Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 7 (2015) 420 – 440

Transportation Research Procedia

www.elsevier.com/locate/procedia

21st International Symposium on Transportation and Traffic Theory

Modeling Lane-Changing Behavior in a Connected Environment: A Game Theory Approach

Alireza Talebpour [a], Hani S. Mahmassani [a,*], Samer H. Hamdar [b]

[a] *Trasnportation Center Northwestern University, 600 Foster St., Chambers Hall, Evanston, IL, 60208*
[b] *School of Engineering and Applied Science, George Washington University, 20101 Academic Way #201-I, Ashburn, VA, 20147*

Abstract

Vehicle-to-Vehicle communications provide the opportunity to create an internet of cars through the recent advances in communication technologies, processing power, and sensing technologies. A connected vehicle receives real-time information from surrounding vehicles; such information can improve drivers' awareness about their surrounding traffic condition and lead to safer and more efficient driving maneuvers. Lane-changing behavior, as one of the most challenging driving maneuvers to understand and to predict, and a major source of congestion and collisions, can benefit from this additional information. This paper presents a lane-changing model based on a game-theoretical approach that endogenously accounts for the flow of information in a connected vehicular environment. A calibration approach based on the method of simulated moments is presented and a simplified version of the proposed framework is calibrated against NGSIM data. The prediction capability of the simplified model is validated. It is concluded the presented framework is capable of predicting lane-changing behavior with limitations that still need to be addressed. Finally, a simulation framework based on the fictitious play is proposed. The simulation results revealed that the presented lane-changing model provides a greater level of realism than a basic gap-acceptance model.

Keywords: lane-changing; game theory; simulated moments; fictitious play

1. Introduction

Game theory has been applied in different disciplines in order to understand, analyze and model decision-making processes (Petrosjan and Mazalov, 2012). In addition to its application in the domains of economics, politics and

* Corresponding author. Tel.: +1-847-491-2276; fax: +1-847-491-3090.
E-mail address: masmah@northwestern.edu

doi:10.1016/j.trpro.2015.06.022

sociology, the theory was adopted in the engineering field to explore the presence of more efficient wireless network communications (Han et al., 2012). One area of interest that bridges the gap between human decision-making and wireless communications is that of connected vehicles systems, possibly with some degree of autonomous driving. Connected vehicles systems require efficient Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication algorithms and appropriate driver responsiveness in order to avoid human errors that could lead to unsafe and congested traffic conditions. Computer scientists and engineers have focused on the security, safety and privacy of vehicular communications (Qianhong et al., 2010; Xiaodong et al., 2007) in connected environments. However, two critical questions regarding the traffic flow aspects need to be answered: (1) How to capture and specify the driving behaviors (tactical and operational) in a connected vehicle environment, and (2) How to translate such behaviors into efficient/practical V2V and V2I communication logic in that environment.

Addressing the above questions requires proper driver behavioral modeling (for which this paper proposes using game theory), efficient algorithmic implementation and accurate trajectory data based calibration. The corresponding efforts should lead to a connected vehicle framework for reducing congestion and decreasing crash rates. Particularly, V2V communications and V2I communications provide the opportunity to create a "network" of vehicles through the recent advances in communication technologies, processing power, and sensing technologies. Connected Vehicles technology enhances the effectiveness and reliability of drivers' strategic, tactical, and operational decisions. Even though the impact of this technology is significant at the strategic level (for instance, accurate real-time information about roadway conditions can improve drivers' route choice decisions), drivers' operational and tactical decisions will also be influenced, and thus constitute the focus of this paper.

Acceleration and lane-changing decisions are drivers' main operational and tactical decisions. Lane changing is considered as one of the most challenging driving maneuvers to understand and to predict, and the corresponding driving decisions are often seen as a major source of congestion and collisions. While acceleration behavior (especially car-following behavior) has been studied extensively since the 1950's and many different models with different assumptions have been proposed to capture drivers' car-following and free-flow behaviors, only few lane-changing models have been presented in the literature. Moreover, most existing lane-changing models are rule-based models (Gipps, 1986; Kesting et al., 2007) that do not take into consideration stochasticity and uncertainty. While some models adopt more realistic utility-based approaches to capture drivers' decision-making processes (Ahmed, 1999), they do not explicitly consider the dynamic interactions among drivers and cognitive decision features. Moreover, most of these models are not formulated to consider the flow of information in a connected environment. To address such shortcomings, the main objective of this paper is to develop a lane-changing model that endogenously accounts for the flow of information in a connected vehicular environment. Towards realizing this objective, a game-theoretical approach inspired by early works of Kita and his colleagues (Kita, 1999; Kita et al., 2002) is adopted. Game theory provides the foundation to capture the dynamic interactions between drivers in a lane-changing maneuver. This approach suggests two game types:

- **Game 1: two-person non-zero-sum non-cooperative game under complete information:** this type of game represents lane-changing decisions in a connected environment. Through V2V communication, we assume that drivers are certain about other drivers' decisions. In addition to the information about other drivers' decisions, V2V communication may reduce the uncertainty related to the game payoffs. Payoffs reflect the drivers' utility gain from choosing different strategies and depend on the drivers' preferences and characteristics.
- **Game 2: two-person non-zero-sum non-cooperative game under incomplete information:** This type of game represents lane-changing decisions that are made when drivers are uncertain about other drivers' decisions. Such uncertainty may lead to mandatory or discretionary lane-changes (Ahmed, 1999) depending on the drivers' willingness to take risks.

Moreover, a calibration approach based on the method of simulated moments (MSM) is presented and the modeling framework is calibrated against Next-Generation Simulation (NGSIM) data (Federal Highway Administration, 2007). MSM possesses good small sample properties and thus provides an unbiased and consistent estimator for a fixed number of simulations. Validation of the proposed approach is also presented. An efficient simulation approach based on fictitious play and learning in games is presented. The approach assumes that drivers play a repeated game until a Nash equilibrium is reached (Nash, 1951).

The remainder of the paper is organized as follows: section 2 presents a brief review of major game-theory applications in transportation engineering along with a review of some essential lane-changing models. The modeling framework is presented in section 3 and the logic behind formulating the two games mentioned earlier is discussed. A

MSM based calibration approach is offered in section 4. The corresponding section is followed by a discussion on the model validation in section 5. Finally, section 6 illustrates the fictitious play based simulation approach before concluding with some summary remarks and future research needs in section 7.

2. Background

In addition to the game theory applications in different science and engineering fields mentioned in Section 1, game theory has been adopted multiple times in transportation analysis (He et al., 2010). The main game theory models used are: (1) ordinary non-cooperative game, (2) generalized Nash equilibrium game, (3) Cournot game, (4) Stackelberg game, (5) bounded rationality game and (6) repeated games. The corresponding transportation analysis types may be mainly classified as: (a) macro-policy analysis (including i- games between travelers and authorities, ii- games between authorities, and iii- games between travelers) and (b) micro-behavior simulation (including i- games between travelers and authorities, and ii- games between travelers). These studies focused mainly on vehicular traffic and on the strategic decision making level (i.e. route choice, departure time choice, destination choice and mode choice) (Bell, 2000; Fisk, 1984; Hollander and Prashker, 2006; Tosin, 2008). As for the shorter tactical and operational decision making time frame, the studies approached the problem from a safety perspective, as the objective was to avoid conflicts and collisions between vehicles (Zhang, 2009; Zhe, 2013).

Such operational level applications motivated the use of game theory to develop improved traffic control strategies (Yu and Faldini, 2004), particularly signalization. The objective was to create decentralized and coordinated traffic signal control systems that respond in real time to the volume fluctuations in a given transportation network (El-Tantawy and Abdulhai, 2010). Other game theory applications involved ramp metering and speed harmonization (Ghods and Kian, 2008; Li and Fan, 2008). Lately, the operational and the tactical level decisions (i.e. acceleration, turning, gap-acceptance, lane-changing and merging) involved in traffic flow control started to be translated into autonomous driving logics via agent-based simulation (Rakha et al., 2013). The corresponding studies however suffer from two major limitations. First, the applicability of game theory to microscopic driving decisions modeling may create computationally slow algorithms rendering the adopted approach impractical for real-time simulation purposes (Hoogendoorn and Bovy, 2009). This limitation is even more manifested when dealing with more complex lane-changing models. Second, to examine the hypothesis that game theory may be applied to capture lane-changing behavior in a robust manner, calibration needs to be performed accurately using high resolution (space and time) trajectory data. Such data may not be readily available in different geographic locations for different roadway geometric and traffic control features. At this stage, the authors assume that such data would be available through newer generation sensing instruments. The main goal of this paper is then to show that game theory may be adopted to construct more robust lane-changing models than currently in use, and that such models may be implemented and simulated efficiently in order to be used in a connected vehicle environment.

Lane-changing decisions are latent in nature and one can just observe the outcome of this decision making process (Ahmed, 1999). The time to make this decision is not observable either. Moreover, the whole process is continuous and the order of events in this process can change (e.g., available gap, in one case, can be a triggering factor for a lane-changing decision while, in another case, a driver may look for an acceptable gap to change lane) (Ahmed, 1999). Most of the lane-changing models in the literature are rule-based models. Gipps' (1986) lane-changing model is a notable example of the rule-based models. It assumes that drivers are willing to keep their own desired speed while using a "correct lane" on their path from an origin to a destination. However, these two objectives may be conflicting; therefore several other adjustments (including the possibility of lane-changing maneuver, presence of heavy vehicles, and vehicles' speed and the target lane's speed) were considered in the model, which determines the possibility, necessity, and attractiveness of a lane-changing maneuver through a series of questions that need to be answered (i.e. if-then rules).

In an effort to capture the underlying behavioral mechanisms of lane-changing decisions, (Ahmed, 1999) presented a utility-based framework. This model divides lane-changing maneuvers into mandatory (MLC) and discretionary (DLC) and adopts a three step approach to model a lane-changing maneuver: decision to change lane, decision to select target lane, and decision to accept the gap in the target lane. This model used a discrete choice modeling framework where lane-changing decisions are made at discrete points in time. Moreover, each lane-changing decision was assumed to be independent from previous lane-changing decisions in DLCs. Choudhury et al. (Choudhury et al., 2006) enhanced this framework by incorporating forced merging in the model structure and calibrated the update framework using NGSIM data. They compared this framework with the original model (Ahmed, 1999) and found that the extended model performs significantly better than the original model.

