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Heat transfer effects on a viscous dissipative fluid flow past a vertical plate in the presence of induced magnetic field

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Viscous dissipation and magnetic Reynolds number

Abstract A theoretical analysis is performed to study induced magnetic field effects on free convection flow past a vertical plate. The $\bar{x}$-axis is taken vertically upwards along the plate, $\bar{y}$-axis normal to the plate into the fluid region. It is assumed that the plate is electrically non-conducting and the applied magnetic field is of uniform strength (H_0) and perpendicular to the plate. The magnetic Reynolds number of the flow is not taken to be small enough so that the induced magnetic field is taken into account. The coupled nonlinear partial differential equations are solved by Perturbation technique and the effects of various physical parameters on velocity, temperature, and induced magnetic fields are studied through graphs and tables. Variations in Skin friction and rate of heat transfer are also studied. It is observed that an increase in magnetic parameter decreases the velocity for both water and air. It is also seen that there is a fall in induced magnetic field as magnetic Prandtl number, and magnetic field parameter increase.

1. Introduction

Induced magnetic forces modify the free stream flow and this in turn, affects the external pressure gradient or the free stream velocity that is imposed on the boundary layer. From the technological point of view, MHD free-convection flows have great significance for the applications in the fields of Stellar and Planetary magnetospheres, Aeronautics, Chemical engineering, and Electronics. The effect of magnetic field on free convection flow of electrically conducting fluid past a plate studied by many investigators [1–12]. MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates was studied by Ravikumar et al. [13]. Effect of aligned Magnetic field on unsteady flow between a stretching sheet and oscillating porous plate with constant suction was studied by Reddy et al. [14]. Hydro magnetic Flow and Heat Transfer of a Heat-Generating Fluid over a Surface Embedded in a Porous Medium was considered by Chamkha

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Nomenclature

C_p	specific heat at constant pressure (J kg^{-1} K)
E_c	Eckert number (–)
g	acceleration due to gravity (m s^{-2})
G_m	mass Grashof number (–)
G_r	thermal Grashof number (–)
H	induced magnetic field (–)
H_0	uniform magnetic field (–)
H_x	induced magnetic field along x-axis (–)
J	current density (–)
M	Hartmann number (–)
P_r	Prandtl number (–)
P_{rm}	magnetic Prandtl number (–)
$\overline{T}$	temperature (K)
$\overline{T}_W$	fluid temperature at the surface (K)
$\overline{T}_\infty$	fluid temperature in the free stream (K)
u	velocity component in x-direction (m s^{-1})
U_0	dimensionless free stream velocity (m s^{-1})
v_0	suction velocity (m s^{-1})

Greek symbols

β	coefficient of volume expansion due to temperature (K^{-1})
μ_0	magnetic diffusivity (–)
v	kinematic viscosity (m^2 s^{-1})
κ	thermal conductivity (W m^{-1} K^{-1})
ρ	Density (kg m^{-3})
σ	electrical conductivity (S m^{-1})
θ	dimensionless fluid temperature (K)

Sub scripts

w	condition at the wall
∞	free stream conditions, primes denote dimensional quantities

[15]. In their study Takhar et al. [16] investigated an unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field. Chamkha and Subaie [17] considered the effects of heat generation or absorption on hydrodynamic buoyancy induced flow of a particular suspension through a vertical pipe. The study of flow through porous medium finds application in geophysics, agricultural engineering and technology. Further the free convection flow in enclosures has become increasingly important in engineering applications in recent years due to fact growth of technology, effecting cooling of electronic equations ranges from individual transistors to mainframe computers and so on. Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid was studied by Hayat et al. [18]. Makinde and Mhone [19], considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Magyari et al. [20] found analytical solution for unsteady free convection flow through porous media. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction was studied by Kim [21]. MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space was investigated by Srinivas and Muthuraj [22].

