provides a definition of an effective QCD running coupling that we will explore here. The inherent systematic uncertainties in this experimental ansatz, and in those of the various theoretical approaches are unknown. Their comparison provides a framework for further analysis.

The coupling constant defined with Eq. (3) still obeys the renormalization group equation $d\alpha_s(k)/d\ln(k) = \beta(\alpha_s(k))$ [13]. The first two terms in Eqs. (2) and (3) series are independent of the choice of gauge and renormalization scheme. Consequently, the effective coupling constant is renormalization scheme and gauge independent, but becomes process-dependent. These process-dependent coupling constants can be related by using "commensurate scale relations" which connect observables without renormalization scheme or scale ambiguity [14]. In this topic, an important relation is the Crewther relation [14,15].

Considering an effective coupling constant yields other advantages beside renormalization scheme/gauge independence: such a procedure improves perturbative expansions [13,14], the effective charge is analytic when crossing quark mass thresholds, is non-singular at $Q = \Lambda_{QCD}$, and is well defined at low Q^2 [16]. The choice of defining the effective charge with Eq. (3) has unique advantages: Low Q^2 data exist and near real photon data will be available soon from JLab [17,18]. Furthermore, sum rules constrain α_{s,g_1} at both low and high Q^2 limits, as will be discussed in the next paragraph. Another advantage is that Γ_1^{p-n} is a quantity well suited to be calculated at any

Q^2 because of various cancellations that simplify calculations [9,19].

Using Eq. (3) and the JLab data, α_{s,g_1} can be formed. The elastic ($x = 1$) contribution is excluded in Γ_1^{p-n}. The resulting $\alpha_{s,g_1}/\pi$ is shown in Fig. 1. Systematic effects dominate the uncertainties, see Ref. [9] for details. The uncertainty from g_A is small. We also used the world data of the Bjorken sum evaluated at $\langle Q^2 \rangle = 5$ GeV^2 to compute $\alpha_{s,g_1}/\pi$. We can also use a model for Γ_1 and, using Eq. (3), form α_{s,g_1}. We chose the Burkert–Ioffe model [20] because of its good match with the experimental data on moments of spin structure functions [1–9]. It is interesting to note the behavior of α_{s,g_1} near $Q^2 = 0$ where it is constrained by the Gerasimov–Drell–Hearn (GDH) sum rule [21] that predicts the derivative of the Bjorken integral with respect to Q^2 at $Q^2 = 0$ [22]:

$$\Gamma_1^{p-n} = \frac{Q^2}{16\pi^2\alpha} \left(GDH^p - GDH^n \right)$$
$$= \frac{-Q^2}{8} \left(\frac{\kappa_p^2}{M_p^2} - \frac{\kappa_n^2}{M_n^2} \right), \tag{4}$$

where κ_p (κ_n) is the proton (neutron) anomalous magnetic moment and α is the QED coupling constant. This, in combination with Eq. (3), yields:

$$\frac{d\alpha_{s,g_1}}{dQ^2} = -\frac{3\pi}{4g_A} \times \left(\frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2} \right). \tag{5}$$

The constraint is shown by the dashed line. At $Q^2 = 0$ the Bjorken sum is zero and $\alpha_{s,g_1} = \pi$, a particular property of the definition of α_{s,g_1}. At larger Q^2, where higher twist effects are negligible, α_{s,g_1} can be evaluated by estimating the right-hand side of Eq. (2) (using α_s^{pQCD} as predicted by pQCD) and equating it to $g_A[1 - \alpha_{s,g_1}/\pi]/6$. The resulting α_{s,g_1} is shown by the gray band.[1] The width of the band is due to the uncertainty in Λ_{QCD}. Finally α_{s,F_3} can be computed using data on the Gross–Llewellyn Smith sum rule [23], which relates the number of valence quarks in the hadron, n_v, to the structure function $F_3(Q^2, x)$ measured in neutrino scattering. At leading twist, the GLS sum rule reads:

$$\int_0^1 F_3(Q^2, x) \, dx = n_v \left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 \right.$$
$$\left. - 20.21 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^3 + \cdots \right]. \tag{6}$$

Using the data taken by the CCFR Collaboration [24], we can apply the same procedure as for the Bjorken sum rule to compute α_{s,F_3}. We expect $\alpha_{s,F_3} = \alpha_{s,g_1}$ at large Q^2, since the leading twist Q^2-dependence of Eqs. (2) and (6) is identical up