Another lane-changing model that focused on interactions between a merging vehicle and through vehicles in an on-ramp location is Kita's model (Kita, 1999; Kita et al., 2002). The corresponding game theoretical approach captures the interactions between modeling framework is a two-player non-zero-sum non-cooperative game where the merging vehicle chooses between merge and stay in the merge lane and the through vehicle chooses between give way and do not give way. The main assumption of this model is that drivers minimize their lane-changing risk (defined based on time-to-collision).

In light of the presented literature review, further work is needed to investigate the suitability of game theory as a basis for lane-changing algorithms in a connected vehicle environment. For this purpose, adequate model formulation and efficient calibration are needed. The corresponding model should then be tested through well-designed simulation sensitivity analysis.

3. Model Formulation

3.1. Modeling Lane-Changing with Inactive V2V Communications

In the absence of communication, drivers' perception of their surrounding traffic condition is subjective. The lack of accurate information may result in inefficient, unreliable, and unsafe driving maneuvers. Drivers are uncertain about the nature of other drivers' lane-changing maneuvers. In this study, two types of lane-changing maneuvers are considered: mandatory lane-changing and discretionary lane-changing. Note that different payoffs are expected for these two types. The lane-changing behavior under inactive V2V communications is modeled as a *two-person non-zero-sum non-cooperative game under **incomplete** information*. Figure 1 shows the schematic of a typical lane-changing maneuver and Tables 1 and 2 show the structure of the discretionary and mandatory games in normal form, respectively. In both of these cases, the target vehicle has two pure strategies (Change Lane and Wait) and the lag vehicle has three pure strategies (Accelerate, Decelerate, and Change Lane). Note that in reality, the lag vehicle can have more pure strategies including Do Nothing (ignoring the lane-changing), Courtesy Yield, and Forced Yield (in case that the target vehicle executes a forced lane-change); however, due to simplicity and calibration issues (*e.g.* lack of data for calibration), this study considers a general deceleration strategy, which contains all of the above cases. In such a game, a strategy of a player is defined based on the probabilities that are assigned to each pure strategy.

Table 1. Discretionary lane-changing game with inactive V2V communication in normal form. P and R denote the payoff for target and lag vehicle, respectively.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	(P_{11}, R_{11})	(P_{12}, R_{12})
	B_2 (Decelerate)	(P_{21}, R_{21})	(P_{22}, R_{22})
	B_3 (Change Lane)	(P_{31}, R_{31})	(P_{32}, R_{32})

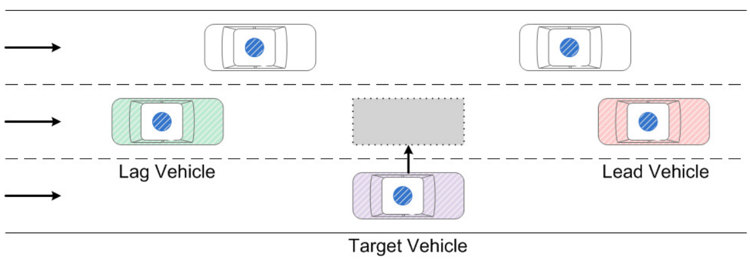

Fig. 1. The schematic of a typical lane-changing maneuver.

Table 2. Mandatory lane-changing game with inactive V2V communication in normal form. P and Q denote the payoff for target and lag vehicle, respectively.

	ACTION	Target Vehicle: A_1 (Change Lane)	Target Vehicle: A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	(P_{11}, Q_{11})	(P_{12}, Q_{12})
	B_2 (Decelerate)	(P_{21}, Q_{21})	(P_{22}, Q_{22})
	B_3 (Change Lane)	(P_{31}, Q_{31})	(P_{32}, Q_{32})

This study adopts Harsanyi transformation (Harsanyi, 1967) to transform a game of incomplete information to a game of imperfect information. This method introduces "nature" as a player who chooses the type of each player. Let p denotes the probability of the target vehicle facing mandatory lane-changing. Consequently, still assuming independence among pure strategies, $1-p$ denotes the probability of the target vehicle engaging in discretionary lane-changing. Figure 2 shows the result of the Harsanyi transformation of the game. In this transformation, from the lag vehicle's standpoint, nature moves first and chooses mandatory lane-changing with probability p and discretionary lane-changing with probability $(1-p)$. Note that both drivers have the same beliefs about these probabilities. However, the target vehicle observes nature's move, while the lag vehicle is unaware of the actual move performed by nature. Combining these two transformed games into a normal form, the target vehicle will have four pure strategies, and the lag vehicle will still have three pure strategies (see Table 3 for more details). The notation $A_i^M A_j^D$ $(i, j = 1,2)$ means the target vehicle can execute a mandatory or a discretionary lane change based on the nature's move.

3.2. Modeling Lane-Changing with Active V2V Communications

In an active V2V Communications network, vehicles receive accurate information from surrounding vehicles. This additional information enhances drivers' inaccurate perception of their surrounding traffic condition leading to safer and reliable execution of lane-changing maneuvers. This study assumes that drivers have information about the nature of the lane-changing maneuver (discretionary vs. mandatory) through V2V communications. Therefore, the lane-changing behavior under active V2V communications is modeled as a *two-person non-zero-sum non-cooperative game under **complete** information*. Table 4 shows the structure of this game in normal form.

Table 3. Lane-changing game with inactive V2V communication in normal form.

	ACTION	Target Vehicle: $A_1^M A_1^D$	Target Vehicle: $A_1^M A_2^D$	Target Vehicle: $A_2^M A_1^D$	Target Vehicle: $A_2^M A_2^D$
Lag Vehicle	B_1 (Accelerate)	$(P_{11}, pQ_{11} + (1-p)R_{11})$	$(pP_{11} + (1-p)P_{21}, pQ_{11} + (1-p)R_{21})$	$(pP_{12} + (1-p)P_{11}, pQ_{12} + (1-p)R_{11})$	$(P_{12}, pQ_{12} + (1-p)R_{12})$
	B_2 (Decelerate)	$(P_{21}, pQ_{21} + (1-p)R_{21})$	$(pP_{21} + (1-p)P_{22}, pQ_{21} + (1-p)R_{22})$	$(pP_{22} + (1-p)P_{21}, pQ_{22} + (1-p)R_{21})$	$(P_{22}, pQ_{22} + (1-p)R_{22})$
	B_3 (Change Lane)	$(P_{31}, pQ_{31} + (1-p)R_{31})$	$(pP_{31} + (1-p)P_{32}, pQ_{31} + (1-p)R_{32})$	$(pP_{32} + (1-p)P_{31}, pQ_{32} + (1-p)R_{31})$	$(P_{32}, pQ_{32} + (1-p)R_{32})$

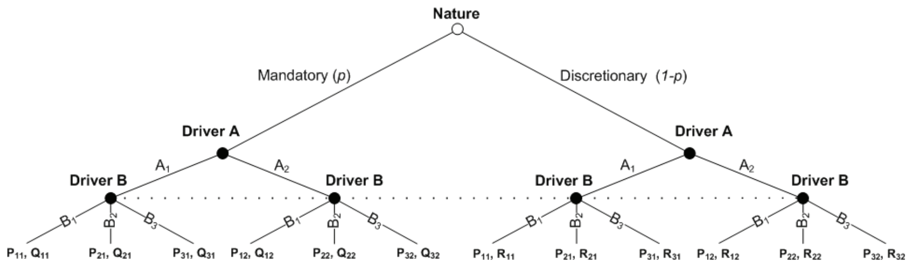

Fig. 2. Lane-changing game with inactive V2V communication in extensive form.

Table 4. Lane-changing game with active V2V communication in normal form.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	$(P_{11}, Q_{11} or R_{11})$	$(P_{12}, Q_{12} or R_{12})$
	B_2 (Decelerate)	$(P_{21}, Q_{21} or R_{21})$	$(P_{22}, Q_{22} or R_{22})$
	B_3 (Change Lane)	$(P_{31}, Q_{31} or R_{31})$	$(P_{32}, Q_{32} or R_{32})$

3.3. Graphical Representation of the Best Response

Calculating players' best responses (Nash Equilibria) has been a topic of interest among economists since the introduction of game theory. Nash equilibrium, in general, is not unique even for a simple two-player bi-matrix game; Finding the entire set of Nash equilibria and identifying players' equilibrium selection mechanisms are challenging in nature. This section presents a graphical representation of the Nash equilibria for the bi-matrix game of Table 4. Note that the game in Table 3 cannot be represented in a 3-dimentional space since the target vehicle has 4 pure strategies; however, the concept of can be transferred from a 3-dimentional space into a 4-dimentional one.