Analytical solutions to the problems of mixed convective flows, which arise in fluids due to the interaction of the force of gravity and the density difference caused by the simultaneous diffusion of thermal energy and chemical species, have been presented by many authors due to their applications in geophysics and engineering. The problems of steady and unsteady mixed convection flows were studied by many authors. Savic and Steinruck [23] studied mixed convection flow past a horizontal plate. Mixed convection over a horizontal plate: self-similar and connecting boundary layer flows were investigated by Steinruck [24]. Siddiqa1 and Hossain [25], considered mixed convection boundary layer flow over a vertical flat plate with radiative heat transfer. Analysis of fully developed opposing mixed convection between inclined parallel plates was studied by Lavine [26]. Bhattacharya et al. [27] investigated a similarity solution of mixed convective boundary layer slip flow over a vertical plate. Raju et al. [28], considered MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating. The problem of combined effects of heat absorption and MHD on convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate with variable temperature and suction, was studied by Ravikumar et al. [29].

- The above studies on convective heat transfer phenomena in different flow geometries in the presence of a magnetic field have been limited to the case when the induced magnetic field is not taken into account. This is due to the fact that the mathematical description as well as solution of such problems involves some less effort.
- Thus, the main aim of this paper is to present the fully developed viscous dissipative, magneto hydrodynamic, steady free convective heat transfer flow over an infinite vertical porous plate in the presence of induced magnetic field.
- The magnetic Reynolds number of the flow is not taken to be small enough so that the induced magnetic field cannot be neglected.

2. Mathematical formulation

The two-dimensional steady magneto-hydrodynamic mixed convective heat transfer flow of a Newtonian, electrically-conducting, viscous incompressible fluid over a porous vertical infinite plate with viscous/magnetic dissipation of energy has been considered. The $\bar{x}$-axis is taken vertically upwards along the plate, $\bar{y}$-axis normal to the plate in the fluid region. It is assumed that the plate is electrically non-conducting and the applied magnetic field is of uniform strength (H_0) and perpendicular to the plate (see Fig. 1). The magnetic Reynolds number of the flow is taken into consideration, so that the presence

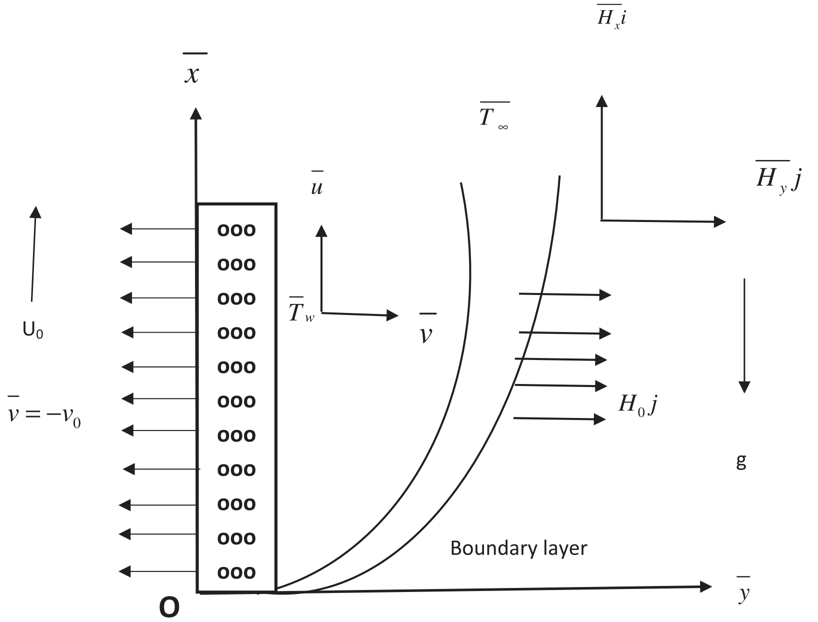

Fig. 1 Physical configuration and coordinate system.