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[1] When forming α_{s,g_1} using the Bjorken sum rule (gray band), we obtain a renormalization scheme dependent quantity, which is in contradiction with our goal. In practice, however, at large Q^2 and for high enough order in α_s^{pQCD}, the renormalization scheme dependence is not significant, which is why, for example, we can compare meaningfully a measured (renormalization scheme independent) Bjorken sum to the (renormalization scheme dependent) pQCD prediction.
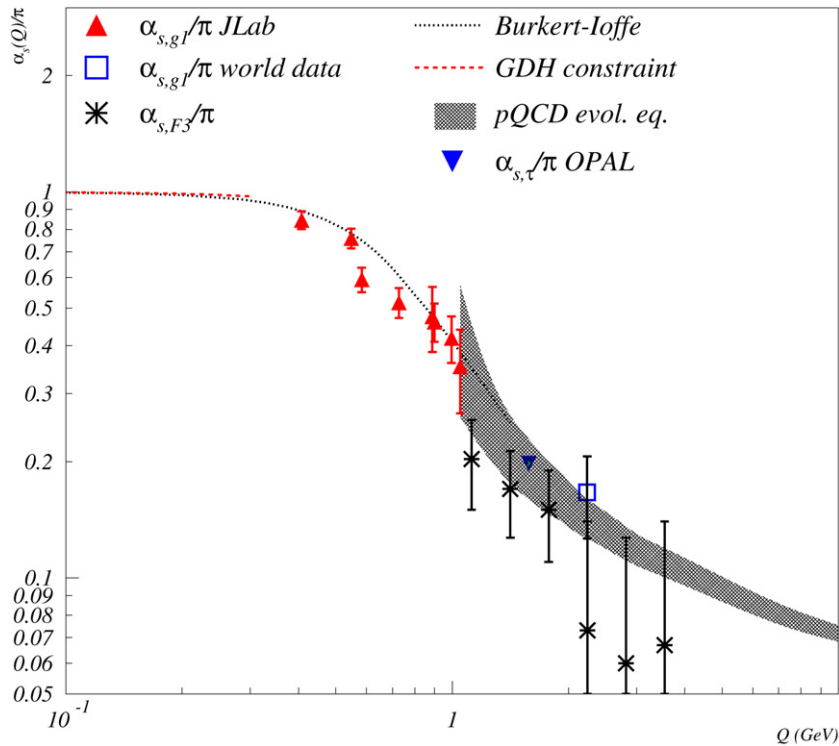
Fig. 1. (Color online.) $\alpha_s(Q)/\pi$ obtained from JLab data (up triangles), the GLS sum result from the CCFR Collaboration [24] (stars), the world Γ_1^{p-n} data (open square), the Bjorken sum rule (gray band) and the Burkert–Ioffe model. $\alpha_s(Q)/\pi$ derived using leading order commensurate scale relations and the $\alpha_{s,\tau}(Q)/\pi$ from OPAL data is given by the reversed triangle. The dashed line is the GDH constraint on the derivative of $\alpha_{s,g_1}/\pi$ at $Q^2 = 0$.

to α_s^3, up to a very small difference at order α_s^3, coming from the light-by-light contribution to the GLS sum.

In principle, the commensurate scale relations derived by Brodsky and Lu in Ref. [14] should be applied when comparing α_{s,g_1} to α_{s,F_3} (and $\alpha_{s,\tau}$ in Fig. 1). In practice, however, the resulting corrections are small; the same scale is used in the α_{s,g_1} and α_{s,F_3} comparison and the correction from the light-by-light contribution decreases the value of α_{s,F_3} by at most 1%. These corrections were neglected. The commensurate scale relations should be used when comparing $\alpha_{s,\tau}(Q)$ extracted from OPAL data on τ-decay [16] to α_{s,g_1}. At leading order (the only order available since only one Q value of $\alpha_{s,\tau}(Q)$ is available) the Q correction necessary to compare α_{s,g_1} to $\alpha_{s,\tau}$ leads to a 20% change on the effective charge. Such Q correction was applied.

In summary, assuming the validity of the GDH and Bjorken sum rules for low and large Q^2 respectively and using the JLab data at intermediate Q^2, we can evaluate α_{s,g_1} at any value of Q^2. The absence of divergence in α_{s,g_1} is obvious since it is defined from finite experimental data. It is interesting to notice that α_{s,g_1} loses its Q^2-dependence at low Q^2. This feature was suggested by the work of Brodsky et al. based on τ-decay data [16] and by other theoretical works, as will be discussed below.

Many theoretical or phenomenological studies of α_s at low-Q^2 are available. See [25,26] for reviews. The theoretical studies comprise Schwinger–Dyson Equations (SDE), lattice QCD, non-perturbative QCD vacuum, and analyticity arguments. Phenomenological studies are based on quark model spectroscopy, low-x–low-Q^2 reactions, parametrization of nucleon and pion form factors, heavy quarkonia decay, and ratio of hadron pro-

duction cross section to $\mu^+\mu^-$ production cross section in e^+e^- annihilation.

Similarly to experimental effective charges, different definitions of the strong coupling constant at low Q^2 [27] are possible in the theoretical calculations. How they are related is not fully known. Furthermore, these calculations should be viewed as indications of the behavior of α_s rather than strict predictions. Although some theoretical uncertainties due to parameterizations are shown, the existence of unknown systematic/model uncertainties should be borne in mind. However, it is interesting to compare the various calculations to our result to see whether they show common features.