In the game illustrated in Table 4, the best response of each driver is the set of probabilities that maximize his/her payoff, considering the set of probabilities chosen by the other driver. Let q_1, q_2, and p denote the probabilities of choosing A_1, B_1, and B_2, respectively. Consequently, assuming independence among alternatives and that the probability axioms hold when modeling drivers' decisions, the probability of choosing A_2 is $(1-p)$ and the probability of choosing B_3 is $(1-q_1-q_2)$. Based on these probabilities, the expected payoff of each driver can be calculated as follows,

$$\begin{aligned} E\left(P^B\right) &= q_1\left[p\left[(P_{11}^B + P_{32}^B) - (P_{12}^B + P_{31}^B)\right] + (P_{12}^B - P_{32}^B)\right] \\ &\quad + q_2\left[p\left[(P_{21}^B + P_{32}^B) - (P_{22}^B + P_{31}^B)\right] + (P_{22}^B - P_{32}^B)\right] + \left[pP_{31}^B + (1-p)P_{32}^B\right] \end{aligned} \tag{1}$$

$$\begin{aligned} E\left(P^A\right) &= p\left[q_1\left[(P_{11}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + q_2\left[(P_{21}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + (P_{31}^A - P_{32}^A)\right] \\ &\quad + \left[q_1(P_{12}^A - P_{32}^A) + q_2(P_{22}^A - P_{32}^A) + P_{32}^A\right] \end{aligned} \tag{2}$$

The best response in this set of equations can be obtained by maximizing the expected payoff for each player:

$$Eq.1 \Rightarrow if\ p < \frac{\left[(P_{22}^B - P_{32}^B) - (P_{12}^B - P_{32}^B)\right]}{\left[(P_{11}^B + P_{32}^B) - (P_{12}^B + P_{31}^B)\right] - \left[(P_{21}^B + P_{32}^B) - (P_{22}^B + P_{31}^B)\right]} \Rightarrow q_1 = 1$$

$$Eq.1 \Rightarrow if\ p > \frac{\left[(P_{22}^B - P_{32}^B) - (P_{12}^B - P_{32}^B)\right]}{\left[(P_{11}^B + P_{32}^B) - (P_{12}^B + P_{31}^B)\right] - \left[(P_{21}^B + P_{32}^B) - (P_{22}^B + P_{31}^B)\right]} \Rightarrow q_1 = 0$$

$$Eq.1 \Rightarrow if\ p > \frac{\left[(P_{22}^B - P_{32}^B) - (P_{12}^B - P_{32}^B)\right]}{\left[(P_{11}^B + P_{32}^B) - (P_{12}^B + P_{31}^B)\right] - \left[(P_{21}^B + P_{32}^B) - (P_{22}^B + P_{31}^B)\right]} \Rightarrow q_1 + q_2 = 1$$

$$Eq.2 \Rightarrow if\left[q_1\left[(P_{11}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + q_2\left[(P_{21}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + (P_{31}^A - P_{32}^A)\right] > 0 \Rightarrow p = 1$$

$$Eq.2 \Rightarrow if\left[q_1\left[(P_{11}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + q_2\left[(P_{21}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + (P_{31}^A - P_{32}^A)\right] = 0 \Rightarrow 0 \le p \le 1$$

$$Eq.2 \Rightarrow if\left[q_1\left[(P_{11}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + q_2\left[(P_{21}^A + P_{32}^A) - (P_{31}^A + P_{12}^A)\right] + (P_{31}^A - P_{32}^A)\right] < 0 \Rightarrow p = 0$$

The best response (obtained through finding the Nash equilibrium) can be determined by finding the point of intersection of these planes in a 3D space (Kita et al., 2002). One should expect multiple intersection points (Nash equilibria) for a single problem. Figure 3 presents an illustration of these intersection points.

3.4. Pay-off Functions Formulation

Kita and his colleagues (Kita, 1999; Kita et al., 2002) formulated the pay-off functions based on drivers' tendency to avoid collisions. However, since collision risk can affect both drivers, formulating pay-offs based on only time-to-collision can lead to unrealistic Nash equiliria (Liu et al., 2007). On the other hand, they assumed that lag vehicle tries to minimize speed variation and target vehicle (merging vehicle in their case) tries to minimize the time spent in the acceleration lane while considering the collision probability. The second approach is expected to produce more realistic results since it considers parameters related to drivers' safety and comfort. However, data analysis shows that sometimes lag vehicles tries to prevent lane-changing by accelerating and closing the gap, which cannot be captures by only comfort related parameters. Moreover, both of the above studies focus on merging section and mandatory lane-changing.

This study focuses on modeling lane-changing behavior in a more general context. Therefore, the pay-offs should be designed to reflect drivers mandatory and discretionary lane-changings. For discretionary lane-changing, it is assumed that the target vehicle (lane-changing vehicle) evaluates whether changing lane is beneficial by comparing the acceleration required to avoid collision (Acc^C) and speed before and after lane-changing. The lag vehicle, however, evaluates whether to avoid interfering in the lane-changing (either by decelerating or changing lane) or to prevent it (by accelerating). Therefore, the lag vehicle compares Acc^C and speed before and after the lane-changing maneuver. Note that in all of the lane-changing instances, it is assumed that: (1) the lead and lag vehicles are in car-following mode before the lane-changing maneuver, (2) both lag and target vehicles are at an equilibrium condition at the time of lane-changing, and (3) both lag and target vehicles can accurately estimate the variables in the pay-off functions.

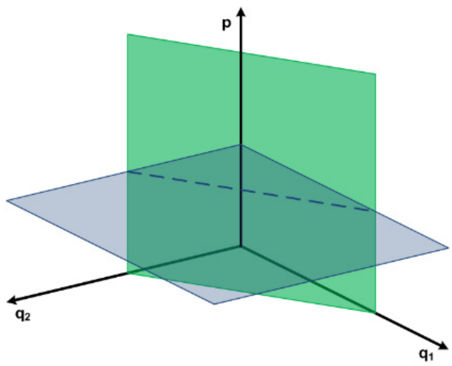

Fig. 3. Illustration of the "Best Response".

3.4.1. Target Vehicle

At the decision time, the target vehicle needs to decide about changing lane. This study considers safety and speed gain as the main decision factors for the target vehicle. Drivers have significant tendency to avoid collision; therefore, before executing the lane-changing maneuver, the target vehicle calculates two values of Acc^C: (1) for the lag vehicle considering the target vehicle as the new leader, and (2) for the target vehicle considering the lead vehicle as the new leader. In other words, target vehicle evaluates the situation from the safety stand point after the lane-changing maneuver. In addition to safety, drivers have a tendency to accelerate until they reach their desired speed. In a car-following regime, the driver's ability to reach the desired speed is limited by the leading vehicle; however, drivers prefer to be as close as possible to the desired speed. Therefore, this study assumes that the target vehicle evaluates the speed gain if he/she decides to change lane.

Table 5 summarizes the resulting pay-off functions. First consider the situation where the target vehicle decides to change lane. If the lag vehicle decides to accelerate or decelerate, the target vehicle needs to calculate Acc^C for itself and the lag vehicle. On the other hand, if the lag vehicle decides to change-lane, the target vehicle does not need to calculate Acc^C for the lag vehicle. In all of these cases, however, the speed gain should be calculated since it is a function of the new and old leaders' speed. In addition to the standard driving situations, there exist very complicated ones, which cannot be captured by Acc^C and speed gain; this study captures the unobserved decision variables by introducing an error term.

$$P_{11} = \alpha_{11}^0 + \alpha_{11}^1 Acc_{Target}^C + \alpha_{11}^2 Acc_{Lead}^C + \alpha_{11}^3 \Delta V + \varepsilon_{11} \tag{3}$$

$$P_{21} = \alpha_{21}^0 + \alpha_{21}^1 Acc_{Target}^C + \alpha_{21}^2 Acc_{Lead}^C + \alpha_{21}^3 \Delta V + \varepsilon_{21} \tag{4}$$

$$P_{31} = \alpha_{31}^0 + \alpha_{31}^1 Acc_{Lead}^C + \alpha_{31}^2 \Delta V + \varepsilon_{31} \tag{5}$$

where

Acc_{Target}^C: Acceleration to prevent collision for the lag vehicle considering the targe vehicle as the leader;

Acc_{Lead}^C: Acceleration to prevent collision for the target vehicle considering the lead vehicle as the leader;

ΔV : Speed difference between the old leader and the new leader (lead vehicle);

$\varepsilon_{11}, \varepsilon_{21}, \varepsilon_{31}$: Error terms to capture the unobserved variables;and

$\alpha_{11}^0, \alpha_{11}^1, \alpha_{11}^2, \alpha_{11}^3, \alpha_{21}^0, \alpha_{21}^1, \alpha_{21}^2, \alpha_{21}^3, \alpha_{31}^0, \alpha_{31}^1, \alpha_{31}^2$: Parameters to be estimated.

Note that it is expected that the target vehicle places different weights on Acc^C when the lag vehicle accelerates or decelerates ($\alpha_{11}^1 \neq \alpha_{21}^1$ and $\alpha_{11}^2 \neq \alpha_{21}^2$).

Alternatively, the target vehicle can select to continue without changing lane. In reality, drivers make decisions about lane-changing by comparing their current situation with their expected situation after the lane-changing maneuver. In other words, they change lane to improve their driving experience (for instance, if the current leader suddenly decelerates, changing lane is an alternative for the target vehicle to avoid collision). Therefore, the variables related to the driving condition prior to the lane-changing maneuver should be included in the pay-off functions. In this case, the target vehicle's pay-offs can be calculated based on the current driving situation at the time of the lane-changing decision.

$$P_{12} = \alpha_{12}^0 + \alpha_{12}^1 Acc^{cf} + \varepsilon_{12} \tag{6}$$

$$P_{22} = \alpha_{22}^0 + \alpha_{22}^1 Acc^{cf} + \varepsilon_{22} \tag{7}$$

$$P_{32} = \alpha_{32}^0 + \alpha_{32}^1 Acc^{cf} + \varepsilon_{32} \tag{8}$$

where Acc^{cf} : *Acceleration based on normal car-following behavior;*

$\varepsilon_{12}, \varepsilon_{22}, \varepsilon_{32}$: Error terms to capture the unobserved variables;and

$\alpha_{12}^0, \alpha_{12}^1, \alpha_{22}^0, \alpha_{22}^1, \alpha_{32}^0, \alpha_{32}^1$: Parameters to be estimated.