of induced magnetic field is also considered. Let the plate be long enough in $\bar{x}$-direction for the flow to be parallel. Let ($\bar{u}$, $\bar{v}$, 0) be the fluid velocity and ($\bar{H}_x$, $\bar{H}_y$,0) be the magnetic induction vector at a point ($\bar{x}$, $\bar{y}$, $\bar{z}$) in the fluid. Since the plate is infinite in length in $\bar{x}$-direction, therefore all the physical quantities except possibly the pressure are assumed to be independent of $\bar{x}$. The wall is maintained at constant temperature $\bar{T}_W$ higher than the ambient temperature $\bar{T}_\infty$. All the gas properties are considered constant except that the influence of density variation with temperature has been considered only in the body force term. The plate is subjected to a constant suction velocity. The equation of conservation of electric charge is $\nabla \cdot J = 0$, where, $J = (J_x, J_y, J_z)$. The direction of propagation is considered only along the $\bar{y}$-axis and does not have any variation along the $\bar{y}$-axis and so $\frac{\partial J_y}{\partial y} = 0$, which gives J_y = constant (see Ahmed [6,9]).

Under above assumptions the flow is governed by the following x-momentum equation.

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}} \qquad (1)$$

The first term in RHS of Eq. (1) shows the mixed convection term. It is assumed that the velocity gradient is very small and hence the viscous term in the above equation is vanished. Therefore at the absence of induced magnetic field we may have

$$\frac{\partial p}{\partial x} = -\rho_\infty g \qquad (2)$$

Eliminating the pressure from Eqs. (1) and (2), and by using the Boussinesq approximation $\rho_\infty - \rho = \rho_\infty \beta(\bar{T} - \bar{T}_\infty)$, Eq. (1) takes the following form

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\mu_0 H_0}{\rho}\frac{\partial \bar{H}_x}{\partial \bar{y}} \qquad (3)$$

Similarly the equations of energy and magnetic induction are given below respectively

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{v}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{1}{\sigma \rho C_p}\left(\frac{\partial \bar{H}_x}{\partial \bar{y}}\right)^2 - \bar{Q}\frac{\partial}{\partial y}(\bar{T}_\infty - \bar{T}) \qquad (4)$$

$$\bar{v}\frac{\partial \bar{H}_x}{\partial \bar{y}} = \frac{1}{\sigma \mu_0}\frac{\partial^2 \bar{H}_x}{\partial \bar{y}^2} + H_0\frac{\partial \bar{u}}{\partial \bar{y}} \qquad (5)$$

The boundary conditions are:

$$\begin{aligned} &\bar{y} = 0 : \bar{u} = 0, \quad \bar{v} = -v_0, \quad \bar{T} = T_W, \quad \bar{H}_x = 0 \\ &\bar{y} \to \infty : \bar{u} \to U_0, \quad \bar{T} \to \bar{T}_\infty, \quad \bar{H}_x \to 0 \end{aligned} \qquad (6)$$

The non-dimensional quantities are:

$$\begin{aligned} &y = \frac{v_0 \bar{y}}{v}, \quad u = \frac{\bar{u}}{U_0}, \quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_W - \bar{T}_\infty}, \\ &\Pr = \frac{\rho v C_p}{\kappa}, \quad Gr = \frac{vg\beta(\bar{T}_W - \bar{T}_\infty)}{U_0 v_0^2}, \\ &H = \sqrt{\frac{\mu_0}{\rho}}\frac{\bar{H}_x}{U_0}, \quad E_c = \frac{U_0^2}{C_p(\bar{T}_W - \bar{T}_\infty)}, \\ &P_{rm} = \sigma v \mu_0, \quad M = \sqrt{\frac{\mu_0}{\rho}}\frac{\bar{H}_x}{v_0}, \quad Q = \frac{Q^*}{v_0} \end{aligned} \qquad (7)$$