In theory, solving the SDE equations can yield α_s. In practice, approximations are necessary and choices of approximation lead to different values of α_s. In the top panels and the bottom left panel of Fig. 2, we compare our data to different approaches [28–32]. The uncertainty in Cornwall's result is due to the uncertainties in parameters that enter the calculation. The uncertainty on the Bloch curve is due to Λ_{QCD}. There is a good agreement between the absolute value obtained from the present ansatz and the results of Bloch [30] and Fisher et al. [29], while the results from Maris–Tandy [31] and Bhagwat et al. [32] do not agree as well. The older calculation from Cornwall [28] disagree with the result of our ansatz. The Godfrey and Isgur curve in the top right panel of Fig. 2 represents the coupling constant used in a quark model [33].

It is interesting to notice that the Q^2-dependence of α_{s,g_1} and the ones of the calculations are similar. A relative comparison reveals that the Q^2-dependence of the Godfrey–Isgur, Cornwall
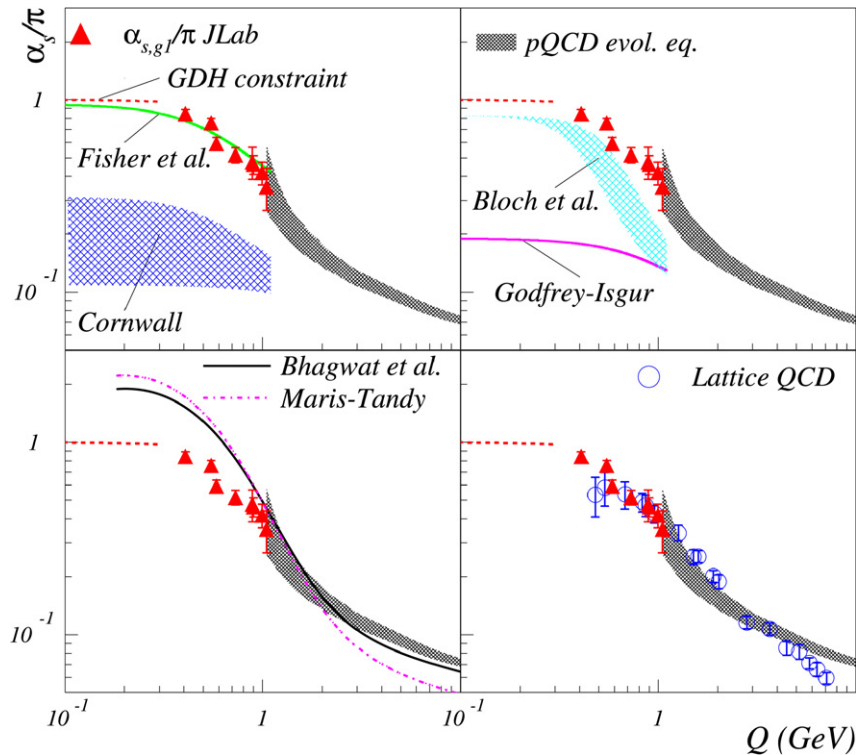
Fig. 2. α_{s,g_1} from JLab data and sum rules compared to various calculations: top left panel: SDE calculations from Fisher et al. and Cornwall; top right panel: Bloch et al. (SDE) and Godfrey–Isgur (quark model); bottom left: Maris–Tandy (SDE) and Bhagwat et al. (SDE); bottom right: Furui and Nakajima (lattice QCD).

and Fisher et al. results agree well with the data while the curves from Maris–Tandy, Bloch et al. and Bhagwat et al. are slightly below the data (by typically one sigma) for $Q^2 > 0.6$ GeV.

One ansatz that has received recent theoretical attention for the QCD coupling at low Q^2 is related to the product of the gluon propagator dressing function with the ghost propagator dressing function squared [29]. Gluon and ghost propagators have been computed in lattice QCD by many groups. The lattice results offer a fairly consistent picture [26] and agree reasonably with the various SDE propagator results [34]. It is calculated in particular in Refs. [26,35]. In the bottom right panel of Fig. 2, we compare α_{s,g_1} to the lattice result from Ref. [35]. We plot the lattice result that is believed to be closer to the continuum limit [36]. There is a good agreement between the lattice QCD calculations and our data.

To conclude, we have formed an effective strong coupling constant α_{s,g_1} at low Q^2. Data, together with sum rules, allow to obtain α_{s,g_1} at any Q^2. The connection between the Bjorken and the GDH sum rules yields a value of α_{s,g_1} equal to π at $Q^2 = 0$. An important feature of α_{s,g_1} is its loss of Q^2-dependence at low Q^2. We compared our result to other coupling constants from different reactions. They agree with each other, although they were defined from different processes. We also compared α_{s,g_1} to SDE calculations, lattice QCD calculations and a coupling constant used in a quark model. Although the relation between the various calculations is not well understood, the data and calculations agree in most cases especially when only considering Q^2-dependences. It will be interesting in the future to pursue the same analysis with lower Q^2 data that will be available both for the neutron [17] and proton [18].

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