Table 5. Pay-off matrix of the target vehicle.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	$\alpha_{11}^{0}+\alpha_{11}^{1}Acc_{Target}^{C}+\alpha_{11}^{2}Acc_{Lead}^{C}+\alpha_{11}^{3}\Delta V+\varepsilon_{11}$	$\alpha_{12}^{0}+\alpha_{12}^{1}Acc^{G}+\varepsilon_{12}$
	B_2 (Decelerate)	$\alpha_{21}^{0}+\alpha_{21}^{1}Acc_{Target}^{C}+\alpha_{21}^{2}Acc_{Lead}^{C}+\alpha_{21}^{3}\Delta V+\varepsilon_{21}$	$\alpha_{22}^{0}+\alpha_{22}^{1}Acc^{G}+\varepsilon_{22}$
	B_3 (Change Lane)	$\alpha_{31}^{0}+\alpha_{31}^{1}Acc_{Lead}^{C}+\alpha_{31}^{2}\Delta V+\varepsilon_{31}$	$\alpha_{32}^{0}+\alpha_{32}^{1}Acc^{G}+\varepsilon_{32}$

3.4.2. Lag Vehicle

At the lane-changing time, the lag vehicle decides whether to give way (decelerate or change lane) or prevent the target vehicle from changing lane (accelerate). Table 6 summarizes the resulting pay-off functions. First consider the situation where the lag vehicle decides to accelerate. If the target vehicle decides to change lane, the lag vehicle should calculate Acc^C with the target vehicle whereas if the target vehicle decides to keep its current lane, the lag vehicle should calculate Acc^C with the lead vehicle. Similar to the pay-off functions of the target vehicle, this study specifies an error term to capture the unobserved decision variables.

$$Q_{11}\ or\ R_{11}=\beta_{11}^{0}+\beta_{11}^{1}Acc_{Target}^{C}+\delta_{11} \tag{9}$$

$$Q_{12}\ or\ R_{12}=\beta_{12}^{0}+\beta_{12}^{1}Acc_{Lead}^{C}+\delta_{12} \tag{10}$$

where δ_{11},δ_{12}: Error terms to capture the unobserved variables;and

$\beta_{11}^{0},\beta_{11}^{1},\beta_{12}^{0},\beta_{12}^{1}$: Parameters to be estimated.

Similar payoff structure is expected if the lag vehicle decides to decelerate. However, instead of calculating the acceleration to prevent collision, the lag vehicle considers a comfortable deceleration to provide a courtesy. Therefore, even if lower deceleration rate is required to prevent collision, the lag vehicle decelerates with higher deceleration rate.

$$Q_{21}\ or\ R_{21}=\beta_{21}^{0}+\beta_{21}^{1}Acc_{Target}^{Y}+\delta_{21} \tag{11}$$

$$Q_{22}\ or\ R_{22}=\beta_{22}^{0}+\beta_{22}^{1}Acc_{Lead}^{Y}+\delta_{22} \tag{12}$$

where Acc_{Target}^{Y}: $\min\{Acc_{Target}^{Y},Acc_{Target}^{C}\}$ *where* $Acc_{Target}^{Y}=-3.05\,{}^{m}\!/_{s^2}$;

Acc_{Lead}^{Y}: $\min\{Acc_{Lead}^{Y},Acc_{Lead}^{C}\}$ *where* $Acc_{Lead}^{Y}=-3.05\,{}^{m}\!/_{s^2}$;

δ_{21},δ_{22}:Error terms to capture the unobserved variables;and

$\beta_{21}^{0},\beta_{21}^{1},\beta_{22}^{0},\beta_{22}^{1}$:Parameterns to be estimated.

Finally, if the lag vehicle decides to give way by changing lane, its pay-off function is similar, in structure, to the target vehicle's pay-off function described in the previous section. Note that the pay-off does not depend on the target vehicle's decision.

$$Q_{31}=Q_{32}\ or\ R_{31}=R_{32}=\beta_{31}^{0}+\beta_{31}^{1}Acc_{Target'}^{C}+\beta_{31}^{2}Acc_{Lead'}^{C}+\beta_{31}^{3}\Delta V+\delta_{31} \tag{13}$$

where

$Acc_{Target'}^{C}$: Acceleration to prevent collision for the *new lag* vehicle considering the lag vehicle as the leader;

$Acc_{Lead'}^{C}$: Acceleration to prevent collision for the lag vehicle considering the *new lead* vehicle as the leader;

δ_{31}: Error terms to capture the unobserved variables;and

$\beta_{31}^{0},\beta_{31}^{1},\beta_{31}^{2},\beta_{31}^{3}$: Parameters to be estimated.

Note that all β_{ij}^{k} ($i, j, k = 1..3$) is different for mandatory and discretionary lane-changing and needs to be estimated separately.

Table 6. Pay-off matrix of the lag vehicle.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	$\beta_{11}^{0} + \beta_{11}^{1} Acc_{Target}^{C} + \delta_{11}$	$\beta_{12}^{0} + \beta_{12}^{1} Acc_{Lead}^{C} + \delta_{12}$
	B_2 (Decelerate)	$\beta_{21}^{0} + \beta_{21}^{1} Acc_{Target}^{I} + \delta_{21}$	$\beta_{22}^{0} + \beta_{22}^{1} Acc_{Lead}^{I} + \delta_{22}$
	B_3 (Change Lane)	$\beta_{31}^{0} + \beta_{31}^{1} Acc_{Target'}^{C} + \beta_{31}^{2} Acc_{Lead'}^{C} + \beta_{31}^{3} \Delta V + \delta_{31}$	

4. Model Calibration

Calibrating games is a challenging task due to the difficulties in finding the entire set of Nash equilibria as well as the unknown nature of the players' equilibrium selection process. Early efforts to find Nash equilibria were limited to specific games with a limited number of players and simple structures. A good example of these early works is a study by Tucker (Tucker, 1960) who proposed a linear programming approach to find a Nash equilibrium in a matrix game. However, most of these methods were not able to calculate the entire set of Nash equilibria. Since these early works, few algorithms have been proposed that are capable of finding the entire set of Nash equilibria; however, to the best of the authors' knowledge, there are only few reliable algorithms in the literature that can effectively find all Nash equilibria in a non-zero-sum game with finite number of players. Govindan and Wilson (Govindan and Wilson, 2003) proposed one of the most effective algorithms for calculating the entire set of Nash equilibria in finite games. Their approach is based on Kohlberg and Mertuens structure theorem (Elon and Mertens, 1986), which indicates the homeomorphic nature of the graph of Nash equilibrium correspondence to the space of games (the space that is determined by the number of pure strategies). In other words, there exist a continuous, invertible transform between these two spaces. This characteristic enables a homotopy method to transform the original game to a much simpler one with a unique equilibrium point. Later, this equilibrium point can be traced back to find an equilibrium point in the original game (Govindan and Wilson, 2003). Another effective method to find all Nash equilibria is called support enumeration (Dickhaut and Kaplan, 1993). This approach is also based on the homeomorphic nature of the graph of Nash equilibrium correspondence to the space of games and solves a set of polynomial equations over the set of strategies with positive selection probability.

In addition to finding the entire set of Nash equilibria, finding the underlying equilibrium selection mechanisms from observations is extremely challenging. Therefore, researchers usually assume a selection mechanism and calibrate the pay-off functions accordingly (Bajari et al., 2010). Note that for simplicity, this study assumes that players only play pure strategies. Adopting more sophisticated equilibrium selection approaches, including selecting Pareto dominated strategies, may improve the models' capability of capturing drivers' lane-changing behavior and is left for the future research.

4.1. Calibration Approach

This study adopts the method of simulated moments (MSM) presented by Bajari et al. (Bajari et al., 2010) to estimate the parameters of the pay-off functions as well as the equilibrium selection mechanism. Consider the two player game of Table 4. Let $A_i, i = 1,2$ denote the set of pure strategies for player i and $a = (a_1, a_2)$ denote a set of action for each player in this two player non-zero-sum non-cooperative game under complete information ($a \in A_1 \times A_2$). The pay-off for player i when a denotes the set of action for each player can be written as follows (Bajari et al., 2010; Bresnahan and Reiss, 1990),

$$u_i(a, x, \theta, \varepsilon) = f_i(a, x; \theta) + \varepsilon(a) \quad (14)$$

where x and θ denote the variables and their associated parameters in the pay-off function for player i, respectively. The pay-off function, u_i, consists of two terms: (1) a function f_i which represents the observable elements in the player i's pay-off function and depends on the set of strategies, and (2) an error term to capture the unobserved decision variables. Following the approach by Bajari et al. (Bajari et al., 2010), this study assumes that the error term is independent and identically distributed (i.i.d.) with a normal distribution. Note that this assumption suggests that a driver's experience does not affect his/her payoff functions. In reality, however, the error terms can be correlated since the driver's previous experience is likely to exert significant effect on the pay-off functions.

Let $u = (u_1, u_2)$ denote the set of pay-offs for both players if a is selected as the set of actions. Let $\Omega(a)$ indicate the set of Nash equilibria for this set of actions (note that there might be more than one Nash equilibrium for one set of actions). The probability of selecting $\omega \in \Omega(u)$, $\lambda(\omega, \Omega(u), \mu)$, can be calculated as follows (Bajari et al., 2010),

$$\lambda(\omega, \Omega(u), \mu) = \frac{e^{\mu.y(\omega,u)}}{\sum_{\omega' \in \Omega(u)} e^{\mu.y(\omega',u)}} \tag{15}$$

where $y(\omega,u) = \begin{cases} 1 & \textit{If } \omega \textit{ is a pure strategy Nash equilibrium} \\ 0 & \textit{Otherwise} \end{cases}$.

By combining the above elements, Bajari et al. (Bajari et al., 2010) formulated the probability of observing a specific set of actions as follows,

$$P(a \mid x, f, \mu) = \int \left[\sum_{\omega \in \Omega(u,x)} \lambda(\omega, \Omega(u), \mu) \left(\prod_{i=1}^{N} \omega(a_i) \right) \right] g(\varepsilon) d\varepsilon \tag{16}$$

where $g(\varepsilon)$ has a normal distribution. Bajari et al. (Bajari et al., 2010) proposed a method of simulated moments (MSM) to solve Eq. 16. MSM, unlike maximum likelihood estimator (MSL), generates an unbiased estimator for any number of iterations (Bajari et al., 2010). The method involves finding the entire set of Nash equilibria. This study utilizes Gambit (McKelvey et al., 2014), which is a free software package that adopted the support enumeration method to calculate the entire set of Nash equilibria.