Using the transformations (7), the non-dimensional governing equations in sets of Ordinary differential equations are as follows:

$$\frac{d^2 u}{dy^2} + \frac{du}{dy} + M\frac{dH}{dy} = -G_r\theta \qquad (8)$$

$$\frac{d^2\theta}{dy^2} + P_r\frac{d\theta}{dy} = -\left(E_C P_r\left(\frac{du}{dy}\right)^2 + \frac{E_C P_r}{P_{rm}}\left(\frac{dH}{dy}\right)^2\right) + Q\frac{d\theta}{dy} \qquad (9)$$

$$\frac{d^2 H}{dy^2} + MP_{rm}\frac{du}{dy} + P_{rm}\frac{dH}{dy} = 0 \qquad (10)$$

The corresponding boundary conditions are:

$$\begin{aligned} &u = 0, \quad \theta = 1, \quad H = 0 \quad \text{at} \quad y = 0 \\ &u \to 1, \quad \theta \to 0, \quad H \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \qquad (11)$$

3. Method of solution

In order to solve the Eqs. (8)–(10) under the boundary conditions (11), we note that E_C 1 for all incompressible fluids and it is assumed the solutions of the equations to be of the form,

$$u(y) = u_0(y) + E_C u_1(y) + O\left(E_C^2\right), \tag{12}$$

$$\theta(y) = \theta_0(y) + E_C\theta_1(y) + O(E_C^2), \tag{13}$$

$$H(y) = H_0(y) + E_C H_1(y) + O\left(E_C^2\right) \tag{14}$$

We now substitute Eqs. (12)–(14) into Eqs. (8)–(11) and equating the coefficients of the same degree terms and neglecting terms of $O\left(E_C^2\right)$, the following Ordinary differential equations are obtained.

$$u_0^{11} + u_0^1 = -MH_0^1 - G_r\theta_0 \tag{15}$$

$$u_1^{11} + u_1^1 = -MH_1^1 - G_r\theta_1 \tag{16}$$

$$\theta_0^{11} + (P_r - Q)\theta_0^1 = 0 \tag{17}$$

$$\theta_1^{11} + (P_r - Q)\theta_1^1 = -P_r\left(u_0^1\right)^2 - \frac{P_r}{P_{rm}}\left(H_0^1\right)^2 \tag{18}$$

$$H_0^{11} + P_{rm}H_0^1 = -MP_{rm}u_0^1 \tag{19}$$

$$H_1^{11} + P_{rm}H_1^1 = -MP_{rm}u_1^1 \tag{20}$$

The boundary conditions reduce to

$$\begin{aligned} &u_0 = 0; \quad u_1 = 0; \quad \theta_0 = 1; \quad \theta_1 = 0; \quad H_0 = 0; \\ &H_1 = 0: \quad \text{at } y = 0 \\ &u_0 = 1; \quad u_1 = 0; \quad \theta_0 = 0; \quad \theta_1 = 0; \quad H_0 = 0; \\ &H_1 = 0: \quad \text{as } y \to \infty \end{aligned} \tag{21}$$

3.1. Skin friction

The boundary layer produces a drag force on the plate due to the viscous stresses which are developed at wall and is defined by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0^1(0) + E_c u_1^1(0) \tag{22}$$

3.2. Rate of heat transfer

The co-efficient of heat transfer can be calculated in non-dimensional form at the plate, which is generally known as Nusselt number as follows:

$$Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \theta_0^1(0) + E_c\theta_1^1(0) \tag{23}$$

4. Results and discussion

In order to assess the effects of the dimensionless thermo physical parameters on the regime calculations have been carried out on velocity field, temperature field, induced magnetic field for various physical parameters like magnetic parameter, Prandtl parameter, magnetic field parameter. The results are represented through graphs in Figs. 2–4.

Fig. 2 depicts the effects of magnetic parameter on induced magnetic field. From this figure it is noticed that there is a fall in induced magnetic field as the magnetic parameter increases. A similar effect is observed from Fig. 3 in the case of magnetic Prandtl number. For all the cases of magnetic parameter and magnetic Prandtl number, induced magnetic field remains negative. This indicates that the induced magnetic flux reversal arises for all distances into the boundary layer, and transverse to the plate. It is also noticed that induced magnetic field peaks a short distance from the plate, and then decays to be zero in the free stream. In these figures, the values of magnetic Prandtl number are set to be less than a unity, which implies that the magnetic diffusion rate exceeds the viscous diffusion rate. When magnetic Prandtl number increases, the momentum diffusivity increases. The magnetic field diffuses in the medium causing a corresponding increase in the induced magnetic field magnitudes.