Figure 4 illustrates the calibration process based on the method of simulated moments. This approach starts with simulating the integral of Eq. 16 using a Monte-Carlo procedure. For this purpose, first, values of ε are drawn from $g(\varepsilon)$ and pay-offs are calculated accordingly. Second, the equilibrium set, $\Omega(u)$, is calculated using Gambit. Third, based on the equilibrium set, the probability of selecting each equilibrium, $\lambda(\omega, \Omega(u), \mu)$, and the probability of observing a set of action, a, is determined. Finally, the integral can be estimated precisely by averaging over a large number of draws. Once $P(a \mid x, f, \mu)$ is simulated ($\hat{P}(a \mid x, f, \mu)$), a comparison between the calculated probabilities and the actual observations can be made. Let $g = 1(a_t = k)$ denote an indicator function ($g = 1$ if the t th set of action is equal to k). Bajari et al. (Bajari et al., 2010) defined a vector of moments similar to the following equation,

$$m_{k,T}(\theta, \mu) = \frac{1}{T} \sum_{t=1}^{T} \left[1(a_t = k) - \hat{P}(k \mid x, \theta, \mu) \right] \tag{17}$$

where T denotes the number of observations in the dataset. The combination of θ and μ that minimize the following equation forms the optimal parameter values (Bajari et al., 2010),

$$(\hat{\theta}, \hat{\mu}) = \arg\min_{\theta, \beta} \left\{ m_{k,T}(\theta, \mu)' \times m_{k,T}(\theta, \mu) \right\} \tag{18}$$

4.2. Calibration Results

The well-known NGSIM data are used for calibration of the presented lane-changing model (Federal Highway Administration, 2007) is used. This dataset was collected on the 15th of June, 2005 on US 101 in Los Angeles, California, USA. The segment length is 2100 ft and has 5 lanes in the southbound direction (see Figure 5). Trajectory data covers three 15 minutes periods during the morning peak: from 7:50 AM to 8:05 AM, from 8:05 AM to 8:20 AM, and from 8:20 AM to 8:35 AM. Figure 5 also demonstrates the speed profiles for the first two data collection periods on US 101 based on the trajectory data. This figure reveals that flow dropped significantly on this segment at the end of the first data collection period. After this point, the overall traffic flow condition remained unchanged. At the same time, the stop and go waves started to form and the frequency of these waves increased over time.

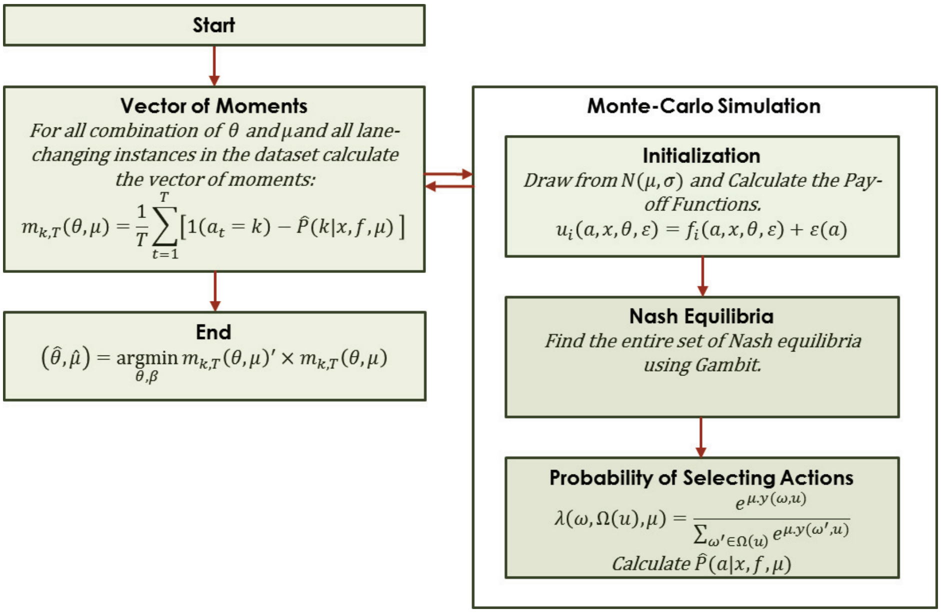

Fig. 4. Calibration process based on the method of simulated moments (Bajari et al., 2010).

To calibrate the proposed model, one needs a number of observations for each combination of pure strategies. For the case that the target vehicle decides to change lane, reasonable number of mandatory and discretionary lane-changing maneuvers are identified in this dataset. Since this segment contains an on-ramp followed by an off-ramp, a lane-changing maneuver is considered mandatory if the corresponding vehicle eventually exits the highway through the off-ramp. Any lane-changing maneuver that does not satisfy this condition, is considered to be discretionary. Note that lane-changing maneuvers to and from the ramp are not considered in the calibration.

Unlike the first case, identifying maneuvers in which drivers decide not to change lane is very challenging. This study proposes an identification based on the visual observation. In other words, the location of a vehicle is monitored throughout the segment; if the vehicle's location deviates from the middle of the lane toward the lower /upper ends of the lane and the vehicle never completely change its current lane, the maneuver is considered as an instance of deciding not to change lane. Indeed, this method can induce significant error into the calculations; however, this is the most straightforward approach considering the data availability. Note that introducing an observations error term could reduce the impact of misidentifications and is left for the future research.

The presented model, in the general form, has 58 parameters. Calibrating the entire set of parameters, in general, is not computationally practical. The following assumptions are made to simplify the model and reduce the number of parameters:

- The constant is removed from all pay-off functions. Note that this assumption put a limit on model's prediction.
- It is assumed that the target vehicle's pay-offs are zero if he/she decides not to change lane. This assumption is reasonable considering the fact that the driving environment has not changed for the target vehicle. The error term can still capture the unobserved decision variables.
- It is assumed that the target vehicle has the same pay-off regardless of the lag vehicle's decision to accelerate or decelerate. This assumption, in general, is not valid; however, in certain cases (including mandatory lane-changing) this can be the dominating behavior.
- It is assumed that the target vehicle only considers the lag vehicle in calculating his/her pay-offs.
- It is assumed that the lag vehicle is only sensitive to collision and not speed variation.
- Since the model parameters are homogenous among drivers, it is assumed that the lag vehicle's pay-off function and its parameters are similar to the target vehicle when the target vehicle decides to change lane.

Table 7 and 8 show the resulting pay-off matrix for the target and lag vehicles, respectively. Note that the decision variables are normalized based on the largest values of decision variables in the entire game. The US-101 dataset from 7:50AM to 8:05AM is used for calibration. Table 9 presents the calibration results for mandatory and discretionary lane-changings. 50 (out of 551) instances of discretionary lane-changing maneuver and 10 (out of 10) instances of mandatory lane-changing instances are used in calibration. It is assumed that drivers are aware of the lane-changing type (mandatory lane-changing in which a driver follows the advised lane-changing maneuver vs. discretionary). The Mean Absolute Error (MAE) is at the acceptable range (below 10%) for the mandatory lane-changing, while the MAE value is above 10% for the discretionary lane-changing. More discussion on these results is presented in the next section.

Table 7. Pay-off matrix of the target vehicle.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	$\eta_1.Acc^{C}_{Target} + \eta_2.\Delta V + \varepsilon_{11}$	$0 + \varepsilon_{12}$
	B_2 (Decelerate)	$\eta_1.Acc^{C}_{Target} + \eta_2.\Delta V + \varepsilon_{21}$	$0 + \varepsilon_{22}$
	B_3 (Change Lane)	$\eta_2.\Delta V + \varepsilon_{31}$	$0 + \varepsilon_{32}$

Table 8. Pay-off matrix of the lag vehicle.

	ACTION	Target Vehicle	
		A_1 (Change Lane)	A_2 (Do not Change Lane)
Lag Vehicle	B_1 (Accelerate)	$\eta_3.Acc^{C}_{Target} + \delta_{11}$	$\eta_3.Acc^{C}_{Lead} + \delta_{12}$
	B_2 (Decelerate)	$\eta_4.Acc^{\iota}_{Target} + \delta_{21}$	$\eta_4.Acc^{\iota}_{Lead} + \delta_{22}$
	B_3 (Change Lane)	$\eta_1.Acc^{C}_{Target'} + \eta_2.\Delta V + \delta_{31}$	

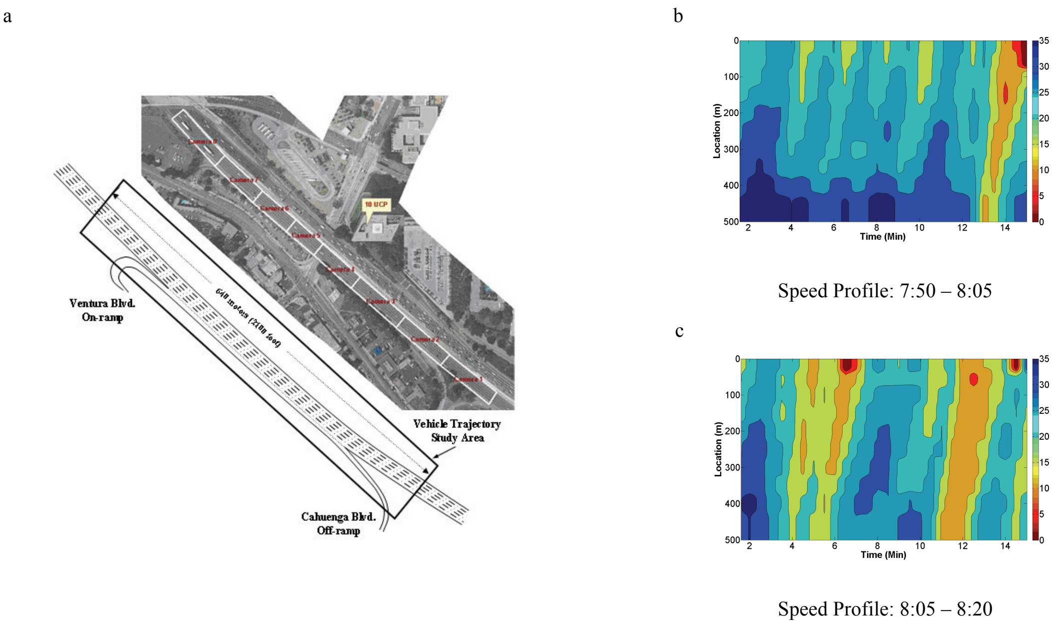

Fig. 5. (a) Schematic illustration of the study area and video coverage for trajectory data collection on US 101 (Federal Highway Administration, 2007) and (b, c) Speed (in mph) profiles during each data collection period.

Table 9. Calibration results for the discretionary and mandatory lane-changing (simplified model).

Discretionary Lane-changing		Mandatory Lane-changing	
Parameter	Calibrated Value	Parameter	Calibrated Value
η_1	-0.750	η_1	-0.875
η_2	0.875	η_2	0.375
η_3	-0.750	η_3	-0.625
η_4	0.125	η_4	0.25
μ	1.000	μ	1.000
Mean Absolute Error (MAE)	0.383	Mean Absolute Error (MAE)	0.059

5. Model Validation

This section discusses the model's capability of predicting lane-changing behavior based on the calibration results. Following the methodology presented in Liu et al. (Liu et al., 2007), in addition to the Mean Absolute Error (MAE), Root Mean Square Error (RMSE) is used to validate the proposed model

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(1(\hat{x}_i - x_i))^2} \tag{19}$$

where n, $\hat{x}$, and x denote the number of observation, model prediction, and actual observation, respectively. $1(\hat{x}_i - x_i)$ is equal to one if $\hat{x}_i = x_i$ and zero otherwise. Note that the validation is based on the US-101 dataset from 8:05AM to 8:20AM. 200 (out of 410) instances of discretionary lane-changing maneuver and 10 (out of 10) instances of mandatory lane-changing instances are used for validation. Table 9 shows the validation results for mandatory and discretionary lane-changing. MAE and RMSE values are very high for both mandatory and discretionary lane-changing. The validation results reveals that the simplified model does not carry a prediction power and is not capable of accurately predicting lane-changing behavior. However, some predicting ability still exists in the model and since the model is simplified to a great extent, the original model is expected to predict the lane-changing behavior more accurately. Further calibration and validation are required and are left for the future research.

Table 10. Validation results for the discretionary and mandatory lane-changing (simplified model).

Discretionary Lane-changing		Mandatory Lane-changing	
Mean Absolute Error (MAE)	0.724	Mean Absolute Error (MAE)	0.645
Root Mean Square Error (RMSE)	0.830	Root Mean Square Error (RMSE)	0.773

6. Simulation Framework

Acceleration modeling and lane changing modeling are core elements of the micro-simulation traffic models. The acceleration models are intended to capture the operational decision making process while the lane changing models aims at capturing the tactical driving decision making process. This paper, following the simulation model of Hamdar et al. (Hamdar et al., 2008), adopts a duration-based framework at the tactical level and part of the acceleration framework of Talebpour and Mahmassani (Talebpour and Mahmassani, 2014) at the operational level. Note that more details on the model formulation can be found in Hamdar and Mahmassani (Hamdar and Mahmassani, 2009; Hamdar, 2009).

6.1. Duration Framework

In the duration framework, the hazard-based duration models are used to capture the tactical decision making process; the driving process is divided into different episodes characterized by a termination probability - given that the episode has not ended before – and an episode duration (the time lapses before the driver enter another episode). The episodes can be divided into car-following episodes and free-flow episodes based on the corresponding inter-vehicle follower-leader interactions.

A car-following episode ends when either the distance between the vehicle and its leader increases to the point that the new episode can be considered as a free-flow episode or the vehicle changes lane (the vehicle can enter another car-following episode or free-flow episode depending on the interaction between the vehicle and the leader). The free-flow episode ends when either the vehicle changes the lane (similar to the car-following episode, the outcome can be either a free-flow episode or a car-following episode) or the distance between the vehicle and its leader decreases to the point that the new episode can be considered as car-following episode.

The hazard at time u is defined as the conditional probability of termination of the current episode at small time period δ after u *(Hamdar, 2009)*,

$$\lambda_{iq} = \lim_{\delta \to 0^+} \frac{P(u \le T_{iq} < u + \delta \mid T_{iq} < u)}{\delta} = \lambda_{0q} \phi(x_{iq}, \beta_q) \tag{20}$$

where i indicates the driver, q represents the exit strategy of an episode, and T_{iq} represents the duration of the episode for driver i and exit strategy q . λ_{0q} is called the base line hazard value at time u , x_{iq} is the vector of explanatory variables for driver i at time u (Hamdar and Mahmassani, 2009), and β_q is the vector of corresponding parameters to be estimated. Hamdar and Mahmassani (Hamdar and Mahmassani, 2009) used the exponential form for the function of exogenous covariates.

6.1.1. Lane-Changing Model

If a driver decides to end his/her current episode through changing lane, the lane-changing model is utilized to simulate the lane-changing behavior. Unlike the calibration process where it was assumed that drivers' are at equilibrium condition at the time of lane-changing decision, it is assumed that drivers seek an equilibrium condition throughout the simulation. In other words, each driver has experience about his/her previous lane-changing maneuvers and he/she starts engaging with the lag vehicles based on this experience. However, at the time of first evaluation, it is assumed that drivers are not at equilibrium. Therefore, a fictitious play approach is adopted to simulate the lane-changing behavior. Figure 6 illustrates this approach. Once a driver decides to change lane, a decision-making and execution period is considered for the driver. It is assumed that this period follows a normal distribution with the mean and standard deviation set to 2 seconds and 1 second, respectively. At each simulation step during this decision-making period, the driver in the target vehicle chooses his/her best response and watches the other driver's reaction. Once this process identify a stable solution (a Nash equilibrium), the driver executes the lane-changing maneuver.

6.2. Acceleration Framework

(Talebpour and Mahmassani, 2014) presented a framework to model driver behavior in a connected environment. In this framework, different acceleration models are used to capture the underlying dynamics of car-following behavior in this new driving environment. This section provides an overview of this acceleration framework.

6.2.1. Modeling Vehicles with No Communication Capability (Regular Vehicles)

The drivers of these vehicles have a rough perception of their surrounding traffic and their acceleration behavior is probabilistic. In general, drivers select their acceleration based on the evaluation of the potential gains and losses. Talebpour et al. (Talebpour et al., 2011) modeled this decision making process using Kahneman and Tversky's prospect theory (Kahneman and Tversky, 1979). Based on this theory, the decision maker first assigns different utilities to different alternatives considering corresponding gain and losses (framing or editing phase); and in then he/she evaluates these alternatives based on the prospect index (evaluation phase). The prospect index is calculated similar to the expected utility using subjective decision weights instead of expected probability of each outcome. Note that this model is an extension to the car-following model of Hamdar et al. (Hamdar et al., 2008). This model recognizes two different driving regimes based on drivers' different perceptions of surrounding traffic condition. Accordingly, they defined two value functions for congested and uncongested traffic regimes:

$$U_{PT}^{UC}(a_n) = \frac{\left[w_m + (1 - w_m)(\tanh(a_n) + 1\right]}{2} \left(\frac{a_n}{a + a_n^{\,2}} \right)^{\gamma} \tag{21}$$

$$U_{PT}^{C}(a_n) = \frac{\left\lfloor w_m^{'} + (1 - w_m^{'})(\tanh(a_n) + 1\right\rfloor}{2}(a_n)^{\gamma^{'}}.Sign(a_n) \tag{22}$$

where U_{PT}^{UC} and U_{PT}^{C} denote the value function for the uncongested traffic conditions and congested traffic condition, respectively. $\gamma > 0$, $\gamma^{'} > 0$, w_m, and $w_m^{'}$ are parameters to be estimated. Drivers employ the corresponding value function based on their perception of surrounding traffic condition. They captured the underlying mechanism of this regime selection using the following binary probabilistic regime selection mechanism:

$$U_{PT}(a_n) = P(C).U_{PT}^{C}(a_n) + P(UC).U_{PT}^{UC}(a_n) \tag{23}$$

where U_{PT} is the expected value function. $P(C)$ is the probabilities of driving in a congested traffic condition, and $P(UC)$ is the probability of driving in an uncongested traffic conditions (see Talebpour and Mahmassani, 2014 for more detail on calculating these probabilities). Once U_{PT} ,is calculated, total utility function of acceleration can be formulated as follows:

$$U(a_n) = (1 - p_{n,i})U_{PT}(a_n) - p_{n,i}w_c k(v, \Delta v) \tag{24}$$

where $p_{n,i}$ reflects the crash probability. $k(v, \Delta v)$ is the crash seriousness term and w_c is a weighting factor. Finally, the logistic functional form specified by Hamdar (Hamdar, 2009) was adopted to calculate the probability density function and to reflect the stochastic response adopted by the drivers:

$$f(a_n) = \begin{cases} \dfrac{e^{(\beta_{PT}.U(a_n))}}{\int_{a_{\min}}^{a_{\max}} e^{(\beta_{PT}.U(a^{'}))}} & a_{\min} \le a_n \le a_{\max} \\ 0 & Otherwise \end{cases} \tag{25}$$

where β_{PT} reflects the sensitivity of choice to the utility $U(a_n)$.

6.2.2. Modeling Vehicles with V2V Communications Capability (Connected Vehicles)

These vehicles are expected to have the capability of communicating with other vehicles in their vicinity. Considering the flow of information in a V2V communications network, drivers are certain about other drivers' behaviors. Moreover, they are aware of driving environment, road condition, and weather condition downstream of their current location. Therefore, a deterministic acceleration modeling framework is suitable for modeling this environment. They utilized Intelligent Driver Model (Kesting et al., 2010) to model this connected environment. While capturing different congestion dynamics, this model provides greater realism than most of the deterministic acceleration modeling frameworks.