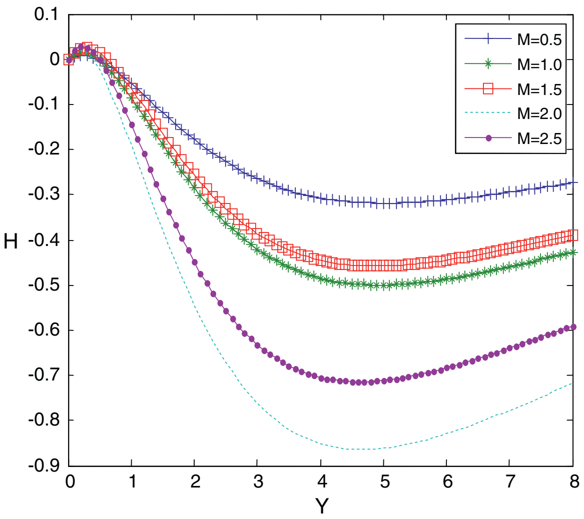

Fig. 2 Induced magnetic field profiles for different M when $P_{rm} = 0.1$; $\Pr = 0.71$; $Gr = 5$, $Q = 0.1$.

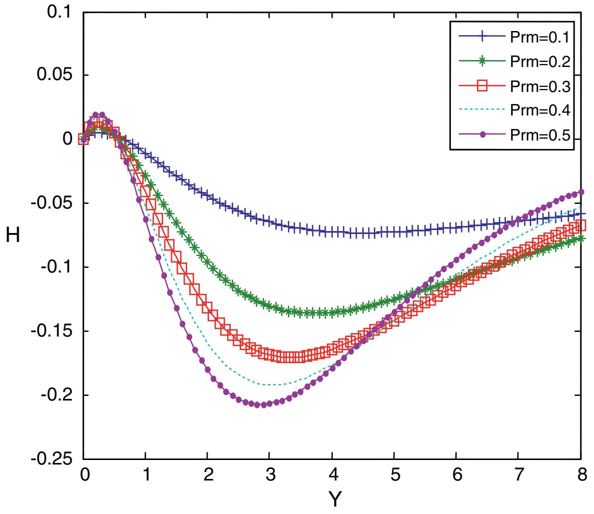

Fig. 3 Induced magnetic field profiles for different P_{rm} when $\Pr = 0.71$; $Gr = 5$; $M = 0.25$, $Q = 0.1$.

Fig. 4 shows the magnetic parameter effect on velocity, it is seen that there is fall in velocity as magnetic parameter increases. This is due to the application of transverse magnetic field, which

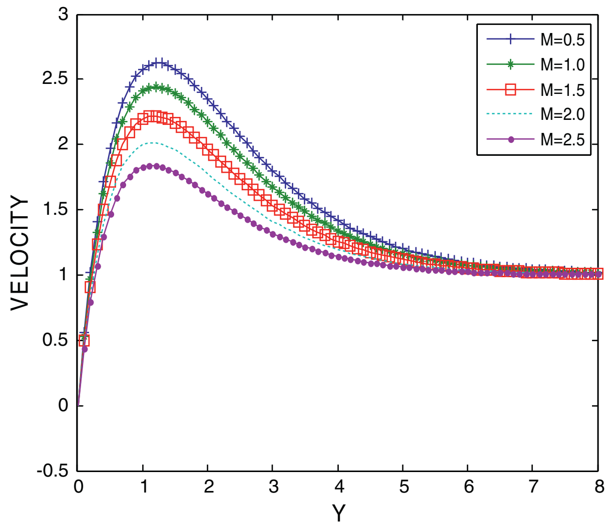

Fig.4 Velocity profiles for different M when $\Pr = 0.71$, $P_{rm} = 0.1$; $E_c = 0.001$; $Gr = 5$, $Q = 0.1$.