IDM specifies a following vehicle's acceleration as a continuous function of the vehicle's current speed, the ratio of the current spacing to the desired spacing, and the difference between the leading and the following vehicles' velocities. Perceptive parameters such as desired acceleration, desired gap size, and comfortable deceleration are considered in this model (Kesting et al., 2010; Treiber et al., 2000):

$$a_{IDM}^{n}(s_n, v_n, \Delta v_n) = a_n \left[1 - \left(\frac{v_n}{v_0^n} \right)^{\delta_n} - \left(\frac{s^*(v_n, \Delta v_n)}{s_n} \right)^2 \right] \tag{25.a}$$

$$s^*(v_n, \Delta v_n) = s_0^n + T_n v_n + \frac{v_n.\Delta v_n}{2\sqrt{a_n b_n}} \tag{25.b}$$

Where δ_n , T_n , s_0^n , a_n , b_n , and v_0^n are parameters to be calibrated. Note that the braking term in the IDM is designed to preclude crashes in the simulation.

6.3. Simulation Results

To investigate the model's capability of simulating real-world situations, a set of simulations is conducted and a comparison between a simple gap-acceptance based lane-changing model and the MOBIL lane-changing model

(Kesting et al., 2007) with the proposed game theory based model is presented. The gap-acceptance model evaluates the lead and lag gap based on the acceleration required to avoid collision. If its value is higher than the maximum deceleration rate (-8 m/s^2), the model allows for lane-changing maneuver. The MOBIL lane-changing model, however, uses a more sophisticated approach. It combines a safety criterion for the lag vehicle with an incentive criterion, which includes the changes in the acceleration values of target vehicle, lag vehicle, and the original follower after the lane-changing maneuver. A politeness factor in this model is set to 0.2 in this study.

A four-lane highway on the eastbound direction of I–290 near Chicago, IL is simulated (see Figure 7). This 3.5-mile long segment has 4 on-ramps and 3 off-ramps each with different characteristics and different merging length. Note that the model parameters are calibrated using NGSIM trajectory data (Federal Highway Administration, 2006). This data was collected on the 13th of April, 2005 on a segment of Interstate I-80 in Emeryville, California, USA from 4:00 PM to 4:15 PM. Driver heterogeneity is considered based on the method of Kim and Mahmassani (Kim and Mahmassani, 2011). They suggested that the parameters of individual drivers in microscopic simulation models are correlated. Therefore, the parameters of each generated vehicle in simulation are the same as a particular vehicle in the NGSIM data. Unfortunately, this trajectory data does not contain any connected or autonomous vehicle; therefore, the calibrated values are adjusted according to the findings of the Talebpour and Mahmassani (2014). It should be noted that V2V communications is assumed to reduce drivers' reaction time by 50%.

Figure 8 shows the fundamental diagram for three different market penetration rates of connected vehicles (0%, 50%, and 100%). This figure reveals that at high and low market penetration rates, the proposed lane-changing model creates less scatter and higher breakdown flow compare with the gap-acceptance model. However, at 50% market penetration rate, the resulting scatter and breakdown flow is not significantly different from the gap-acceptance model. Figure 9 illustrates the speed profile from simulation for 50% market penetration rate of connected vehicles. In the simulation segment, an on-ramp with a high flow rate is located between mile markers 2.0 and 2.5. The lane-changing maneuvers at this merging section result in shockwave formation and propagation. Note that the inflow rates at other on-ramps are not significant. This figure indicates that the proposed lane-changing model creates more realistic shockwave formation at the merging sections compared to the gap-acceptance model. The gap-acceptance model results in an unrealistic congestion pattern (shockwaves are formed along the segment at all on-ramp locations). The MOBIL, however, shows better performance and a more reasonable congestion pattern compared to the gap-acceptance model. A comparison between the performance of the proposed lane-changing model and MOBIL reveals the importance of considering the flow of information in a connected environment. In general, accurate information is expected to reduce some unnecessary interactions among drivers and smooth out the traffic flow. Similar pattern can be observed in Figures 9.b and 9.c. In Figure 9.b, two major speed drop points can be identified (mile markers 1.75 and 2.25). However, in Figure 9.c, with the addition of information, only one major speed drop point can be identified (mile marker 2.25).

7. Conclusion

Connected Vehicles technology is expected to improve drivers' strategic, tactical, and operation decisions. At the operation level, this technology improves drivers' awareness about their surrounding traffic condition. Acceleration and lane-changing decisions are drivers' main operational decisions. However, unlike acceleration models, only few lane-changing models have been presented in the literature. Most of these models are rule-base models and do not explicitly take into consideration the dynamic interactions among drivers and the stochastic nature of lane-changing maneuver. Moreover, most of these models are not framed to consider the flow of information in a connected environment. Adopting a game-theoretical approach, this paper presents a lane-changing model that considers the flow of information in a connected vehicular environment. Accordingly, two game types are considered for modeling lane-changing behavior: a two-person non-zero-sum non-cooperative game under complete information in the presence of connected vehicle technology and a two-person non-zero-sum non-cooperative game under incomplete information in its absence. A calibration approach based on the method of simulated moments (Bajari et al., 2010) is adapted and presented. Since calibrating the presented framework is computationally burdensome, a simplified version of the lane-changing model is presented and calibrated; and developing a computationally efficient calibration approach has been left for future research. The validation results revealed the limited prediction capability of the simplified model.

Finally, a simulation framework based on fictitious play is proposed and a segment in Chicago, IL is simulated based on this framework. The simulation results revealed that the presented lane-changing model provides a greater level of realism than a basic gap-acceptance model and MOBIL. However, further analysis should be performed, the proposed model should be calibrated and included in the simulation framework. Moreover, considering "cognitive decision" features in this framework is essential and can be implemented through reformulating the game as an evolutionary game in which the outcome of a game depends on the drivers' experience. The final goal is to develop a robust simulation framework to capture driving behavior in a connected vehicular environment.

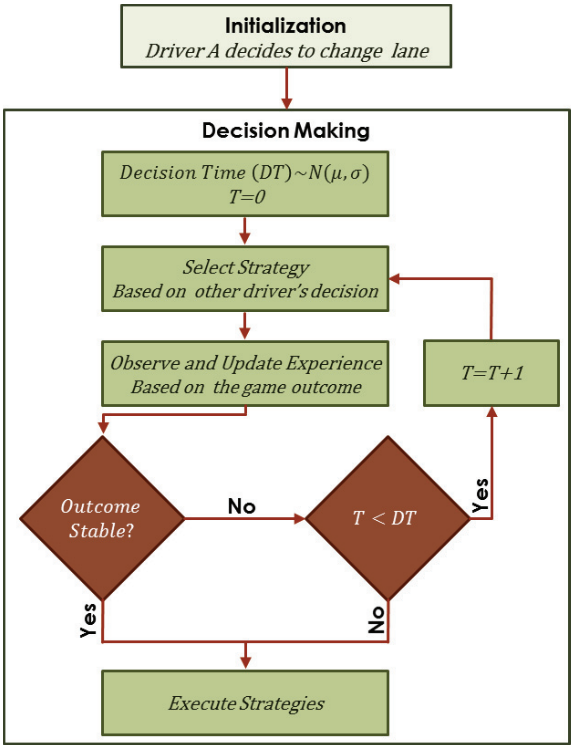

Fig. 6. Lane-changing simulation approach based on fictitious play.

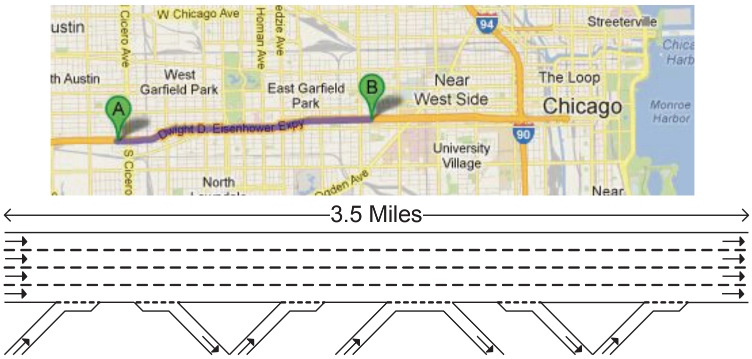

Fig. 7. Geometric characteristics of the selected segment in Chicago, IL.

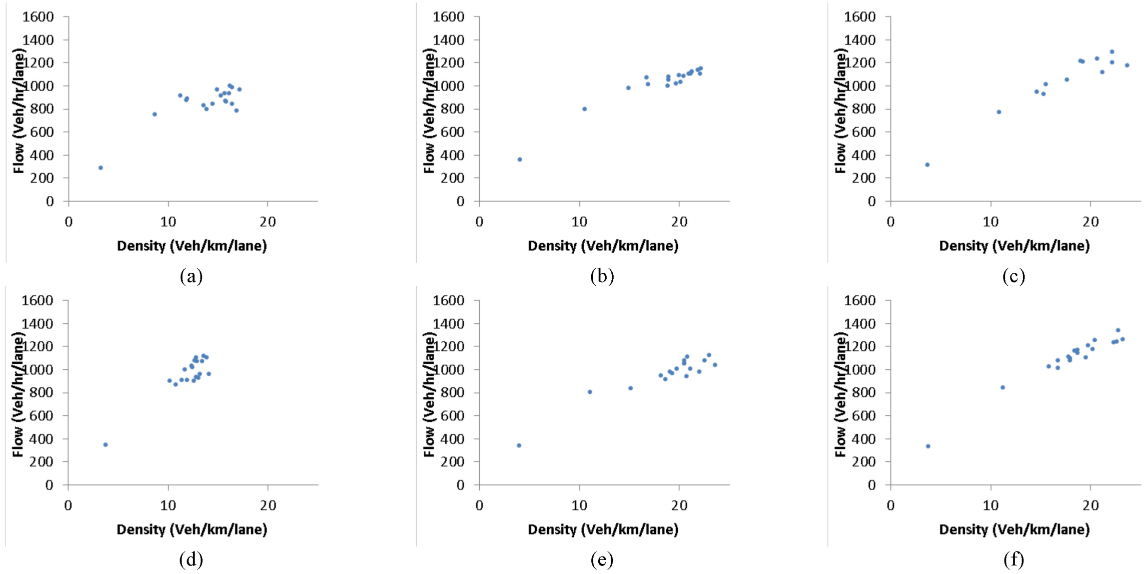

Fig. 8. Fundamental diagram for different market penetration rates of connected vehicles: (a) 0% market penetration rate with gap-acceptance based lane-changing model, (b) 50% market penetration rate with gap-acceptance based lane-changing model, (c) 100% market penetration rate with gap-acceptance based lane-changing model, (d) 0% market penetration rate with game theory based lane-changing model, (e) 50% market penetration rate with game theory based lane-changing model, and (f) 100% market penetration rate with game theory based lane-changing model.

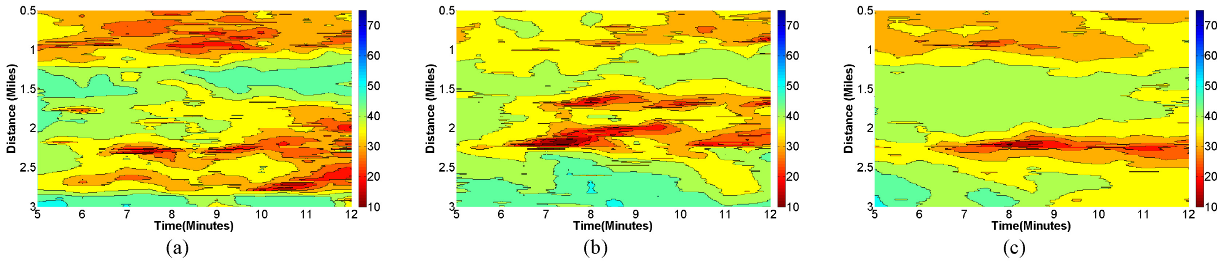

Fig. 9. Speed profile from simulation (50% market penetration rate of connected vehicles) using (a) gap-acceptance based lane-changing model, (b) MOBIL lane-changing model, and (c) game theory based lane-changing model.

References

Ahmed, K.I., 1999. Modeling drivers' acceleration and lane changing behavior, Civil and Environmental Engineering. Massachusetts Institute of Technology, Massachusetts, p. 189.

Bajari, P., Hong, H., Ryan, S.P., 2010. Identification and Estimation of a Discrete Game of Complete Information. Econometrica 78(5), 1529-1568.

Bell, M.G.H., 2000. A game theory approach to measuring the performance reliability of transport networks. Transportation Research Part B: Methodological 34(6), 533-545.

Bresnahan, T.F., Reiss, P.C., 1990. Entry in Monopoly Markets. The Review of Economic Studies 57(4), 531-553.

Choudhury, C., Ben-Akiva, M., Toledo, T., Rao, A., Lee, G., 2006. NGSIM Cooperative Lane Changing and Forced Merging Model. Federal Highway Administration, p. 167.

Dickhaut, J., Kaplan, T., 1993. A Program for Finding Nash Equilibria, in: Varian, H. (Ed.), Economic and Financial Modeling with Mathematica®. Springer New York, pp. 148-166.

El-Tantawy, S., Abdulhai, B., 2010. An agent-based learning towards decentralized and coordinated traffic signal control, Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on, pp. 665-670.

Elon, K., Mertens, J.-F., 1986. On the Strategic Stability of Equilibria. Econometrica 54(5), 1003-1037.

Federal Highway Administration, 2006. Next Generation Simulation: Interstate 80 Freeway Dataset.
Federal Highway Administration, 2007. Next Generation Simulation: US101 Freeway Dataset, in: Inc., C.S. (Ed.), Massachusetts.
Fisk, C.S., 1984. Game theory and transportation systems modelling. Transportation Research Part B: Methodological 18(4–5), 301-313.
Ghods, A.H., Kian, A., 2008. A game theory approach to optimal coordinated ramp metering and variable speed limits, Control and Decision Conference, 2008. CCDC 2008. Chinese, pp. 91-96.
Gipps, P.G., 1986. A model for the structure of lane-changing decisions. Transportation Research Part B: Methodological 20(5), 403-414.
Govindan, S., Wilson, R., 2003. A global Newton method to compute Nash equilibria. Journal of Economic Theory 110(1), 65-86.
Hamdar, S., Mahmassani, H., 2009. Life in the Fast Lane. Transportation Research Record: Journal of the Transportation Research Board 2124(-1), 89-102.
Hamdar, S., Treiber, M., Mahmassani, H., Kesting, A., 2008. Modeling Driver Behavior as Sequential Risk-Taking Task. Transportation Research Record: Journal of the Transportation Research Board 2088(-1), 208-217.
Hamdar, S.H., 2009. Modeling Driver Behavior as a Stochastic Hazard-Based Risk-Taking Process, Civil and Environmental Engineering. Northwestern University, Evanston, Illinois, p. 197.
Han, Z., Nivato, D., Saad, W., Basar, T., Hiorungnes, A., 2012. Game Theory in Wireless and Communication Networks: Theory, Models, and Applications. Cambridge University Press.
Harsanyi, J.C., 1967. Games with Incomplete Information Played by "Bayesian" Players, I-III. Part I. The Basic Model. Management Science 14(3), 159-182.
He, Z., Yuelong, S., Lihui, P., Danya, Y., 2010. A review of game theory applications in transportation analysis, Computer and Information Application (ICCIA), 2010 International Conference on, pp. 152-157.
Hollander, Y., Prashker, J., 2006. The applicability of non-cooperative game theory in transport analysis. Transportation 33(5), 481-496.
Hoogendoorn, S., Bovy, P., 2009. Generic Driving Behavior Modeling by Differential Game Theory, in: Appert-Rolland, C., Chevoir, F., Gondret, P., Lassarre, S., Lebacque, J.-P., Schreckenberg, M. (Eds.), Traffic and Granular Flow '07. Springer Berlin Heidelberg, pp. 321-331.
Kahneman, D., Tversky, A., 1979. Prospect Theory: An Analysis of Decision under Risk. Econometrica 47(2), 263-291.
Kesting, A., Treiber, M., Helbing, D., 2007. General Lane-Changing Model MOBIL for Car-Following Models. Transportation Research Record: Journal of the Transportation Research Board 1999(-1), 86-94.
Kesting, A., Treiber, M., Helbing, D., 2010. Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 368(1928), 4585-4605.
Kim, J., Mahmassani, H., 2011. Correlated Parameters in Driving Behavior Models. Transportation Research Record: Journal of the Transportation Research Board 2249(-1), 62-77.
Kita, H., 1999. A merging–giveway interaction model of cars in a merging section: a game theoretic analysis. Transportation Research Part A: Policy and Practice 33(3–4), 305-312.
Kita, H., Tanimoto, K., Fukuyama, K., 2002. A game theoretical analysis of merging-giveway interaction: a joint estimation model, Transportation and traffic theory in the 21st century : proceedings of the 15th International Symposium on Transportation and Traffic Theory, Adelaide, Australia, pp. 503-518.
Li, Z., Fan, L., 2008. Optimal Coordination of Variable Speed Control and Ramp Metering Based on Stackelberg Game, Networking, Sensing and Control, 2008. ICNSC 2008. IEEE International Conference on, pp. 666-670.
Liu, H.X., Xin, W., Adams, Z.M., Ban, J.X., 2007. A Game Theoretical Approach for Modelling Merging and Yielding Behavior at Freeway On-Ramp Sections, INTERNATIONAL SYMPOSIUM ON TRANSPORTATION AND TRAFFIC THEORY. Elsevier, London , England.
McKelvey, R.D., McLennan, A.M., Turocy, T.L., 2014. Gambit: Software Tools for Game Theory, 13.1.2 ed.
Nash, J., 1951. Non-Cooperative Games. Annals of Mathematics 54(2), 286-295.
Petrosjan, L., Mazalov, V., 2012. Game Theory and Applications. Nova Publishers.
Qianhong, W., Domingo-Ferrer, J., Gonzalez-Nicolas, U., 2010. Balanced Trustworthiness, Safety, and Privacy in Vehicle-to-Vehicle Communications. Vehicular Technology, IEEE Transactions on 59(2), 559-573.
Rakha, H., Zohdy, I.S., Kamalanathsharma, R.K., 2013. Agent-Based Game Theory Modeling for Driveless Vehicles at Intersections. Mid-Atlantic University Transportation Center.

Talebpour, A., Mahmassani, H., Hamdar, S., 2011. Multiregime Sequential Risk-Taking Model of Car-Following Behavior. Transportation Research Record: Journal of the Transportation Research Board 2260(-1), 60-66.
Talebpour, A., Mahmassani, H.S., 2014. Modeling Acceleration Behavior in a Connected Environment, Midyear Meetings and Symposium Celebrating 50 Years of Traffic Flow Theory, Portland, OR.
Tosin, A., 2008. Discrete Kinetic and Stochastic Game Theory for Vehicular Traffic: Modeling and Mathematical Problems. Plitecnico Di Torina, Massachusetts.
Treiber, M., Hennecke, A., Helbing, D., 2000. Congested traffic states in empirical observations and microscopic simulations. Physical Review E 62(2), 1805-1824.
Tucker, A.W., 1960. Solving a matrix game by linear programming. IBM J. Res. Dev. 4(5), 507-517.
Xiaodong, L., Xiaoting, S., Pin-Han, H., Xuemin, S., 2007. GSIS: A Secure and Privacy-Preserving Protocol for Vehicular Communications. Vehicular Technology, IEEE Transactions on 56(6), 3442-3456.
Yu, J., Faldini, E., 2004. A Game Theoretic Approach to Traffic Control. Research Paper in Artificial Intelligence, Katholieke Universiteit Leuven.
Zhang, Z., 2009. Analysis of the Traffic Behavior Based on Game Theory, International Conference on Transportation Engineering 2009, pp. 501-506.
Zhe, D., 2013. Emergency Modeling in Transportation via queuing and game theory. Rutgers University.