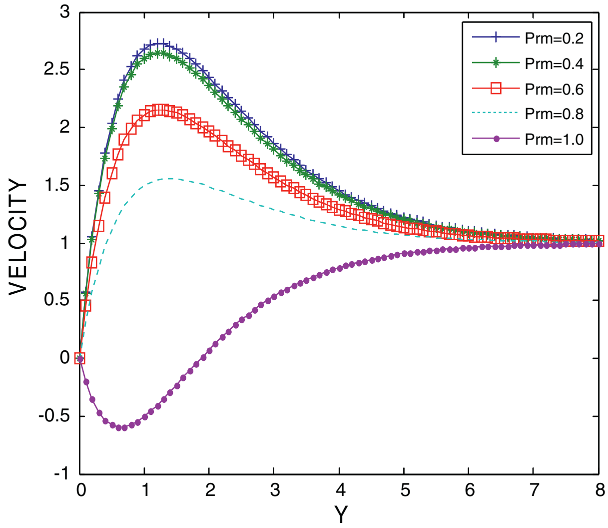

Fig. 5 Velocity profiles for different P_{rm} when $\Pr = 0.71$; $M = 0.25$; $Gr = 5$, $Q = 0.1$.

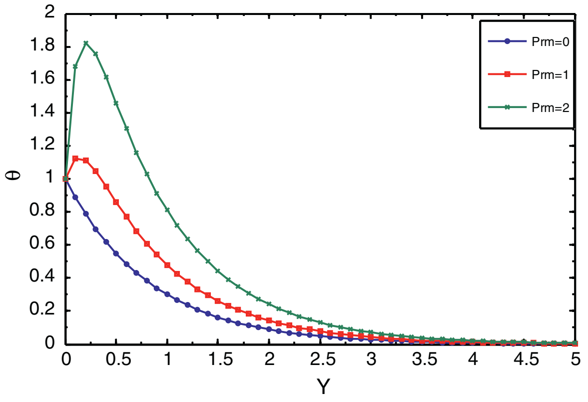

Fig. 6 Temperature profiles for different values of P_{rm}.

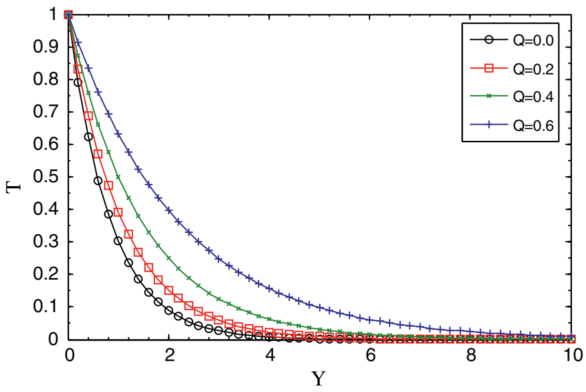

Fig. 7 Temperature profiles for different values of Q.

Table 1 Skin-friction for cooling of the plate ($Gr > 0$).

Pr	P_{rm}	E_c	M	Gr	Skin-friction
0.71	1.0	0.001	0.25	10.0	−17.9977
7.00	1.0	0.001	0.25	10.0	2.5799
0.71	2.0	0.001	0.25	10.0	4.4052
0.71	3.0	0.001	0.25	10.0	1.2976
0.71	3.0	0.005	0.25	10.0	1.0599
0.71	3.0	0.001	0.25	10.0	−1.2207
0.71	3.0	0.001	0.5	10.0	−1.6311
0.71	3.0	0.001	0.25	10.0	4.6209
0.71	3.0	0.001	0.25	20.0	1.8074

Table 2 Skin-friction for heating of the plate ($Gr < 0$).

Pr	P_{rm}	E_c	M	Gr	Skin-friction
0.71	1.0	0.001	0.25	−10.0	19.4686
7.00	1.0	0.001	0.25	−10.0	−0.05894
0.71	2.0	0.001	0.25	−10.0	−4.3102
0.71	3.0	0.001	0.25	−10.0	−4.5073
0.71	3.0	0.005	0.25	−10.0	−1.0279
0.71	3.0	0.001	0.25	−10.0	1.2240
0.71	3.0	0.001	0.5	−10.0	1.6885
0.71	3.0	0.001	0.25	−10.0	−4.5073
0.71	3.0	0.001	0.25	−20.0	−1.78059

Table 3 Nusselt number.

Pr	P_{rm}	E_c	M	Gr	Nusselt number
0.71	1.0	0.001	0.25	10.0	−0.7103
7.00	1.0	0.001	0.25	10.0	−7.1386
0.71	2.0	0.001	0.25	10.0	5.2208
0.71	3.0	0.001	0.25	10.0	7.6742
0.71	2.0	0.002	0.25	12.0	16.5323
0.71	2.0	0.001	0.25	12.0	7.85361

in turn acts as a Lorentz force that retards the flow. Fig. 5 displays magnetic Prandtl number (P_{rm}) effect on the velocity field, it is noticed that there is a fall in the velocity as magnetic Prandtl number increases. Fig. 6 depicts the effect of magnetic Prandtl number on temperature distribution. This figure reveals that

Table 4 Comparision of values of the induced magnetic field with Ahmed [6] at the absence of magnetic dissipation and Heat source, when $P_{rm} = 0.20$, $Gr = 5$.

	Results of Ahmed				Results of present study			
y	$M = 0.25$	$M = 0.50$	$M = 1$	$M = 2$	$M = 0.25$	$M = 0.50$	$M = 1$	$M = 2$
0.2	0.1596	0.04986	−0.4301	−4.4498	0.1603	0.0505	−0.4278	−4.5664
0.4	0.1636	0.0925	−2.2468	−3.4998	0.1642	0.0939	−0.2479	−3.4558
0.6	0.1311	0.0825	−0.1902	−2.7886	0.1312	0.0832	−0.1903	−2.7948
0.8	0.1135	0.0685	−0.1542	−2.2838	0.1137	0.0690	−0.1543	−2.2840
1.0	0.0925	0.0556	−0.1256	−1.8678	0.0932	0.0566	−0.1261	−1.8694

temperature increases when P_{rm} increases. In Fig. 7, variations in temperature are shown, from this figure it is noticed that temperature increases with an increase in heat source parameter. Variations in Skin friction are shown in Tables 1 and 2 for the cases of heating and cooling respectively, for different values of various physical parameters. From these tables it is noticed that, Skin friction increases with an increase in Prandtl number Pr, where as it shows reverse effect in the case of magnetic Prandtl number, Eckert number, Magnetic parameter and grashof number. In Table 3, Nusselt number is studied. From this table it is observed that Nusselt number decreases with an increase in the values of Prandtl number and Grashof number, whereas it shows opposite reaction in the case of magnetic Prandtl number and Eckert number. For the validity of our work we have compared our results with the existing results of Ahmed [6,9] at the absence of magnetic dissipation and heat source. Our results appears to be in good agreement with the existing results (see Table 4).

5. Conclusions

A theoretical analysis is performed to study induced magnetic field effects on free convection flow of dissipative fluid past a vertical plate. The coupled nonlinear partial differential equations are solved by Perturbation technique and the effects of various physical parameters like, magnetic parameter (M), Prandtl parameter (Pr), Eckert number (E_c), Grashof number, Skin friction and rate of heat transfer on velocity field, temperature field, induced magnetic field and have been presented through graphs and tables. It is observed that with an increase in magnetic field parameter (M), velocity decreases for both water ($\mathrm{Pr} = 7.0$) and air (0.71). It is seen that there is fall in induced magnetic field (H) as magnetic Prandtl number (P_{rm}), heat source parameter (Q) and magnetic field parameter (M) increases. The present study is confined to only Newtonian fluids and therefore it can be used in all steady state transport Phenomenon especially in electric power generation and distribution.

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